Estimating the Sea Surface Dynamic Topography from Geosat Altimetry Data

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Optimal interpolation method is applied to Geosat altimetry data both to remove orbit error and to separate temporal mean sea surface dynamic topography (SSDT) from temporal fluctuations around the mean. Loss of long-wavelength oceanic signals at orbit error reduction procedure is smaller in this method than that in conventional collinear methods, but the areal average height over the study domain is still removed as the orbit error. The fluctuation SSDT is quantitatively evaluated by sea level data from tide gauge stations at Japanese islands. The correlation coefficient of the two sea-level variations is 0.83 when the loss of the areal average is compensated by the seasonal variation of the areal average height determined from the climatological monthly-mean SSDT. In addition, the improvement of the geoid model by combined use of Seasat altimetry data and hydrographic data is validated through the estimated temporal mean SSDT. In a local area where hydrographic data contemporary with the Seasat mission exist, the geoid model has been significantly improved so that the absolute SSDT can be determined from combination of the altimetry data and geoid model; the absolute SSDT describes the onset event of a quasi-stationary large meander of the Kuroshio south of Japan very well. Outside this local area, however, errors of several tens of centimeters still remain in the improved geoid model.

1. Introduction

There would be no doubt that satellite altimetry is one of the most fruitful measurement techniques in physical oceanography, but some special procedures are still required for its usage. Most serious one is the removal of the geoid error. In order to determine geostrophic velocity field at the sea surface, the sea surface height (hereinafter abbreviated as SSH) observed by an altimetry system should be converted to the sea surface dynamic topography (SSDT) by removing equi-geopotential heights near the sea surface (or geoid heights) as well as sea-surface height variations (e.g. tides) whose frequencies are higher than the Coriolis parameter. The geoid models have been improved year by year, but they still have errors larger than expected oceanic signals, except for models in some local regions (e.g. Rapp and Wang, 1994) and models of very large scales (e.g. Nerem et al., 1990). A simple solution to exclude the geoid error is to limit analyses to the temporal fluctuation part of the SSDT since the geoid can be considered as steady. So-called collinear method is widely used to separate temporal fluctuations from the mean (e.g. Cheney et al., 1983), but it has several problems for accurate determination of the fluctuation part of the SSDT. One of the problems is the limitation of applicable data. Since the method is based on the assumption that SSH at a given position is repeatedly observed by an altimeter, it cannot
be applied to altimetry data which do not follow exactly repeating orbits, such as during some periods of Seasat and ERS (European Remote Sensing Satellite)-1 observations.

Another procedure required for altimetry data processing is orbit error reduction. The SSH observed by a satellite altimetry system consists of two distance determinations; distance between the satellite and the sea surface at nadir is measured by the altimeter, while height of the satellite is independently calculated. Owing to the refinement of gravitational field models and to the progress in satellite tracking systems, determination of the latter has been improved so that its error (radial orbit error) has been decreased from a few meters to several tens of centimeters, or even several centimeters for higher-altitude TOPEX/POSEIDON satellite (Koblinsky et al., 1992; Wagner and Tai, 1994). Uncertainty of several tens of centimeters, however, is still similar to or larger than the magnitude of expected oceanic signals, so that the orbit error should be removed before temporal averaging. Since the temporally variant part of this error has been known to have a spectral peak at a frequency of once per satellite revolution around the Earth (e.g. Lerch et al., 1982), it has been usually removed from SSH data as a long-wavelength component along a subsatellite track. Oceanic signals of long wavelengths, however, are also removed by this procedure.

More satellites carrying altimeters are expected to be launched in the near future, but there is no guarantee that all of them will take exactly repeating orbits nor that their orbit calculations are accurate enough. Therefore, instead of the conventional collinear method, we should construct an accurate and robust method which can consistently handle any altimetry data sets of different accuracies and data sampling patterns. Use of the optimal interpolation is one of the candidates satisfying the required conditions. The optimal interpolation produces least-squared-error linear estimates and their estimated errors statistically at arbitrary positions from noisy input data, provided that covariance functions of the signal and noise are known in advance (Bretherton et al., 1976). The method has been widely used to produce maps of the fluctuation part of SSDT from irregularly distributed along-track altimetry observations after the radial orbit errors are removed by conventional methods, but it can also be applied both to orbit error removal (Wunsch and Zlotnicki, 1984; Mazzega and Houry, 1989) and to separation of the temporal mean from fluctuations (Ichikawa and Imawaki, 1992). Although the number of data points is practically limited in this method since it requires fairly big computer to process, the method permits us to treat non-exactly-repeating altimetry data and to remove orbit errors accurately by utilizing knowledge of radial orbit errors. For example, the optimal interpolation was applied to Seasat altimetry data southeast of Japan, and both the temporal mean elevation field during the Seasat three-month period and the fluctuation part of SSDT around the mean were successfully determined (Ichikawa and Imawaki, 1992).

In addition, improvement of the present geoid model is required in order to fully utilize future altimetry data free from limitation to the temporal fluctuation part of the SSDT. Apart from increasing observations of the gravitational field (e.g. Paik et al., 1988), geoid models can be improved by the combined use of altimetry data and hydrographic data. In general, the temporal mean SSDT determined from altimetry data cannot be used because of the large error in the geoid model. This guarantees, on the contrary, that if we can obtain the true SSDT through any other method, the geoid error can be determined as the discrepancy between the true SSDT and the altimetric SSDT so that the geoid model used in the analysis can be improved (Glenn et al., 1991; Imawaki et al., 1991). Once the geoid model is improved with altimetry data by this method, it can be applied to any future altimetry data set to produce absolute SSDT with less geoid error. However, none has yet applied the geoid model improved through this method to another
Estimating SSDT from Altimetry Data

altimetry data set to produce the absolute SSDT, therefore, the extent to which this method can improve the geoid model has not been investigated.

In the present paper, we extend the optimal interpolation method used in Ichikawa and Imawaki (1992) as to treat a large number of data, and apply it to one-year long Geosat altimetry data for southeast of Japan, together with the geoid model improved by combined use of Seasat altimetry data and in situ hydrographic data (Imawaki et al., 1991). One of the objectives of the present paper is to quantitatively evaluate the optimal interpolation method; the practical performance of the method is first checked by analysis of simulated altimetry data, and then the estimated fluctuation SSDT is compared with tide gauge records at Japanese islands. The other objective is to investigate the improvement of the geoid model through combined use of Seasat altimetry data and hydrographic data; this will be carried out by evaluating the estimated temporal mean elevation field during the Geosat period from the viewpoint whether the field is depart from the expected mean SSDT because of the large geoid error. Note that the fluctuation SSDT can be used to be combined with the climatological mean SSDT to produce the composite SSDT, which describes variations of the Kuroshio and the Kuroshio Extension and rings separated from them vividly; the composite SSDT is described in a separated paper (Ichikawa and Imawaki, 1994).

The method we used is explained in Section 2; general description of the optimal interpolation method is summarized in Subsection 2.1, and its practical performances are studied in Subsection 2.2. The data used in the present analysis is described in Section 3, while the results are described in Section 4, and discussed in Section 5. Finally, concluding remarks are summarized in Section 6.

2. Method

2.1 Background

The instantaneous sea surface height $S(r, t)$ observed by an altimeter at time $t$ can be written as

$$S(r, t) = ζ(r, t) + \left\{ N(r) + ε_N \right\} + \left\{ ε_r + ε_s \right\} + ε_m (t), \quad (1)$$

where $r$ is the horizontal position vector of the observation point on the sea surface, $ζ(r, t)$ is the SSDT, $N(r)$ is the geoid height in the best available model which has an unknown error $ε_N$, $ε_r + ε_s$ are the systematic and random orbit errors, and $ε_m(t)$ is the random measurement error. Here, it is understood that $S(r, t)$ has been corrected for distance measurement errors (several path length corrections) and that high frequency fluctuations have been eliminated (e.g. tide corrections); the errors of all these corrections as well as the altimeter sensor error are included in the random measurement error $ε_m(t)$. As explained in the previous section, we need to separate the instantaneous SSDT $ζ(r, t)$ into the temporal mean SSDT $ζ(\bar{r})$ over the entire period and the deviation $ζ′(r, t)$ from the mean, or $ζ(r, t) = ζ(\bar{r}) + ζ′(r, t)$. Equation (1) is then rewritten as

$$R(r, t) = H(\bar{r}) + E(\bar{r}, t), \quad (2)$$

where
Here, the errors of random nature, $\varepsilon_r(t)$ and $\varepsilon_m(t)$, are assumed to be negligible when they are averaged. Regarding $H(r)$ as the signal and $E(r, t)$ as the noise of the instantaneous observation $R(r, t)$, the mean elevation $H(x)$ at an arbitrary position $x$ can be estimated by the optimal interpolation from the entire input data set \{R(r, t)\} (where the formula \{R\} denotes the ensemble of $R$ for all $t$). Note that the temporal fluctuation $\zeta'(r, t)$ of the SSDT from the mean is regarded here as the noise.

Practically, however, the method cannot be applied directly to a large number of input data because it needs to operate a matrix of (number of data points) $\times$ (number of data points) elements. In order to relax this computational limitation, we further divide the entire period into several subperiods since the coverage of altimetry measurements is generally repeated. The instantaneous SSDT $\zeta(x, t_q)$ at an arbitrary position $x$ and time $t_q$ which belongs to a subperiod $q$ is now separated into the mean SSDT over the given subperiod $q$, or $\bar{\zeta}(x) + \zeta'_q(x)$, and the departure $\zeta''_q(x, t_q)$ from it; here, $\zeta'_q(x)$ is equivalent to the temporal mean of the deviation SSDT $\zeta'(x, t)$ over the subperiod $q$. Then Eq. (2) at an arbitrary observation time $t_q$ in the subperiod $q$ can be rewritten as

$$R(x, t_q) = H_q(x) + E_q(x, t_q), \quad (3)$$

where

$$R(x, t_q) = S(x, t_q) - N(x),$$

$$H_q(x) = \bar{\zeta}(x) + \zeta'_q(x) + \varepsilon_N(x) + \varepsilon_s(x),$$

$$E_q(x, t_q) = \zeta''_q(x, t_q) + \varepsilon_r(t_q) + \varepsilon_m(t_q).$$

Since the duration of the subperiod $q$ is chosen to satisfy the computational limitation, we can now estimate the mean elevation $H_q(x)$ over the given subperiod $q$ by the optimal interpolation from the input data set \{R(r, t_q)\} during the subperiod $q$, regarding the deviation SSDT $\zeta''_q(r, t_q)$ from the subperiod mean $\bar{\zeta}(r) + \zeta'_q(r)$ as the noise. Note that the data coverage \{r\} during each subperiod $q$ may be similar to each other, but it does not have to be exactly the same. Denoting the duration of a subperiod $q$ as $T_q$, the mean elevation $H(x)$ over the entire period can be estimated simply by the weighted mean

$$H(x) = \frac{\Sigma_q T_q H_q(x) / \varepsilon_q(x)}{\Sigma_q T_q / \varepsilon_q(x)}, \quad (4)$$
where $\epsilon_q(x)$ is the estimated error of $H_q(x)$ which is provided by the optimal interpolation.

Using the estimated mean elevation $H(x)$ over the entire period, Eq. (2) for an arbitrary observation time $t_p$ in a subperiod $p$ can be further transformed as

$$R'(z, t_p) = H'_{p}(z) + E'_{p}(z, t_p),$$

where

$$R'(z, t_p) = S(z, t_p) - N(z) - H(z),$$

$$H'_{p}(z) = \zeta'_{p}(z),$$

$$E'_{p}(z, t_p) = \zeta''_{p}(z, t_p) + \epsilon_{r}(t_p) + \epsilon_{m}(t_p).$$

Now we can estimate the time invariant component $H'_{p}(x)$ during the subperiod $p$ using the optimal interpolation regarding the data set $\{R'(r, t_p)\}$ as the input data and $E'(r, t_p)$ as their noise. Note that the time invariant component $H'_{p}(x)$ consists only of the subperiod-mean deviation SSDT $\zeta'_{p}(x)$ from the entire-period-mean SSDT $\zeta_{p}(x)$. Provided that the duration of the subperiod $p$ is sufficiently short, the component $\zeta'_{p}(x)$ would represent the quasi-instantaneous temporal fluctuation of SSDT and hence is called hereinafter the fluctuation SSDT. One would notice that durations of the subperiods in Eq. (3) and ones in Eq. (5) are not necessarily be the same. In general, the subperiod duration $T_q$ in Eq. (3) should be long to increase spatial resolution of the mean elevation field $H_q(x)$ over the subperiod, whereas those in Eq. (5) should be short to increase temporal resolution of the fluctuation SSDT as far as the observation point distribution is not sparse. An exception is the case when the data are in an exactly repeating mission, in which the observation point distribution cannot be denser once the duration of the subperiod $T_q$ exceeds the exact repeating cycle (e.g. 17 days for Geosat).

Covariance functions of both the signal and noise must be given in advance in the optimal interpolation. In practice, however, it is too difficult to generalize these covariances since they may have complicated anisotropic and inhomogeneous characters. Therefore, in the present analysis, we decided to use simple functions to represent those covariances rather than complicated statistical functions. For the covariance function of the signal, we chose the negative squared exponential, or the Gaussian shape spatial function, which is widely used to represent aperiodic fields in the atmospheric sciences (Thiébaux and Pedder, 1987); the spatial covariance function $W(|s|)$ of the signal is given by

$$W(|s|) = w_0^2 \exp\left[-\left(|s| / L\right)^2\right].$$

where $|s|$ is the horizontal distance between two positions on the sea surface, $L$ the spatial decorrelation length scale of the field, and $w_0$ the magnitude of the signal. Note that we do not need to consider the time dependency of the covariance function since the signal is chosen to be temporally invariant both in Eqs. (3) and (5).
Use of the Gaussian shape covariance function, however, may induce two possible problems. One is that data distant from an estimation point \( x \) with respect to the decorrelation scale \( L \) are not utilized in reconstruction of the signal, no matter how strong they actually correlate with the signal at \( x \). The other is that the Gaussian shape covariance function cannot correctly handle the data with negative correlations to the signal; this would be serious especially when the data close to the estimation point with respect to \( L \) have negative correlations (Thiébaux and Pedder, 1987). Ideally, the choice of the decorrelation scale \( L \) should be a compromise of these two possible problems; \( L \) should be determined to maximize the index \( Q(x) \) of correct data utilization in reconstruction of the signal at position \( x \), which can be expressed as

\[
Q(x) = \sum_{i=1}^{N} \frac{C(s_i; x)W(|s_i|)}{C(0; x)W(0)},
\]

where \( C(s_i; x) \) is the unknown true covariance at the point \( x \), \( s_i = r_i - x \) the horizontal position vector of \( i \)-th observation point \( r_i \) with respect to \( x \), and \( N \) the number of observations. Practically, however, the best \( L \) cannot be determined because of difficulty in obtaining \( C(s_i; x) \) for all points \( x \), and hence, smaller \( L \) would be secure in order to avoid negative \( Q(x) \) at any \( x \), although we may loose large contribution from distant data points for some \( x \). Therefore, we choose as small \( L \) as possible as long as several data points can be referred in reconstructing the signal; we set \( L \) as 150 km, about 1.5 times the distance between adjacent tracks. In addition, we decide to avoid using estimated signals in data-sparse areas where the estimates strongly depend on the choice of covariance functions (Thiébaux and Pedder, 1987).

The noise covariance function \( \phi(\Delta t) \) is here given by

\[
\phi(\Delta t) = \sigma_0^2 \delta(\Delta t) + \sigma_1^2 \exp\left[-\left(\frac{\Delta t}{T_1^2}\right)^2\right] \cos\left[2\pi\left(\frac{\Delta t}{T_0}\right) / T_0\right],
\]

where the first term comes from the random measurement error \( \epsilon_m(t) \) of the altimeter as well as from the deviation SSDT \( \zeta''(r, t) \), and the second term comes from the random radial orbit error \( \epsilon(t) \) (Wunsch, 1986; Wunsch and Zlotnicki, 1984). Here \( \sigma_0 \) represents the magnitude of the measurement error at a data point, \( \delta \) the Dirac-delta function, \( \sigma_1 \) the rms (root-mean-squared) amplitude of the random radial orbit error, \( \Delta t \) the time difference between two observations, \( T_0 \) the period of the dominant component of the random radial orbit error (namely the revolution period of the satellite orbit) and \( T_1 \) the decorrelation time scale for the random radial orbit error, which is related to the band-width of the frequency peak of the dominant component of the orbit error.

Parameters of signal and noise covariance functions (6) and (8) are chosen empirically as follows. For the covariance function of the noise, \( \sigma_0 = 0.2 \) m, \( \sigma_1 = 1 \) m, \( T_0 = 6041 \) s (100.7 min), and \( T_1 = 20T_0 \) (see Lerch et al., 1982; Tapley et al., 1982; Wunsch and Zlotnicki, 1984; Haines et al., 1990). For the covariance function of the mean elevation field in Eq. (3), \( w_0 \) is 0.4 m, whereas for that of the fluctuation SSDT field in Eq. (5), \( w_0 \) is 0.2 m; Subsection 2.2 is to be referred for more detailed discussions of the choice of \( w_0 \). As noted above, we did not use estimated values when the estimated error \( \epsilon_q(x) \) exceeds 0.3 m in Eq. (3) or \( \epsilon_p(x) \) exceeds 0.16 m in Eq. (5) in order to exclude the data-sparse area.
2.2 Performance tests

In order to understand the performance of the covariance functions used in the present analysis, we made test analyses for artificial observation data and compared the estimated field with the known true field. We first produce artificial observation data from a given true field by extracting its values at simulated altimetry data points and by adding artificial noises, then the field is reconstructed by the optimal interpolation from those artificial observation data, and finally, it is evaluated by comparing with the known true field. For the convenience of following discussions, we first focus on the Gaussian shape signal covariance function (6) and the Dirac-delta noise covariance function as the first term of function (8). Then the performance of the orbit error noise covariance function as the second term of function (8) is considered.

For the test of the Gaussian shape signal covariance and the Dirac-delta noise covariance function, an artificial altimetry observation at a point \( r = (x, y) \) and time \( t \) is given by the formula

\[
\alpha_{i} \cos 2\pi \left( \frac{x}{L_{x}} + \theta_{i} \right) \cos 2\pi \left( \frac{y}{L_{y}} \right) + \theta'_{i} + \beta R(t),
\]

in which the first term represents the true field and the second the noise; \( N \) is the number of wave components, \( \alpha \) the amplitude of the \( i \)-th wave component, \( (L_{x})_{i} \) and \( (L_{y})_{i} \) the wavelengths in \( x \) and \( y \) directions, respectively, for the \( i \)-th wave component, \( \theta \) and \( \theta' \) arbitrary phase constants, \( R(t) \) the normalized random number function, and \( \beta \) the strength of the noise. Then the true field is reconstructed from the artificial data set following the same procedures used in the present paper (see Subsection 2.1 and Section 3); namely, 10 data points along tracks are averaged and the height field is estimated by the optimal interpolation using the Gaussian shape covariance function with \( L = 150 \) km for the signal covariance and the Dirac-delta function for the noise covariance. An example is shown in Fig. 1. We made several tests for different combinations of \( N, L_{x}, L_{y}, \alpha \) and \( \beta \) including \( L_{x} \) or \( L_{y} = \infty \) cases, and for various parameters \( w_{0} \) and \( \sigma_{0} \).

For variations of \( L_{x} \) and \( L_{y} \), the results are summarized in Fig. 2. The larger the scale of the true field is, the smaller is the rms difference between the true field and the reconstructed field is, namely, the reconstruction by the optimal interpolation is more accurate. When the wavelength \( L_{x} \) or \( L_{y} \) becomes less than approximately 2\( L \) (300 km), however, the field cannot be reconstructed correctly; the estimated field results in pseudo structures having wavelengths different from those of the true field. This is somewhat reasonable since the length of a packet of positive or negative values in the true field (see Fig. 1a) is shorter than the decorrelation length scale \( L \) for these cases; note that the index \( Q(x) \) in Eq. (7) would be negative for structures whose \( L_{x} \) or \( L_{y} \) is shorter than 2\( L \). Although the rms height of such deformed structures are reduced from that of the true field as indicated by crosses in Fig. 2, it is concluded that small-scale structures of strong magnitude should be excluded from the signal to be reconstructed. Note that this conclusion holds for fields composed by several waves (\( N \geq 2 \)), and therefore, it would be adopted to arbitrary shapes of the input field taking the Fourier decomposition into account. A technical suggestion can be led from these results; when intense small-scale structures are known to be included in the signal, such as mean SSDT \( \zeta(r) \) variations associated with the Kuroshio and Kuroshio Extension in Eq. (3), some pre- and post-processes are necessary to protect them to be lost or deformed through the optimal interpolation. Namely, a speculated field of those structures should be removed from input data as a “first guess” before the optimal interpolation and is then added back to the estimated field after the interpolation. In the present analysis, we used climatological mean
Fig. 1. An example of optimal interpolation performance tests. The true field superimposed on data points (after 10-points averaging) (a) and the reconstructed field (b) are shown. Note that the structure in panel (a) is almost correctly reconstructed in panel (b) with only slight distortion. Parameters in formula (9) for panel (a) are $N = 1$, $\alpha = 0.4$ m (rms amplitude of the signal is 0.2 m), $L_x = 490$ km, $L_y = 600$ km and $\beta = 0.02$ m (rms magnitude of the noise is 0.02 m). Parameters for the optimal interpolation for panel (b) are $w_0 = 0.2$ m, $L = 150$ km and $\sigma_0 = 0.02$ m. Contour and shading intervals are 0.1 m and lower values are shaded more heavily; zero level is indicated by thick contour lines and negative values are shown with dotted contour lines. Resolution of both panels is $0.25^\circ \times 0.25^\circ$. In panel (b), contours and shading are omitted at points where the estimated error exceeds 0.05 m in order to exclude unreliable estimates.

Fig. 2. The rms difference (solid line) between the true field and the reconstructed field plotted against various $L_y$ (in km) keeping the ratio $L_x/L_y = 0.82$; other parameters used are the same as Fig. 1. Rms differences for cases of $L_y$ smaller than 300 km are not plotted since their reconstructed fields are dominated by structures whose wavelengths are different from those of the true field. Also plotted are rms amplitudes of the reconstructed field (crosses). In calculations of both rms amplitude and rms difference, unreliable estimates indicated by higher estimated errors are not used.
SSDT as the first guess of the signal $H_q(x)$ in Eq. (3); therefore, the Gaussian shape covariance function is not for the mean elevation field itself, but for its deviation from the first guess.

Figure 3 summarizes the results of various combinations of $\alpha$, $\beta$, $w_0$ and $\sigma_0$. In the figure, rms differences between the true field and the reconstructed field are plotted against various combinations of parameters $w_0$ and $\sigma_0$ used in the optimal interpolation; these calculations are conducted for two input data sets with same signal strength $\alpha = 0.2$ m but different noise magnitudes of $\beta = 0.02$ m (solid line) and $\beta = 0.2$ m (dotted line). In general, as the signal-to-noise (SN) ratio given in the optimal interpolation ($w_0/\sigma_0$) decreases, the estimated field becomes smoother because of excluding noises in the observation data by smoothing. On the contrary, when too high SN ratio is provided in the optimal interpolation, the estimated field becomes very rough by inclusion of noises of random nature due to overconfidence of input data. As a result, the most accurate reconstruction of the field is expected when the ratio $w_0/\sigma_0$ is chosen close to the true SN ratio of the input data (indicated by triangles in Fig. 3). Errors in reconstruction of the field induced by invalid choices of $w_0/\sigma_0$ ratio, however, do not seem to be very severe, as far as the order of the ratio is the same as the true SN ratio. Therefore, height error would be small even when a constant $w_0/\sigma_0$ value is used over the study domain including both energetic western boundary currents and quiet interior regimes. Conversely, the $w_0/\sigma_0$ values used in the present method for the real altimetry data (Subsection 2.1) are empirically determined so that the estimates would not be significantly altered by increasing or decreasing the chosen values.

In order to study the performance of the radial orbit error removal, we made another set of test data which includes artificial orbit errors. For this series of tests, the sea surface height observation at a point $r$ and time $t$ is given by

![Graph](image)

Fig. 3. The rms difference between the true field and the reconstructed field plotted against the ratio $w_0/\sigma_0$. Solid line is for artificial input observation data with $\alpha = 0.2$ m and $\beta = 0.02$ m, whereas dotted line is for data with $\alpha = 0.2$ m and $\beta = 0.2$ m. Other parameters used are the same as Fig. 1. Signal to noise ratios of the input data (after 10 point averaging) are indicated on the abscissa by open (for solid line) and closed (for dotted line) triangles. In calculations of rms difference, unreliable estimates indicated by higher estimated errors are not used.
Table 1. Comparison of rms amplitudes (in cm) of the fields reconstructed by the present optimal interpolation method and those by the conventional collinear orbit reduction method; rms differences (in cm) between the true and reconstructed fields are also shown in brackets. Case A is for a zonally-inclined field (or $B = C = 0$ in the formula (10)), Case B for a meridionally-inclined field ($A = C = 0$), and Case C for a constant field ($A = B = 0$); the rms amplitude of the true field is fixed as 20 cm.

<table>
<thead>
<tr>
<th>Case</th>
<th>Present method</th>
<th>Conventional method</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>17.1 (2.6)</td>
<td>0.5 (19.9)</td>
</tr>
<tr>
<td>B</td>
<td>18.4 (2.1)</td>
<td>0.5 (19.9)</td>
</tr>
<tr>
<td>C</td>
<td>2.6 (18.2)</td>
<td>0.5 (20.0)</td>
</tr>
</tbody>
</table>
when input data are known to include significant small-scale oceanic signals, additional pre- and post-processings are required. Namely, rough approximation of the field (first guess) should be removed from the input data before the optimal interpolation, and then the first-guess field is added back to the estimated field after the interpolation; this is the case for the mean field.

3. Data
We used Geosat altimetry data for the area southeast of Japan \((20^\circ-45^\circ N, 120^\circ-150^\circ E)\) during the first year of the Exact Repeat Mission (ERM) (from November 8, 1986, to November 17, 1987). We did not use the data in the second and third years of ERM since the data distribution in the study area during those periods was sparse compared with that of the first year. Geosat altimetry data used in the present analysis were distributed as Geophysical Data Records (GDR) by the National Oceanic and Atmospheric Administration (NOAA) (Cheney et al., 1987). Separately distributed orbit height data were also used (Haines et al., 1990). After correcting measurement errors supplied or suggested in GDR (ionospheric free-electron, tropospheric dry-air and tropospheric water vapor, solid and ocean tides and Electro-Magnetic bias corrections) and carefully excluding extreme or doubtful data which were judged by eye, we averaged the data over 10 data points along tracks (67 km) to reduce measurement errors and small-scale fluctuations as well as the total amount of data. Here, marginal seas such as the Japan Sea, Yellow Sea, Okhotsk Sea and part of the East China Sea were excluded because ocean tide corrections are known inaccurate in those areas. An example of data point distribution in the present selected area is shown in Fig. 4 for the cycle of densest data coverage (Cycle 2); the number of the data points is 1015. Distances between adjacent tracks are approximately 100 km at these latitudes, and data points are almost uniformly distributed. Number of data points in each cycle is 774 in average with the standard deviation of 153 points.

Fig. 4. Data points distribution for Geosat ERM Cycle 2. Dotted line indicates the boundary of the study area.
The geoid model used here was originally obtained from marine gravity data (Ganeko, 1983) and improved by Seasat altimetry data combined with sea surface geopotential anomaly data (Imawaki et al., 1991). We also used climatological mean SSDT as a first guess of the mean elevation field \( H(x) \) in Eq. (3) in order to reduce loss or distortion of small scale structures by the optimal interpolation. The climatological mean SSDT was calculated from climatological mean geopotential anomalies (on \( 1^\circ \times 1^\circ \) grid) at the sea surface relative to the 1000 dbar surface estimated from hydrographic observation data compiled since 1907 by the Japan Oceanographic Data Center.

We prepared two different *in situ* observation data sets to evaluate the geoid improvement and the fluctuation SSDT. The first data set is daily averaged sea levels recorded at tide gauge stations in the Japanese archipelago provided by the Japan Meteorological Agency and Hydrographic Department of the Maritime Safety Agency, Japan. First, we determine deviations from the temporal mean sea level over the first year of the Geosat ERM for each station, and then, we average them over the same periods as the Geosat 17-day repeat cycles to produce 17-day averaged fluctuation part of sea levels which is equivalent to the altimetric fluctuation SSDT \( \zeta_p' \).

The second data set is location maps of the Kuroshio axis south of Japan, inferred both from acoustic Doppler current profiler (ADCP) surface velocities and horizontal temperature distributions in upper layers. These maps have been provided in “Prompt Report on Oceanographic Conditions” issued semimonthly by the Hydrographic Department.

4. Results

4.1 Evaluation of the fluctuation SSDT

The temporal mean elevation field \( H(x) \) relative to the improved geoid model (Imawaki et al., 1991) in Eq. (4) is estimated by the optimal interpolation method described in Section 2 from Geosat altimetry data for southeast of Japan for the first year of ERM (Fig. 5); the estimated mean elevation field \( H(x) \) is discussed in Subsection 4.2. Using this mean elevation field \( H(x) \), the fluctuation SSDT relative to the one-year mean is then estimated for 22 repeat cycles of 17-day duration. As an example, the fluctuation SSDT \( \zeta_p' \) for Cycle 2 is shown in Fig. 6 with its estimated error; the error is around 0.1 m except for the boundary where less altimetry data are available.

In order to quantitatively evaluate the fluctuation SSDT, we compared it with tide gauge records. For eight tide gauge stations south of Japan (Fig. 7), the tide gauge fluctuation SSDT (hereinafter denoted by \( \zeta_p'|_{\text{tide}} \)) for each Geosat 17-day cycle is calculated as described in Section 3. On the other hand, values of the altimetric SSDT (\( \zeta_p' \)) and its estimated error are extracted at locations of the tidal stations by spatial bilinear interpolation using values from the closest four grid points. Figure 8 shows the comparison of the fluctuation SSDT determined from the tide gauge records \( \zeta_p'|_{\text{tide}} \) (dots) and that from the altimetry data \( \zeta_p' \) (circles); basic statistics of those comparisons are summarized in Table 2. Here, values of the altimetric fluctuation SSDT are not used when any one of the four closest grid points has an estimated error larger than 0.16 m. Uncertainty index (or an error bar in the figure) is chosen as the estimated error for the altimetric fluctuation SSDT \( \zeta_p' \), whereas the standard deviation of the 17-day average is used for the tide gauge fluctuation SSDT \( \zeta_p'|_{\text{tide}} \).

In general, most of pairs of altimetric and tide gauge fluctuation SSDT data are within ranges of error bars of each other; the correlation coefficient of 0.47 for 140 comparisons is significant for \( t \)-test of 99.9% confidence level. This value is, however, smaller than those determined in similar comparisons at island tidal stations in the tropical Pacific, 0.65–0.68 (Cheney et al., 1989;
Fig. 5. Temporal mean elevation field $H(x)$ relative to the improved geoid model (Imawaki et al., 1991) estimated on a 0.5° × 0.5° grid. Contour and shading intervals are both 0.2 m, but they are gapped by 0.1 m; lower values are shaded more heavily. Inside the thick broken line, contemporary hydrographic observation data were used in the geoid model improvement.

Fig. 6. An example (Cycle 2) of the fluctuation SSDT estimated on a 0.5° × 0.5° grid (a) and its estimated error (b). Contour and shading intervals for (a) are 0.1 m and lower values are shaded more heavily; zero level is indicated by thick contour lines and negative values are shown with dotted contour lines. On the other hand, contour and shading intervals for (b) are 0.01 m, and higher values are shaded more heavily.
Fig. 7. Locations of tide gauge stations used in comparison of the altimetric and tide gauge fluctuation SSDTs in Fig. 8. They are Ishigaki (A), Naha (B), Naze (C), Nishino-omote (D), Kushimoto (E), Minami-izu (F), Hachijo-jima (G) and Chichi-jima (H).

Table 2. Statistics of the fluctuation SSDT determined from tide gauge records ($\zeta_p'$|tide) and from Geosat altimetry data ($\zeta_p'$) (Fig. 8) at each tide gauge station (Fig. 7). They include number of data used in this comparison ($n$), standard deviation of $\zeta_p'$|tide ($VT$), that of $\zeta_p'$ ($VA$), rms difference between $\zeta_p'$|tide and $\zeta_p'$ ($VD$), mean error (error bars in Fig. 8) of $\zeta_p'$|tide ($ET$), that of $\zeta_p'$ ($EA$), correlation coefficient between $\zeta_p'$|tide and $\zeta_p'$ ($r$), tilt ($a$) and bias ($b$) of the regression line: $\zeta_p' = a \times \zeta_p'$|tide $+ b$. All values are in centimeter except for nondimensional values of $n$, $r$ and $a$.

<table>
<thead>
<tr>
<th>Station</th>
<th>$n$</th>
<th>$VT$</th>
<th>$VA$</th>
<th>$VD$</th>
<th>$ET$</th>
<th>$EA$</th>
<th>$r$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19</td>
<td>11.2</td>
<td>11.6</td>
<td>10.3</td>
<td>4.4</td>
<td>10.9</td>
<td>0.58</td>
<td>1.11</td>
<td>1.6</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>10.4</td>
<td>6.8</td>
<td>6.7</td>
<td>3.7</td>
<td>11.4</td>
<td>0.79</td>
<td>0.55</td>
<td>–1.6</td>
</tr>
<tr>
<td>C</td>
<td>18</td>
<td>10.1</td>
<td>9.3</td>
<td>8.7</td>
<td>4.2</td>
<td>10.6</td>
<td>0.59</td>
<td>0.69</td>
<td>–3.4</td>
</tr>
<tr>
<td>D</td>
<td>14</td>
<td>10.4</td>
<td>11.1</td>
<td>10.3</td>
<td>5.6</td>
<td>12.7</td>
<td>0.48</td>
<td>0.81</td>
<td>–4.0</td>
</tr>
<tr>
<td>E</td>
<td>19</td>
<td>7.0</td>
<td>12.1</td>
<td>12.7</td>
<td>6.3</td>
<td>12.2</td>
<td>0.05</td>
<td>–0.07</td>
<td>–7.5</td>
</tr>
<tr>
<td>F</td>
<td>6</td>
<td>3.3</td>
<td>8.7</td>
<td>7.4</td>
<td>6.4</td>
<td>12.9</td>
<td>0.60</td>
<td>3.77</td>
<td>3.7</td>
</tr>
<tr>
<td>G</td>
<td>22</td>
<td>16.2</td>
<td>13.9</td>
<td>16.2</td>
<td>12.0</td>
<td>11.2</td>
<td>0.43</td>
<td>0.69</td>
<td>–1.6</td>
</tr>
<tr>
<td>H</td>
<td>22</td>
<td>9.1</td>
<td>7.8</td>
<td>8.5</td>
<td>4.4</td>
<td>10.7</td>
<td>0.50</td>
<td>0.74</td>
<td>–0.0</td>
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<tr>
<td>Total</td>
<td>140</td>
<td>10.8</td>
<td>10.5</td>
<td>10.9</td>
<td>5.9</td>
<td>11.4</td>
<td>0.47</td>
<td>0.88</td>
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<tr>
<td>Subtotal for A–C and H</td>
<td>79</td>
<td>10.2</td>
<td>9.0</td>
<td>8.6</td>
<td>4.2</td>
<td>10.9</td>
<td>0.59</td>
<td>0.79</td>
<td>–0.9</td>
</tr>
</tbody>
</table>

Shibata and Kitamura, 1990). One of the reasons for this lower correlation is considered that the evaluation of the present analysis includes stations where the altimetric fluctuation SSDT cannot be obtained accurately. Some stations (Stations D–F) are located on or close to large islands (Honshu and Kyushu) so that the amount of altimetry data near those "coastal" stations is small.
Fig. 8. Comparison of fluctuation SSDT’s estimated from the tide gauge records $\zeta_{p}'_{\text{tide}}$ (dots) and from the Geosat altimetry data $\zeta_{p}'$ (circles). Vertical lines are error bars. Estimates of $\zeta_{p}'$ are not shown when any of the estimated errors at the closest four grid points exceeds 0.16 m, and estimates of $\zeta_{p}'_{\text{tide}}$ are not shown when the cycle mean is estimated from less than 8-day records. Numerals in the abscissa are Geosat ERM cycle numbers, and those in the ordinate are height in cm.
compared with that of island stations in open ocean such as Stations A–C, G, and H or those in the tropical Pacific. At those coastal tidal stations, both the mean SSDT $\zeta$ and the fluctuation SSDT $\zeta_{p}'$ determined from the altimetry data are less reliable (e.g. see Fig. 6b); they can be marked by higher mean error values (12–13 cm) of the altimetric fluctuation SSDT $\zeta_{p}'$ (EA) in Table 2. Note also that stations on smaller islands are much representative of open ocean conditions due to lack of continental shelves and the associated boundary phenomena (Mitchum, 1994). Another reason for relatively poor comparisons is that time scale of dominant phenomena at some tidal stations may be too small to be resolved in the present altimetric fluctuation SSDT ($\zeta_{p}'$). This is the case especially for Station G, which is marked with a distinctively high mean error value (12 cm) of the tide gauge fluctuation SSDT $\zeta_{p}'|_{\text{tide}}$ (ET) in Table 2, indicating rapid variations of sea levels within 17 days. The combined effect of rapid sea level variations with intermittent altimetry observations can be clearly shown in the panel of Station G in Fig. 8 during Cycles 18–20. During those periods, a strong cyclonic ring moving westward is known to have been coalesced to the Kuroshio at 33°N (Cycles 18–19) and advected to the east (Cycles 19–20) (Ichikawa and Imawaki, 1994). Namely, fast-moving small-scale structure was dominated in variations of the fluctuation SSDT at Station G, which results in rapid decrease of the tide gauge fluctuation SSDT $\zeta_{p}'|_{\text{tide}}$ from Cycles 18 to 20 and increase from Cycles 20 to 22, with large error bars in Cycles 19–21. Meanwhile, most of the altimeter’s descending along-track observations were missing during these periods so that the altimetric observations close to Station G were made only along an ascending track on the 14th day of the 17-day repeating cycle. As a result, the altimetric fluctuation SSDT $\zeta_{p}'$ was strongly weighted toward the latter half of the cycle; the altimetric fluctuation SSDT $\zeta_{p}'$ (circle) for each cycle during Cycles 18–20 in Fig. 8 is much closer to the tide gauge fluctuation SSDT $\zeta_{p}'|_{\text{tide}}$ (dot) of the next cycle rather than that of the same cycle. After Cycle 21 when data from a descending track on the 8th day of the cycle became also available and when the small-scale cyclonic ring left the station completely, the error bar for the tide gauge fluctuation SSDT $\zeta_{p}'|_{\text{tide}}$ is relatively small, and the agreement between the two sea level variations is better.

The other four stations (A–C and H), whose comparisons would be as reliable as those of the island tidal stations in the tropical Pacific, show good agreements; correlation coefficient for these four stations is 0.59 (Table 2). However, the tilt ($a$) of the regression line ($\zeta_{p}' = a \times \zeta_{p}'|_{\text{tide}} + b$), 0.79, seems to be small even if the effect of spatial smoothing on the altimetric fluctuation SSDT $\zeta_{p}'$ is taken into account, suggesting the existence of some systematic discrepancies still remained. Careful readers may have recognized the tendency that circles (altimetric SSDT) in Fig. 8 at any stations are generally higher than the corresponding dots (tide gauge SSDT) in the first half of the entire period, whereas the situation is opposite in the second half. This tendency is more clearly shown in Fig. 9, a time series of differences ($\zeta_{p}'|_{\text{tide}} - \zeta_{p}'$) between a pair of the fluctuation SSDT’s determined from the tide gauge records and from the altimetry data. The figure indicates that the differences are spatially systematic, and that they oscillate with a one-year period having a peak in late summer; the seasonal variation determined from all data plotted in Fig. 9 by the harmonic analysis is shown by a solid curve in the figure. This seasonal variation of the systematic differences is considered as the temporal variation of the areal average height which is included in the tide gauge fluctuation SSDT $\zeta_{p}'|_{\text{tide}}$ but is lost in the altimetric fluctuation SSDT $\zeta_{p}'$ through the optimal interpolation method (see Subsection 2.2). In order to compensate the lost areal averages, seasonal variation of the areal average height over the present study field is calculated from climatological monthly-mean geopotential anomaly data (Teague et al., 1990); it is plotted as dotted curve in Fig. 9. Its good agreement with the solid curve both
Estimating SSDT from Altimetry Data

in amplitude and phase confirms that the loss of the areal average is the dominant cause of the seasonal variation of the systematic discrepancies. When the seasonal variation of the areal average height determined from the climatological monthly-mean SSDT (dotted curve in Fig. 9) is added back to the altimetric fluctuation SSDT $\zeta_p'$ to compensate the loss of the areal averages, correlation coefficient ($r$) for those four stations increases up to 0.83, and the tilt ($a$) and bias ($b$) of the regression curve (Fig. 10) are improved to be 1.04 and −1.1 cm, respectively; the rms difference between the tide gauge and altimetric fluctuation SSDT's ($VD$) decreases to 6.1 cm.

Except for the areal average height, oceanic signals are expected to be retained by the present method with the orbit error and the observation noise removed. In order to illustrate these results, we calculate a spatial covariance function over the domain ($C_{opt}(s)$) from the estimated fluctuation SSDT field, which includes contributions only from $\zeta_p'$; here $s$ denotes the distance lag. Then it is compared with another covariance function of the fluctuation SSDT processed through the conventional orbit error reduction method; namely, the covariance function $\hat{C}_{conv}(s)$ is calculated from input altimetry data $\mathbf{R}'(r, t_p)$ in Eq. (5) after the conventional along-track tilt-and-bias orbit error removal procedure is applied (hereinafter, a symbol $\hat{}$ indicates "with the tilt and bias removed"). Since the procedure also excludes tilt and bias of oceanic signals, contributions included in $\hat{C}_{conv}(s)$ are considered to be from oceanic signals of mid- and short-wavelength variations $\hat{\zeta}_p'(r) + \hat{\zeta}_p''(r, t_p)$ and from the random observation noise $\hat{\mathbf{E}}_m(t_p)$ in Eq. (5) assuming that the orbit error was removed somewhat correctly by the conventional method. Both covariance functions are calculated for Cycle 2 when the areal average height is expected to be negligible (Fig. 9); for the calculation of $C_{opt}(s)$, values of the fluctuation SSDT field are extracted at the same data points as $\hat{C}_{conv}(s)$ (Fig. 4). No data are used, however, along short (less than 20 data points) subsatellite tracks; the total number of data used to calculate the covariance is 777. Those covariance functions are plotted in Fig. 11 with normalization by the variance $C_{opt}(0)$ of (0.12 m)$^2$. The variance $C_{opt}(0)$ is of reasonable value since the variance calculated from eight
tide gauge records (total VT in Table 2) is (0.11 m)². As seen in the figure, both covariance functions $C_{opt}(s)$ (solid line) and $\hat{C}_{conv}(s)$ (dotted line) behave in considerably different manner; for example, $\hat{C}_{conv}(s)$, which is twice as large as $C_{opt}(0)$ at distance lag $s$ of 0 km, suddenly decreases as lag $s$ increases, while $C_{opt}(s)$ marks positive correlations for longer lag $s$ so that the zero-crossing correlation length for $C_{opt}(s)$ of 800 km is four times larger than that for $\hat{C}_{conv}(s)$ of 200 km.

For the convenience of comparison, we dare to exclude along-track tilt-and-bias oceanic signals from the fluctuation SSDT field $\zeta_p'$ and calculate another covariance function $\hat{C}_{opt}(s)$ so that it includes contributions from $\hat{\zeta}_p'' + \hat{\varepsilon}_m$; the calculated covariance function is plotted by the broken line in Fig. 11. Comparing $\hat{C}_{conv}(s)$ and $\hat{C}_{opt}(s)$ which are expected to differ with contributions of $\hat{\zeta}_p'' + \hat{\varepsilon}_m$, their discrepancy is found only at lag $s$ of 0 km. This discrepancy is explained by the inclusion of contributions of $\hat{\zeta}_p'' + \hat{\varepsilon}_m$ in the covariance function $\hat{C}_{conv}(s)$; since these terms are expected to have random nature, their covariance function would behave like the Dirac-delta function at lag of 0 km for the present spatial resolution (67 km). Note that the variance of these components of (0.13 m)² estimated from $\hat{C}_{conv}(0) - \hat{C}_{opt}(0)$ is larger than that of $\zeta_p''$ of (0.06 m)² calculated from the eight tide gauge records (total ET in Table 2), which indicates that the observation noise $\varepsilon_m$ is of the magnitude of 0.12 m. On the other hand, the similarity of $\hat{C}_{conv}(s)$ and $\hat{C}_{opt}(s)$ for longer lag $s$ suggests that the optimal interpolation does not alter the oceanic signal of mid-wavelength $\zeta_p'$ significantly, as expected from the test analysis in Subsection 2.2, and also that the discrepancy of $C_{opt}(s)$ and $\hat{C}_{conv}(s)$ for $s \neq 0$ described above is caused by the removal of the tilt and bias of oceanic signals in the conventional method.

Fig. 10. Scatter plots of the tide gauge and altimetric fluctuation SSDT’s for Stations A (circles), B (squares), C (triangles) and H (crosses) before (a) and after (b) the compensation of the seasonal variation of the areal average height determined from the climatological monthly-mean SSDT. Regression lines are also drawn in the figures.
latter indicates that statistics such as covariance functions would strongly depend on the accuracy of the orbit error reduction procedure, and thus, the use of the present optimal interpolation method is advantageous.

4.2 Error assessment of the geoid model improvement

Improvement of the geoid model is assessed in this subsection using the temporal mean elevation field \( H(x) \) relative to the improved geoid model (Fig. 5). The model was improved by Seasat altimetry data combined with the mean SSDT determined from \textit{in situ} hydrographic data, which is a hybrid of the mean SSDT (Imawaki et al., 1991); in the local area south of Japan inside the thick dotted line in Fig. 5, the mean SSDT is determined from contemporary hydrographic observations during the three-month Seasat mission, while climatological mean SSDT is basically used for the outside of the area. Therefore, the improvement of the geoid model should be independently discussed for each areas.

In the local area south of Japan inside the thick dotted line in Fig. 5, the mean elevation field \( H(x) \) seems to represent the mean SSDT \( \tilde{\zeta}(x) \) correctly. Namely, the strong gradient of mean elevation field \( H(x) \) south of Japan corresponds well to the expected structure of the mean SSDT \( \tilde{\zeta}(x) \) associated with the meandering Kuroshio, except for unrealistic longitudinal gradient at 28–30°N, 133–136°E which is considered to be affected by the extreme low value centered just outside the boundary at 28°N, 133°E. Therefore in this region, qualitative but synoptic comparison with hydrographic observations can be made in terms of the absolute SSDT determined by combining the mean elevation \( H(x) \) and the fluctuation SSDT \( \zeta_p'(x) \).

Time series of the absolute SSDT from Cycles 1 to 6 are shown in Fig. 12 together with maps of estimated Kuroshio axis during corresponding periods determined by \textit{in situ} observations. Both
Fig. 12. Comparison of the absolute SSDT and simultaneous in situ oceanographic observations in the local region south of Japan. Maps of the Kuroshio axis determined from in situ oceanographic observations for early November, 1986 (a) to early February, 1987 (g) are shown on the left-hand side (partial copies of “Prompt Report on Oceanographic Conditions” issued semimonthly by the Hydrographic Department); small arrows indicate ADCP surface velocities. Maps of the absolute SSDT (the mean elevation $H(x)$ plus fluctuation SSDT $\zeta_p'(x)$) from Cycle 1 (C1) to Cycle 6 (C6) are shown on the right-hand side; contour and shading intervals are 0.2 m with 0.1 m gap for each other, and lower values are shaded more heavily. Positions of panels are shifted from top to bottom according to the central date of each observation period.
left and right panels, Figs. 12(a)–(g) and (C1)–(C6), clearly describe the onset event of a large southward quasi-stationary meander of the Kuroshio. Namely, a small meander off Kii Peninsula centered at 33.5°N, 136–137°E in November, 1986 ((a)–(b) and (C1)–(C2)) rapidly grew in mid December to become a large narrow meander of approximately 150 km width, reaching its tip at 31°N, 140°E ((d) and (C3)). After the meander grew, it then gradually increased its width up to 400 km in January, 1987, which is remarked by the southward shift of the Kuroshio axis along 136°E ((f) and (C5)). The trail of the narrow meander can still be seen in Fig. 12(C5) as a small southward distortion of the Kuroshio meander at the tip (31.5°N, 139.5°E), but it was dismissed in that region in February, 1987 ((g) and (C6)). The small southward distortion of the meander in January, 1987 is not clear in the Kuroshio axis estimated from in situ observations (Fig. 12(f)), but a strong south-southeastward ADCP velocity was recorded at 31°N, 139°E, which agrees well with the existence of the southward distortion of the axis. These good correspondences between the absolute SSDT and in situ observations reveal that the temporal mean elevation field $H(x)$ can be accurately used as the mean SSDT $\zeta(x)$ in this region, at least qualitatively. Note that geostrophic velocities determined from the absolute SSDT in this region show quantitatively good agreement with surface velocities determined from trajectory data of a satellite-tracked drifting buoy, although the comparison is limited both in space and time; these comparisons are described in a separated paper (Ichikawa et al., 1995).

Outside the local area south of Japan, errors in the geoid model improvement would be roughly assessed by subtracting the climatological mean SSDT from $H(x)$ with an accuracy of the extent to which the one-year mean SSDT during Geosat mission is approximated by the climatological mean. Here, the unknown systematic orbit error $\epsilon_s(x)$ in Eq. (2) is neglected; this will be discussed in Section 5. The error, shown in Fig. 13, has a minimum value of −0.9 m in the Kuroshio Extension region (36°N, 145°E) and a maximum value of 1.3 m in the Oyashio region (43°N, 149.5°E), and is within a range of ±1 m as a whole; overall rms amplitude is 0.39 m. Considering that the error of the original geoid model before the improvement exceeds a range of ±3 m (Imawaki et al., 1991), the model has been significantly improved from the original one even in the region where the climatological mean SSDT is used instead of the contemporary mean SSDT. In general, however, the geoid error $\epsilon_s(x)$ still remaining is too large for the mean elevation field $H(x)$ to be used as the mean SSDT $\zeta(x)$.

Closer look to Fig. 13 indicates that some local extreme values are superimposed on a spatially relatively smooth area where variations are within a range of ±0.3 m (e.g. 25°–30°N, 125°–145°E); namely, local extremes are found in the Kuroshio Extension region (36°N, 145°E; 34.5°N, 148°E), in the Oyashio region (43°N, 149.5°E), around Tsugaru Strait (41.5°N, 141.5°E), near Taiwan (22°N, 120.5°E; 24.5°N, 121.5°E), and in the southeastern area of the present study domain (20.5°N, 145.5°E; 24°N, 142.5°E; 26.5°N, 148.5°E). Among those regions, the extreme error in the Kuroshio Extension region (36°N, 145°E) is considered to be caused by substituting the climatological mean for three-month mean during the Seasat mission. The axis of the Kuroshio Extension at 145°E was shifted northward during the Seasat mission (Ichikawa and Imawaki, 1992), and hence the three-month mean SSDT in this region is higher than the climatological mean by approximately 1 m; substitution of the climatological mean would, therefore, produce the negative extreme error. Note that errors induced by substitution of the climatological mean can be assessed here since Geosat one-year mean should be generally closer to the climatological mean than Seasat three-month mean is. The other regions may also include such errors due to the substitution of long-term mean, but rms variability of SSDT is known to be less than 0.15 m (e.g. Aoki et al., 1995) so that the large error exceeding 0.5 m would...
not be explained by difference of averaging period alone. Rather, two common features are remarked for all those regions; namely, they are the areas where the geoid changes rapidly in space, and they are located near the boundary of the study domain. Both of those features suggest that the large error in these regions are caused by an insufficient accuracy of estimates of the mean elevation field $H(x)$ during the Seasat mission used in the geoid model improvement.

5. Discussion

The estimated fluctuation SSDT $\zeta_p'(x)$ is quantitatively compared with island tide gauge records whose locations and time scale of sea-level variations are proper to evaluating $\zeta_p'(x)$. The comparison indicates the existence of a seasonal variation of systematic discrepancies between them. Errors in path length corrections such as tropospheric water vapor corrections may cause some seasonally-varying discrepancies, and aliased tidal errors in the altimetry data may result in seasonal variations (e.g. Schlax and Chelton, 1994). However, good correspondence of a seasonal variation of the areal average heights determined from the climatological monthly-mean SSDT (dotted curve in Fig. 9) with that of the systematic discrepancies (solid curve) suggests that the loss of the areal average in the present optimal interpolation method is the dominant cause of the systematic discrepancies. Note that the seasonal variations determined from the climatological mean data and the one-year long data result in almost the same; this is probably because variations of average heights over a relatively wide area are dominated by the thermal expansion, which would not have large interannual variations. Also note, however, that seasonal variations of the areal average heights would be negligible for a wider study area so that compensation of the lost areal average height would not be a necessary procedure in that case.

In addition, the areal average of the SSDT field has nothing to do with calculation of geostrophic velocities.

Fig. 13. Possible errors in the improved geoid model for the area outside the thick broken line in Fig. 5. Contour interval is 0.1 m, and shading interval is 0.2 m; lower values are shaded more heavily.
The geoid model improvement by combined use of Seasat altimetry data and in situ hydrographic data is excellent only in a local area south of Japan where contemporary observations during the three-month mission of Seasat exist. In the region outside of the local area, errors of several tens of centimeters still remain in the model. Two reasons are considered to cause the errors: One is substitution of the climatological mean for the three-month mean, and the other is relatively inaccurate determination of the Seasat mean elevation field in the area where the geoid changes rapidly in space. Since more altimetry data are expected to be obtained in the near future, neither of these two will be a severe problem; opportunities to obtain contemporary observation data would increase, while distribution of altimetric observation points would become dense enough to produce more accurate mean elevation field. On the other hand, the accuracy of determination of the true SSDT, such as an assumption of the “level of no motion” in dynamic height calculations, should also be considered for more accurate improvement of the geoid model.

The unknown systematic orbit error \( \varepsilon_s(x) \) is provided to be small in the present paper, but this may not be proper. For example, extreme discrepancies between the one-year mean elevation field \( H(x) \) and the climatological mean SSDT (Fig. 13) may be related to the systematic orbit error \( \varepsilon_s(x) \) which is mainly caused by insufficient knowledge of gravity anomalies in the calculation of the satellite orbit height. The effect of the systematic orbit error would be estimated by determining the mean elevation field during the same period using another orbit height data set and comparing it with that determined in the present paper.

The tip of the large narrow meander of the Kuroshio seems to have been moved northeastward (Figs. 12(C4)–(C5)) and dismissed in that region in February, 1987 (Fig. 12(C6)). Time series of the fluctuation SSDT around 30°N for Cycles 4–11 (Fig. 14) indicates that the tip was truncated from the meander and kept eastward movement with its magnitude being reduced; in the figure, this can be recognized as a somewhat fast (9 cm/s) eastward propagation of negative values (shown as a chain-line). This eastward propagation is very peculiar since most of the fluctuation SSDT tends to show westward propagation (Tai and White, 1990; Aoki et al., 1995), but the reason is not well understood.

As exhibited in the comparison for the tide gauge station G in Fig. 8, fast-moving small-scale features may cause spatio-temporal distortion of the estimated fluctuation SSDT field. It is impossible, however, to increase both temporal and spatial resolutions at the same time for any altimetry data set from a single satellite. Only way to increase both resolutions is to analyze altimetry data sets from several satellites, such as combined use of ERS-1 and TOPEX/POSEIDON. Note that the present optimal interpolation has no problem to consistently process these data sets of different accuracies and sampling patterns.

6. Summary

An optimal interpolation method is applied to Geosat altimetry data both to remove radial orbit error and to separate the temporal mean sea surface dynamic topography (SSDT) from fluctuations around the mean. The reliability of the method is first tested by artificial observation data, and it is found that the method can accurately reconstruct the SSDT field except for small scale structures and the areal average component over the study area.

The temporally fluctuating part of SSDT (fluctuation SSDT) is quantitatively evaluated by eight tide gauge records on Japanese islands. The correlation coefficient between the tide gauge and altimetric fluctuation SSDT’s is 0.47, which is significant for \( t \)-test of 99.9% confidence level. These comparisons include, however, stations unfavorable for evaluation of the fluctuation
SSDT, such as coastal stations where amount of ambient altimetry data is relatively small and stations where dominant sea-level variations are too rapid to be resolved by the present altimetry data set. Excluding such stations and recovering the areal averages lost in the optimal interpolation by seasonal variations determined from the climatological monthly-mean SSDT, the correlation coefficient increases up to 0.83, and the tilt of the regression line becomes nearly the unity (1.04).

Geoid model improvement by combined use of Seasat altimetry data and \textit{in situ} hydrographic
data (Imawaki et al., 1991) is assessed with the use of the estimated temporal mean elevation field. In a local area south of Japan where hydrographic observations contemporary to the Seasat altimetry mission exist, the improvement was excellent so that the absolute SSDT can be determined from altimetry data and the geoid model. Time series of the absolute SSDT describes the onset event of a quasi-stationary large meander of the Kuroshio south of Japan very well; namely, a small meander off Kii Peninsula rapidly grew to form a large narrow meander at the first stage, which gradually increased its width. The tip of the narrow meander moved eastward and seems to have been truncated from the large meander; time series of the fluctuation SSDT around 30°N indicates that the truncated tip of the meander kept eastward movement at somewhat fast propagation speed of 9 cm/s.

Outside the local area south of Japan, however, the geoid model still includes errors of several tens of centimeters. Substitution of the climatological mean for the three-month mean during Seasat mission is one of the reasons of those errors, and relatively inaccurate determination of the mean elevation field during the Seasat mission in the areas where the geoid changes rapidly in space and the areas close to the boundary of the study domain would be another one.

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