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What Is the Signal Processing Toolbox?

This section describes how to begin using the Signal Processing Toolbox. It explains how to use this manual and points you to additional books for toolbox installation information.

The Signal Processing Toolbox is a collection of tools built on the MATLAB® numeric computing environment. The toolbox supports a wide range of signal processing operations, from waveform generation to filter design and implementation, parametric modeling, and spectral analysis. The toolbox provides two categories of tools:

- Signal processing functions
- Graphical, interactive tools

The first category of tools is made up of functions that you can call from the command line or from your own applications. Many of these functions are MATLAB M-files, series of MATLAB statements that implement specialized signal processing algorithms. You can view the MATLAB code for these functions using the statement

```
    type function_name
```

or by opening the M-file in the MATLAB Editor/Debugger. You can change the way any toolbox function works by copying and renaming the M-file, then modifying your copy. You can also extend the toolbox by adding your own M-files.

Second, the toolbox provides a number of interactive tools that let you access many of the functions through a graphical user interface (GUI). The GUI-based tools provide an integrated environment for filter design, analysis, and implementation, as well as signal exploration and editing. For example, with the graphical user interface tools you can:

- Use the mouse to graphically edit the magnitude response of a filter or measure the slope of a signal with onscreen rulers.
- Play a signal on your system's audio hardware by selecting a menu item or pressing a corresponding keystroke combination.
- Customize the parameters and method of computing the spectrum of a signal.
How to Use This Manual

If you are a new user. Begin with Chapter 1, “Signal Processing Basics.” This chapter introduces the MATLAB signal processing environment through the toolbox functions. It describes the basic functions of the Signal Processing Toolbox, reviewing its use in basic waveform generation, filter implementation and analysis, impulse and frequency response, zero-pole analysis, linear system models, and the discrete Fourier transform.

When you feel comfortable with the basic functions, move on to Chapter 2 and Chapter 3 for a more in-depth introduction to using the Signal Processing Toolbox:

• Chapter 2, “Filter Design,” for a detailed explanation of using the Signal Processing Toolbox in infinite impulse response (IIR) and finite impulse response (FIR) filter design and implementation, including special topics in IIR filter design.

• Chapter 3, “Statistical Signal Processing,” for how to use the correlation, covariance, and spectral analysis tools to estimate important functions of discrete random signals.

Once you understand the general principles and applications of the toolbox, learn how to use the interactive tools.

• Chapter 5, “Interactive Tools,” for an overview of the interactive GUI environment and examples of how to use it for signal exploration, filter design and implementation, and spectral analysis.

Finally, see the following chapter for a discussion of specialized toolbox functions.

• Chapter 4, “Special Topics,” for specialized functions, including filter windows, parametric modeling, resampling, cepstrum analysis, time-dependent Fourier transforms and spectrograms, median filtering, communications applications, deconvolution, and specialized transforms.

If you are an experienced toolbox user. See Chapter 5, “Interactive Tools,” for an overview of the interactive GUI environment and examples of how to use it for signal viewing, filter design and implementation, and spectral analysis.
All toolbox users. Use Chapter 6, “Reference,” for locating information on specific functions. Reference descriptions include a synopsis of the function’s syntax, as well as a complete explanation of options and operations. Many reference descriptions also include helpful examples, a description of the function’s algorithm, and references to additional reading material.

Use this manual in conjunction with the software to learn about the powerful features that MATLAB provides. Each chapter provides numerous examples that apply the toolbox to representative signal processing tasks.

Some examples use MATLAB’s random number generation function \texttt{randn}. In these cases, to duplicate the results in the example, type

\begin{verbatim}
    randn('seed',0)
\end{verbatim}

before running the example.
Installation

To install this toolbox on a workstation, see the MATLAB Installation Guide for UNIX. To install the toolbox on a PC, see the MATLAB PC Installation Guide.

To determine if the Signal Processing Toolbox is already installed on your system, check for a subdirectory named signal within the main toolbox directory or folder.
## Typographical Conventions

<table>
<thead>
<tr>
<th>To Indicate</th>
<th>This Manual Uses</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example code</td>
<td>Monospace type.</td>
<td>To assign the value 5 to A, enter</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 5</td>
</tr>
<tr>
<td>MATLAB output</td>
<td>Monospace type.</td>
<td>MATLAB responds with</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A = 5</td>
</tr>
<tr>
<td>Function names</td>
<td>Monospace type.</td>
<td>The cos function finds the cosine of each array element.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New terms</td>
<td>Italics.</td>
<td>An array is an ordered collection of information.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keys</td>
<td><strong>Boldface</strong> with an initial capital letter.</td>
<td>Press the <strong>Return</strong> key.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Menu names, items, and GUI controls</td>
<td><strong>Boldface</strong> with an initial capital letter.</td>
<td>Choose the <strong>File</strong> menu.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical expressions</td>
<td>Variables in italics. Functions, operators, and constants in standard type.</td>
<td>This vector represents the polynomial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p = x^2 + 2x + 3</td>
</tr>
</tbody>
</table>
Technical Notations
This manual and the Signal Processing Toolbox functions use the following technical notations:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
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<tr>
<td>Nyquist frequency</td>
<td>One-half the sampling frequency. Most toolbox functions normalize this value to 1.</td>
</tr>
<tr>
<td>$x(1)$</td>
<td>The first element of a data sequence or filter, corresponding to zero lag.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Analog frequency in radians per second.</td>
</tr>
<tr>
<td>$\omega$ or $w$</td>
<td>Digital frequency in radians per second.</td>
</tr>
<tr>
<td>$f$</td>
<td>Digital frequency in Hertz.</td>
</tr>
<tr>
<td>$[x, y)$</td>
<td>The interval from $x$ to $y$, including $x$ but not including $y$.</td>
</tr>
</tbody>
</table>
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Signal Processing Basics

Signal Processing Toolbox Central Features

This chapter describes how to begin using MATLAB and the Signal Processing Toolbox for your signal processing applications. It assumes a basic knowledge and understanding of signals and systems, including such topics as filter and linear system theory and basic Fourier analysis.

Many examples throughout the chapter demonstrate how to apply toolbox functions. If you are not already familiar with MATLAB’s signal processing capabilities, use this chapter in conjunction with the software to try examples and learn about the powerful features available to you.

The Signal Processing Toolbox functions are algorithms, expressed mostly in M-files, that implement a variety of signal processing tasks. These toolbox functions are a specialized extension of the MATLAB computational and graphical environment.

Filtering and FFTs

Two of the most important functions for signal processing are not in the Signal Processing Toolbox at all, but are built-in MATLAB functions:

- `filter` applies a digital filter to a data sequence.
- `fft` calculates the discrete Fourier transform of a sequence.

The operations these functions perform are the main computational workhorses of classical signal processing. Both are described in this chapter. The Signal Processing Toolbox uses many other standard MATLAB functions and language features, including polynomial root finding, complex arithmetic, matrix inversion and manipulation, and graphics tools.

Signals and Systems

The basic entities that toolbox functions work with are signals and systems. The functions emphasize digital, or discrete, signals and filters, as opposed to analog, or continuous, signals. The principal filter type the toolbox supports is the linear, time-invariant digital filter with a single input and a single output. You can represent linear time-invariant systems using one of several models (such as transfer function, state-space, zero-pole-gain, and second-order section) and convert between representations.
Key Areas: Filter Design and Spectral Analysis
In addition to its core functions, the toolbox provides rich, customizable support for the key areas of filter design and spectral analysis. It is easy to implement a design technique that suits your application, design digital filters directly, or create analog prototypes and discretize them. Toolbox functions also estimate power spectral density and cross spectral density, using either parametric or nonparametric techniques. Chapters 2 and 3, respectively, detail toolbox functions for filter design and spectral analysis.

There are functions for computation and graphical display of frequency response, as well as functions for system identification; generating signals; discrete cosine, chirp-z, and Hilbert transforms; lattice filters; resampling; time-frequency analysis; and basic communication systems simulation.

Graphical User Interface (GUI)
The power of the Signal Processing Toolbox is greatly enhanced by its easy-to-use graphical user interface. The GUI provides an integrated set of interactive tools for performing a wide variety of signal processing tasks. These tools enable you to use the mouse and menus to manipulate a rich graphical environment for signal viewing, filter design and implementation, and spectral analysis.

Extensibility
Perhaps the most important feature of the MATLAB environment is that it is extensible: MATLAB lets you create your own M-files to meet numeric computation needs for research, design, or engineering of signal processing systems. Simply copy the M-files provided with the Signal Processing Toolbox and modify them as needed, or create new functions to expand the functionality of the toolbox.
Representing Signals

The central data construct in MATLAB is the numeric array, an ordered collection of real or complex numeric data with two or more dimensions. The basic data objects of signal processing (one-dimensional signals or sequences, multichannel signals, and two-dimensional signals) are all naturally suited to array representation.

Vector Representation

MATLAB represents ordinary one-dimensional sampled data signals, or sequences, as vectors. Vectors are 1-by-n or n-by-1 arrays, where n is the number of samples in the sequence. One way to introduce a sequence into MATLAB is to enter it as a list of elements at the command prompt. The statement

\[
x = [4 \
3 \
7 \
-9 \
1]
\]

creates a simple five-element real sequence in a row vector. Transposition turns the sequence into a column vector

\[
x = x'
\]

resulting in

\[
x = \\
4 \\
3 \\
7 \\
-9 \\
1
\]

Column orientation is preferable for single channel signals because it extends naturally to the multichannel case. For multichannel data, each column of a matrix represents one channel. Each row of such a matrix then corresponds to a sample point. A three-channel signal that consists of x, 2x, and x/π is

\[
y = [x 
2x 
x/\pi]
\]
This results in

\[ y = \begin{bmatrix}
4.0000 & 8.0000 & 1.2732 \\
3.0000 & 6.0000 & 0.9549 \\
7.0000 & 14.0000 & 2.2282 \\
-9.0000 & -18.0000 & -2.8648 \\
1.0000 & 2.0000 & 0.3183
\end{bmatrix} \]
Waveform Generation: Time Vectors and Sinusoids

A variety of toolbox functions generate waveforms. Most require you to begin with a vector representing a time base. Consider generating data with a 1000 Hz sample frequency, for example. An appropriate time vector is

\[ t = (0:0.001:1)'; \]

where MATLAB’s colon operator creates a 1001-element row vector that represents time running from zero to one second in steps of one millisecond. The transpose operator (') changes the row vector into a column; the semicolon (;) tells MATLAB to compute but not display the result.

Given \( t \) you can create a sample signal \( y \) consisting of two sinusoids, one at 50 Hz and one at 120 Hz with twice the amplitude:

\[ y = \sin(2\pi \cdot 50 \cdot t) + 2 \cdot \sin(2\pi \cdot 120 \cdot t); \]

The new variable \( y \), formed from vector \( t \), is also 1001 elements long. You can add normally distributed white noise to the signal and graph the first fifty points using

\[ yn = y + 0.5 \cdot \text{randn(size}(t)); \]
\[ \text{plot}(t(1:50),yn(1:50)) \]
Common Sequences: Unit Impulse, Unit Step, and Unit Ramp

Since MATLAB is a programming language, an endless variety of different signals is possible. Here are some statements that generate several commonly used sequences, including the unit impulse, unit step, and unit ramp functions:

\[
\begin{align*}
t &= (0:0.001:1)'; \\
y &= [1; \text{zeros}(99,1)]; \quad \% \text{impulse} \\
y &= \text{ones}(100,1); \quad \% \text{step (filter assumes 0 initial cond.)} \\
y &= t; \quad \% \text{ramp} \\
y &= t.^2; \\
y &= \text{square}(4*t); 
\end{align*}
\]

All of these sequences are column vectors – the last three inherit their shapes from \( t \).

Multichannel Signals

Use standard MATLAB array syntax to work with multichannel signals. For example, a multichannel signal consisting of the last three signals generated above is

\[
z = [t \ t.^2 \ \text{square}(4*t)];
\]

You can generate a multichannel unit sample function using the outer product operator. For example, a six-element column vector whose first element is one, and whose remaining five elements are zeros, is

\[
a = [1 \ \text{zeros}(1,5)'];
\]

To duplicate column vector \( a \) into a matrix without performing any multiplication, use MATLAB’s colon operator and the \( \text{ones} \) function.

\[
c = a(:,\text{ones}(1,3));
\]

Common Periodic Waveforms

The toolbox provides functions for generating widely used periodic waveforms:
• `sawtooth` generates a sawtooth wave with peaks at ±1 and a period of $2\pi$. An optional `width` parameter specifies a fractional multiple of $2\pi$ at which the signal’s maximum occurs.

• `square` generates a square wave with a period of $2\pi$. An optional parameter specifies duty cycle, the percent of the period for which the signal is positive.

To generate 1.5 seconds of a 50 Hz sawtooth wave with a sample rate of 10 kHz and plot 0.2 seconds of the generated waveform, use

```plaintext
Fs = 10000;
t = 0:1/Fs:1.5;
x = sawtooth(2*pi*50*t);
plot(t,x), axis([0 0.2 -1 1])
```

![Graph of a sawtooth wave](image)

### Common Aperiodic Waveforms

The toolbox also provides functions for generating several widely used aperiodic waveforms:

• `gauspuls` generates a Gaussian-modulated sinusoidal pulse with a specified time, center frequency, and fractional bandwidth. Optional parameters
return in-phase and quadrature pulses, the RF signal envelope, and the
cutoff time for the trailing pulse envelope.

- chirp generates a linear swept-frequency cosine signal. An optional
  parameter specifies alternative sweep methods. An optional parameter phi
  allows initial phase to be specified in degrees.

To compute 2 seconds of a linear chirp signal with a sample rate of 1 kHz, that
starts at DC and crosses 150 Hz at 1 second, use

```matlab
t = 0:1/1000:2;
y = chirp(t,0,1,150);
```

To plot the spectrogram, use

```matlab
specgram(y,256,1000,256,250)
```

---

**The pulstran Function**

The pulstran function generates pulse trains from either continuous or
sampled prototype pulses. The following example generates a pulse train
consisting of the sum of multiple delayed interpolations of a Gaussian pulse.
The pulse train is defined to have a sample rate of 50 kHz, a pulse train length
of 10 ms, and a pulse repetition rate of 1 kHz; \( D \) specifies the delay to each pulse repetition in column 1 and an optional attenuation for each repetition in column 2. The pulse train is constructed by passing the name of the `gauspuls` function to `pulstran`, along with additional parameters that specify a 10 kHz Gaussian pulse with 50% bandwidth:

\[
T = 0:1/50E3:10E-3; \\
D = [0:1/1E3:10E-3;0.8.^(0:10)]; \\
Y = pulstran(T,D,'gauspuls',10E3,0.5); \\
plot(T,Y)
\]

![Graph of Gaussian pulse train](image)

**The Sinc Function**

The `sinc` function computes the mathematical sinc function for an input vector or matrix \( x \). The sinc function is the continuous inverse Fourier transform of the rectangular pulse of width \( 2\pi \) and height 1:

The sinc function has a value of 1 where \( x \) is zero, and a value of

\[
\frac{\sin(\pi x)}{\pi x}
\]

for all other elements of \( x \).
To plot the sinc function for a linearly spaced vector with values ranging from -5 to 5,

```matlab
x = linspace(-5,5);
y = sinc(x);
plot(x,y)
```

The Dirichlet Function

The toolbox function `diric` computes the Dirichlet function, sometimes called the periodic sinc or aliased sinc function, for an input vector or matrix `x`. The Dirichlet function is

\[
diric(x) = \begin{cases} 
-k(n-1) & x = 2\pi k, \quad k = 0, \pm 1, \pm 2, \ldots \\
\sin(nx/2) & \text{otherwise}
\end{cases}
\]

where `n` is a user-specified positive integer. For `n` odd, the Dirichlet function has a period of `2\pi`; for `n` even, its period is `4\pi`. The magnitude of this function is \((1/n)\) times the magnitude of the discrete-time Fourier transform of the n-point rectangular window.
To plot the Dirichlet function over the range 0 to $4\pi$ for $n = 7$ and $n = 8$, use

```matlab
x = linspace(0,4*pi,300);
plot(x,diric(x,7))
plot(x,diric(x,8))
```
Working with Data

The examples in the preceding sections obtain data in one of two ways:

- By direct input, that is, entering the data manually at the keyboard
- By using a MATLAB or toolbox function, such as \( \sin \), \( \cos \), \( \text{sawtooth} \), \( \text{square} \), or \( \text{sinc} \)

Some applications, however, may need to import data from outside MATLAB. Depending on your data format, you can do this in the following ways:

- Load data from an ASCII file or MAT-file with MATLAB's \texttt{load} command
- Read the data into MATLAB with a low-level file I/O function, such as \texttt{fopen}, \texttt{fread}, and \texttt{fscanf}
- Develop a MEX-file to read the data

Other resources are also useful, such as a high-level language program (in Fortran or C, for example) that converts your data into MAT-file format—see the MATLAB Application Programming Interface reference manual for details. MATLAB reads such files using the \texttt{load} command.

Similar techniques are available for exporting data generated within MATLAB. See Using MATLAB for more details on importing and exporting data, and see the online MATLAB Function Reference for descriptions of file loading and I/O routines.
Filter Implementation and Analysis

This section describes how to filter discrete signals using MATLAB’s `filter` function and other functions in the Signal Processing Toolbox. It also discusses how to use the toolbox functions to analyze filter characteristics, including impulse response, magnitude and phase response, group delay, and zero-pole locations.

Convolution and Filtering

The mathematical foundation of filtering is convolution. MATLAB’s `conv` function performs standard one-dimensional convolution, convolving one vector with another.

```matlab
conv([1 1 1],[1 1 1])
```

```
ans =
    1     2     3     2     1
```

**NOTE** Convolve rectangular matrices for two-dimensional signal processing using the `conv2` function.

A digital filter’s output \( y(n) \) is related to its input \( x(n) \) by convolution with its impulse response \( h(n) \).

\[
y(n) = h(n) * x(n) = \sum_{m=-\infty}^{\infty} h(n-m)x(m)
\]

If a digital filter’s impulse response \( h(n) \) is finite length, and the input \( x(n) \) is also finite length, you can implement the filter using `conv`. Store \( x(n) \) in a vector \( x \), \( h(n) \) in a vector \( h \), and convolve the two.

```matlab
x = randn(5,1);   % a random vector of length 5
h = [1 1 1 1]/4;  % length 4 averaging filter
y = conv(h,x);
```
Filter Implementation and Analysis

Filters and Transfer Functions

In general, the z-transform $Y(z)$ of a digital filter’s output $y(n)$ is related to the z-transform $X(z)$ of the input by

$$Y(z) = H(z)X(z) = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na}}X(z)$$

where $H(z)$ is the filter’s transfer function. Here, the constants $b(i)$ and $a(i)$ are the filter coefficients and the order of the filter is the maximum of $na$ and $nb$.

**NOTE** The filter coefficients start with subscript 1, rather than 0. This reflects MATLAB’s standard indexing scheme for vectors.

MATLAB stores the coefficients in two vectors, one for the numerator and one for the denominator. By convention, MATLAB uses row vectors for filter coefficients.

Filter Coefficients and Filter Names

Many standard names for filters reflect the number of $a$ and $b$ coefficients present:

- When $nb = 0$ (that is, $b$ is a scalar), the filter is an Infinite Impulse Response (IIR), all-pole, recursive, or autoregressive (AR) filter.
- When $na = 0$ (that is, $a$ is a scalar), the filter is a Finite Impulse Response (FIR), all-zero, nonrecursive, or moving average (MA) filter.
- If both $na$ and $nb$ are greater than zero, the filter is an IIR, pole-zero, recursive, or autoregressive moving average (ARMA) filter.

The acronyms AR, MA, and ARMA are usually applied to filters associated with filtered stochastic processes.
Filtering with the filter Function

It is simple to work back to a difference equation from the z-transform relation shown earlier. Assume that \( a(1) = 1 \). Move the denominator to the left-hand side and take the inverse z-transform.

\[
y(n) + a(2)y(n - 1) + \cdots + a(na + 1)y(n - na) = b(1)x(n) + b(2)x(n - 1) + \cdots + b(nb + 1)x(n - nb)
\]

In terms of current and past inputs, and past outputs, \( y(n) \) is

\[
y(n) = b(1)x(n) + b(2)x(n - 1) + \cdots + b(nb + 1)x(n - nb) - a(2)y(n - 1) - \cdots - a(na + 1)y(n - na)
\]

This is the standard time-domain representation of a digital filter, computed starting with \( y(1) \) and assuming zero initial conditions. This representation's progression is

\[
\begin{align*}
y(1) &= b(1)x(1) \\
y(2) &= b(1)x(2) + b(2)x(1) - a(2)y(1) \\
y(3) &= b(1)x(3) + b(2)x(2) + b(3)x(1) - a(2)y(2) - a(3)y(1) \\
\vdots &= \vdots
\end{align*}
\]

A filter in this form is easy to implement with the \texttt{filter} function. For example, a simple single-pole filter (lowpass) is

\[
b = 1; \quad \% \text{ numerator} \\
a = [1 -0.9]; \quad \% \text{ denominator}
\]

where the vectors \( b \) and \( a \) represent the coefficients of a filter in transfer function form. To apply this filter to your data

\[
y = \text{filter}(b,a,x);
\]

\texttt{filter} gives you as many output samples as there are input samples, that is, the length of \( y \) is the same as the length of \( x \). If the first element of \( a \) is not 1, \texttt{filter} divides the coefficients by \( a(1) \) before implementing the difference equation.
**filter Function Implementation and Initial Conditions**

*filter* is implemented as a transposed direct form II structure

$$
\begin{align*}
    y(m) &= b(1)x(m) + z_1(m-1) \\
    z_1(m) &= b(2)x(m) + z_2(m-1) - a(2)y(m) \\
    &\vdots \quad = \quad \vdots \\
    z_{n-2}(m) &= b(n-1)x(m) + z_{n-1}(m-1) - a(n-1)y(m) \\
    z_{n-1}(m) &= b(n)x(m) - a(n)y(m)
\end{align*}
$$

where \( n-1 \) is the filter order. This is a canonical form that has the minimum number of delay elements.

At sample \( m \), *filter* computes the difference equations

\[
\begin{align*}
    y(m) &= b(1)x(m) + z_1(m-1) \\
    z_1(m) &= b(2)x(m) + z_2(m-1) - a(2)y(m) \\
    &\vdots = \vdots \\
    z_{n-2}(m) &= b(n-1)x(m) + z_{n-1}(m-1) - a(n-1)y(m) \\
    z_{n-1}(m) &= b(n)x(m) - a(n)y(m)
\end{align*}
\]

In its most basic form, *filter* initializes the delay outputs \( z_i(1), i = 1, \ldots, n-1 \) to 0. This is equivalent to assuming both past inputs and outputs are zero. Set the initial delay outputs using a fourth input parameter to *filter*, or access the final delay outputs using a second output parameter.

\[
[y, zf] = \text{filter}(b, a, x, zi)
\]

Access to initial and final conditions is useful for filtering data in sections, especially if memory limitations are a consideration. Suppose you have collected data in two segments of 5000 points each.

\[
x1 = \text{randn}(5000,1); \quad \% \text{two random sequences to}
\]
\[
x2 = \text{randn}(5000,1); \quad \% \text{serve as simulated data}
\]

Perhaps the first sequence, \( x1 \), corresponds to the first 10 minutes of data and the second, \( x2 \), to an additional 10 minutes. The whole sequence is \( x = [x1; x2] \). If there is not sufficient memory to hold the combined sequence,
filter the subsequences \(x_1\) and \(x_2\) one at a time. To ensure continuity of the filtered sequences, use the final conditions from \(x_1\) as initial conditions to filter \(x_2\).

\[
[y_1,zf] = \text{filter}(b,a,x_1);
\]
\[
y_2 = \text{filter}(b,a,x_2,zf);
\]

The \text{filtic} function generates initial conditions for \text{filter}. \text{filtic} computes the delay vector to make the behavior of the filter reflect past inputs and outputs that you specify. To obtain the same output delay values \(zf\) as above using \text{filtic}

\[
zf = \text{filtic}(b,a,\text{flipud}(y_1),\text{flipud}(x_1));
\]

This can be useful when filtering short data sequences, as appropriate initial conditions help reduce transient startup effects.
Other Functions for Filtering

In addition to filter, several other functions in the Signal Processing Toolbox perform the basic filtering operation. These functions include upfirdn, which performs FIR filtering with resampling, filtfilt, which eliminates phase distortion in the filtering process, fftfilt, which performs the FIR filtering operation in the frequency domain, and latcfilt, which filters using a lattice implementation.

Multirate Filter Bank Implementation

The function upfirdn alters the sampling rate of a signal by an integer ratio $P/Q$. It computes the result of the cascade of three systems: (1) upsampling (zero insertion) by integer factor $p$, (2) filtering by FIR filter $h$, and (3) downsampling by integer factor $q$.

For example, to change the sample rate of a signal from 44.1 kHz to 48 kHz, we first find the smallest integer conversion ratio $p/q$.

\[
d = \gcd(48000, 44100);
p = 48000/d;
q = 44100/d;
\]

where we find that $p = 160$ and $q = 147$. Sample rate conversion is then accomplished by $y = \text{upfirdn}(x, h, p, q)$. This cascade of operations is implemented in an efficient manner using polyphase filtering techniques, and it is a central concept of multirate filtering (see reference [1] for details on multirate filter theory). Note that the quality of the resampling result relies on the quality of the FIR filter $h$.

Filter banks may be implemented using upfirdn by allowing the filter $h$ to be a matrix, with one FIR filter per column. A signal vector is passed independently through each FIR filter, resulting in a matrix of output signals.

Other functions that perform multirate filtering (with fixed filter) include resample, interp, and decimate.
Anti-Causal, Zero-Phase Filter Implementation

In the case of FIR filters, it is possible to design linear phase filters that, when applied to data (using filter or conv), simply delay the output by a fixed number of samples. For IIR filters, however, the phase distortion is usually highly nonlinear. The \texttt{filtfilt} function uses the information in the signal at points before and after the current point, in essence “looking into the future,” to eliminating phase distortion.

To see how \texttt{filtfilt} does this, recall that if the \textit{z}-transform of a real sequence \(x(n)\) is \(X(z)\), the \textit{z}-transform of the time reversed sequence \(x(n)\) is \(X(1/z)\).

Consider the processing scheme

\[
\begin{align*}
X(z) \rightarrow H(z) \rightarrow \text{Time Reverse} \rightarrow H(z) \rightarrow \text{Time Reverse} \rightarrow X(z)H(1/z)H(z)
\end{align*}
\]

When \(|z| = 1\), that is \(z = \exp(j\omega)\), the output reduces to \(X(e^{j\omega})|H(e^{j\omega})|^2\). Given all the samples of the sequence \(x(n)\), a doubly filtered version of \(x\) that has zero-phase distortion is possible.

For example, a one-second duration signal sampled at 100 Hz, composed of two sinusoidal components at 3 Hz and 40 Hz, is

\[
\begin{align*}
Fs &= 100; \\
t &= 0:1/Fs:1; \\
x &= \sin(2\pi t*3) + .25*\sin(2\pi t*40);
\end{align*}
\]
Now create a 10-point averaging FIR filter, and filter \( x \) using both \texttt{filter} and \texttt{filtfilt} for comparison:

\[
\begin{align*}
    b &= \text{ones}(1,10)/10; \quad \% \text{ 10 point averaging filter} \\
    y &= \text{filtfilt}(b,1,x); \quad \% \text{ non-causal filtering} \\
    yy &= \text{filter}(b,1,x); \quad \% \text{ normal filtering} \\
    \text{plot}(t,x,t,y,\'-\',t,yy,':')
\end{align*}
\]

Both filtered versions eliminate the 40 Hz sinusoid evident in the original, solid line. The plot also shows how \texttt{filter} and \texttt{filtfilt} differ; the dashed (\texttt{filtfilt}) line is in phase with the original 3 Hz sinusoid, while the dotted (\texttt{filter}) line is delayed by about five samples. Also, the amplitude of the dashed line is smaller due to the magnitude squared effects of \texttt{filtfilt}.

\texttt{filtfilt} reduces filter startup transients by carefully choosing initial conditions, and by prepending onto the input sequence a short, reflected piece of the input sequence. For best results, make sure the sequence you are filtering has length at least three times the filter order and tapers to zero on both edges.
**Frequency Domain Filter Implementation**

Duality between the time domain and the frequency domain makes it possible to perform any operation in either domain. Usually one domain or the other is more convenient for a particular operation, but you can always accomplish a given operation in either domain.

To implement general IIR filtering in the frequency domain, multiply the discrete Fourier transform (DFT) of the input sequence with the quotient of the DFT of the filter,

\[
\begin{align*}
    n &= \text{length}(x); \\
    y &= \text{ifft}(\text{fft}(x) \cdot \text{fft}(b,n) / \text{fft}(a,n));
\end{align*}
\]

This computes results that are identical to `filter`, but with different startup transients (edge effects). For long sequences, this computation is very inefficient because of the large zero-padded FFT operations on the filter coefficients, and because the FFT algorithm becomes less efficient as the number of points \( n \) increases.

For FIR filters, however, it is possible to break longer sequences into shorter, computationally efficient FFT lengths. The function

\[
y = \text{fftfilt}(b,x)
\]

uses the overlap add method (see reference [1] at the end of this chapter) to filter a long sequence with multiple medium-length FFTs. Its output is equivalent to `filter(b,1,x)`. 
Impulse Response

The impulse response of a digital filter is the output arising from the input sequence

\[ x(n) = \begin{cases} 
    1, & n = 1 \\
    0, & n \neq 1
\end{cases} \]

In MATLAB, you can generate an impulse sequence a number of ways; one straightforward way is

\[ \text{imp} = [1; \text{zeros}(49,1)]; \]

The impulse response of the simple filter \( b = 1 \) and \( a = [1 \ -0.9] \) is

\[ h = \text{filter}(b,a,\text{imp}); \]

The \texttt{impz} function in the toolbox simplifies this operation, choosing the number of points to generate and then making a stem plot (using the \texttt{stem} function).

\[ \text{impz}(b,a) \]

The plot shows the exponential decay \( h(n) = 0.9^n \) of the single pole system.
Frequency Response

The Signal Processing Toolbox enables you to perform frequency domain analysis of both analog and digital filters.

Digital Domain

freqz uses an FFT-based algorithm to calculate the z-transform frequency response of a digital filter. Specifically, the statement

\[ [h,w] = \text{freqz}(b,a,n) \]

returns the n-point complex frequency response, \( H(e^{j\omega}) \), of the digital filter.

\[
H(e^{j\omega}) = \frac{b(1) + b(2)e^{-j\omega} + \ldots + b(nb + 1)e^{-j\omega(nb)}}{a(1) + a(2)e^{-j\omega} + \ldots + a(na + 1)e^{-j\omega(na)}}
\]

In its simplest form, freqz accepts the filter coefficient vectors \( b \) and \( a \), and an integer \( n \) specifying the number of points at which to calculate the frequency response. freqz returns the complex frequency response in vector \( h \), and the actual frequency points in vector \( w \) in radians/second.

freqz can accept other parameters, such as a sampling frequency or a vector of arbitrary frequency points. The example below finds the 256-point frequency response for a 12th-order Chebyshev type I filter. The call to freqz specifies a sampling frequency \( Fs \) of 1000 Hz.

\[
[b,a] = \text{cheby1}(12,0.5,200/500);
\]

\[
[h,f] = \text{freqz}(b,a,256,1000);
\]

Because the parameter list includes a sampling frequency, freqz returns a vector \( f \) that contains the 256 frequency points between 0 and \( Fs/2 \) used in the frequency response calculation.
**Frequency Normalization**  This toolbox uses the convention that unit frequency is the Nyquist frequency, defined as half the sampling frequency. The cutoff frequency parameter for all basic filter design functions is normalized by the Nyquist frequency. For a system with a 1000 Hz sampling frequency, for example, 300 Hz is $300/500 = 0.6$. To convert normalized frequency to angular frequency around the unit circle, multiply by $\pi$. To convert normalized frequency back to Hertz, multiply by half the sample frequency.

If you call `freqz` with no output arguments, it automatically plots both magnitude versus frequency and phase versus frequency. For example, a ninth-order Butterworth lowpass filter with a cutoff frequency of 400 Hz, based on a 2000 Hz sampling frequency, is

\[ [b, a] = \text{butter}(9, 400/1000); \]

Now calculate the 256-point complex frequency response for this filter, and plot the magnitude and phase with a call to `freqz`.

\[ \text{freqz}(b, a, 256, 2000) \]
freqz can also accept a vector of arbitrary frequency points for use in the frequency response calculation. For example,

```matlab
w = linspace(0,pi);
h = freqz(b,a,w);
```
calculates the complex frequency response at the frequency points in `w` for the filter defined by vectors `b` and `a`. The frequency points can range from 0 to $2\pi$. To specify a frequency vector that ranges from zero to your sampling frequency, include both the frequency vector and the sampling frequency value in the parameter list.

**Analog Domain**

`freqs` evaluates frequency response for an analog filter defined by two input coefficient vectors `b` and `a`. Its operation is similar to that of `freqz`; you can specify a number of frequency points to use (by default, the function uses 200), supply a vector of arbitrary frequency points, and plot the magnitude and phase response of the filter.

**Magnitude and Phase**

MATLAB provides functions to extract magnitude and phase from a frequency response vector `h`. The function `abs` returns the magnitude of the response; `angle` returns the phase angle in radians. To extract and plot the magnitude and phase of a Butterworth filter.

```matlab
[b,a] = butter(6,300/500); [h,w] = freqz(b,a,512,1000);
m = abs(h); p = angle(h);
semilogy(w,m);
plot(w,p*180/pi)
```

![Graphs](image-url)
The `unwrap` function is also useful in frequency analysis. `unwrap` unwraps the phase to make it continuous across 360° phase discontinuities by adding multiples of ±360°, as needed. To see how `unwrap` is useful, design a 25th-order lowpass FIR filter.

```matlab
h = fir1(25,0.4);
```

Obtain the filter’s frequency response with `freqz`, and plot the phase in degrees.

```matlab
[H,f] = freqz(h,1,512,2);
plot(f,angle(H)*180/pi); grid
```

It is difficult to distinguish the 360° jumps (an artifact of the arctangent function inside `angle`) from the 180° jumps that signify zeros in the frequency response.
Use `unwrap` to eliminate the 360° jumps.

```matlab
plot(f,unwrap(angle(H))*180/pi); grid
```

### Delay

The group delay of a filter is a measure of the average delay of the filter as a function of frequency. It is defined as the negative first derivative of a filter’s phase response. If the complex frequency response of a filter is \( H(e^{j\omega}) \), then the group delay is

\[
\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}
\]

where \( \theta \) is the phase angle of \( H(e^{j\omega}) \). Compute group delay with

```matlab
[gd,w] = grpdelay(b,a,n)
```

which returns the \( n \)-point group delay, \( \tau_g(\omega) \), of the digital filter specified by \( b \) and \( a \), evaluated at the frequencies in vector \( w \).

The phase delay of a filter is the negative of phase divided by frequency

\[
\tau_p(\omega) = -\frac{\theta(\omega)}{\omega}
\]
To plot both the group and phase delays of a system on the same graph.

```matlab
[b,a] = butter(10,200/1000);
gd = grpdelay(b,a,128);
[h,f] = freqz(b,a,128,2000);
pd = -unwrap(angle(h))*(2000/(2*pi))./f;
plot(f,gd,'-',f,pd,'--')
axis([0 1000 -30 30])
legend('Group Delay', 'Phase Delay')
```
Zero-Pole Analysis

The `zplane` function plots poles and zeros of a linear system. For example, a simple filter with a 0 at -1/2 and a complex pole pair at $0.9e^{j2\pi(0.3)}$ and $0.9e^{-j2\pi(0.3)}$ is

```matlab
zer = -0.5;
pol = .9*exp(j*2*pi*[–0.3 .3]');
```

The zero-pole plot for the filter is

```matlab
zplane(zer,pol)
```

For a system in zero-pole form, supply column vector arguments `z` and `p` to `zplane`.

```matlab
zplane(z,p)
```

For a system in transfer function form, supply row vectors `b` and `a` as arguments to `zplane`.

```matlab
zplane(b,a)
```

In this case `zplane` finds the roots of `b` and `a` using the `roots` function and plots the resulting zeros and poles.
See “Linear System Models” on page 1-32 for details on zero-pole and transfer function representation of systems.
Linear System Models

The Signal Processing Toolbox provides several models for representing linear time-invariant systems. This flexibility lets you choose the representational scheme that best suits your application and, within the bounds of numeric stability, convert freely to and from most other models. This section provides a brief overview of supported linear system models and describes how to work with these models in MATLAB.

Discrete-Time System Models

The discrete-time system models are representational schemes for digital filters. MATLAB supports several discrete-time system models:

- Transfer function
- Zero-pole-gain form
- State-space form
- Partial fraction expansion
- Second-order section form
- Lattice structure form
- Convolution matrices

Transfer Function

The transfer function is a basic z-domain representation of a digital filter, expressing the filter as a ratio of two polynomials. It is the principal discrete-time model for this toolbox. The transfer function model description for the z-transform of a digital filter’s difference equation is

$$Y(z) = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na}}X(z)$$

Here, the constants $b(i)$ and $a(i)$ are the filter coefficients, and the order of the filter is the maximum of $na$ and $nb$. In MATLAB, you store these coefficients in two vectors (row vectors by convention), one row vector for the numerator and one for the denominator. See “Filters and Transfer Functions” on page 1-15 for more details on the transfer function form.
Zero-Pole-Gain

The factored or zero-pole-gain form of a transfer function is

\[
H(z) = \frac{q(z)}{p(z)} = k \frac{(z - q(1))(z - q(2)) \cdots (z - q(n))}{(z - p(1))(z - p(2)) \cdots (z - p(n))}
\]

By convention, MATLAB stores polynomial coefficients in row vectors and polynomial roots in column vectors. In zero-pole-gain form, therefore, the zero and pole locations for the numerator and denominator of a transfer function reside in column vectors. The factored transfer function gain \( k \) is a MATLAB scalar.

The \texttt{poly} and \texttt{roots} functions convert between polynomial and zero-pole-gain representations. For example, a simple IIR filter is

\[
b = [2 3 4];
a = [1 3 3 1];
\]

The zeros and poles of this filter are

\[
q = \text{roots}(b)
q =
-0.7500 + 1.1990i
-0.7500 - 1.1990i
\]

\[
p = \text{roots}(a)
p =
-1.0000
-1.0000 + 0.0000i
-1.0000 - 0.0000i
\]

\[
k = b(1)/a(1)
k =
2
\]
Returning to the original polynomials,

\[ bb = k \cdot \text{poly}(q) \]
\[ bb = \begin{bmatrix} 2.0000 & 3.0000 & 4.0000 \end{bmatrix} \]

\[ aa = \text{poly}(p) \]
\[ aa = \begin{bmatrix} 1.0000 & 3.0000 & 3.0000 & 1.0000 \end{bmatrix} \]

Note that \( b \) and \( a \) in this case represent the transfer function

\[ H(z) = \frac{2 + 3z^{-1} + 4z^{-2}}{1 + 3z^{-1} + 3z^{-2} + z^{-3}} = \frac{2z^3 + 3z^2 + 4z}{z^3 + 3z^2 + 3z + 1} \]

For \( b = [2 \ 3 \ 4] \), the \texttt{roots} function misses the zero for \( z \) equal to 0. In fact, it misses poles and zeros for \( z \) equal to 0 whenever the input transfer function has more poles than zeros, or vice versa. This is acceptable in most cases. To circumvent the problem, however, simply append zeros to make the vectors the same length before using the \texttt{roots} function, for example, \( b = [b \ 0] \).

**State-Space**

It is always possible to represent a digital filter, or a system of difference equations, as a set of first-order difference equations. In matrix or state-space form, you can write the equations as

\[ x(n+1) = Ax(n) + Bu(n) \]
\[ y(n) = Cx(n) + Du(n) \]

where \( u \) is the input, \( x \) is the state vector, and \( y \) is the output. For single-channel systems, \( A \) is an \( m \)-by-\( m \) matrix where \( m \) is the order of the filter, \( B \) is a column vector, \( C \) is a row vector, and \( D \) is a scalar. State-space notation is especially convenient for multichannel systems where input \( u \) and output \( y \) become vectors, and \( B \), \( C \), and \( D \) become matrices.

State-space representation extends easily to the MATLAB environment. In MATLAB, \( A \), \( B \), \( C \), and \( D \) are rectangular arrays; MATLAB treats them as individual variables.
Taking the z-transform of the state-space equations and combining them shows the equivalence of state-space and transfer function forms.

\[ Y(z) = H(z)U(z), \quad \text{where } H(z) = C(zI - A)^{-1}B + D \]

Don’t be concerned if you are not familiar with the state-space representation of linear systems. Some of the filter design algorithms use state-space form internally but do not require any knowledge of state-space concepts to use them successfully. If your applications use state-space based signal processing extensively, however, consult the Control System Toolbox for a comprehensive library of state-space tools.

**Partial Fraction Expansion (Residue Form)**

Each transfer function also has a corresponding partial fraction expansion or residue form representation, given by

\[
\frac{b(z)}{a(z)} = \frac{r(1)}{1 - p(1)z^{-1}} + \cdots + \frac{r(n)}{1 - p(n)z^{-1}} + k(1) + k(2)z^{-1} + \cdots + k(m-n+1)z^{-(m-n)}
\]

provided \(H(z)\) has no repeated poles. Here, \(n\) is the degree of the denominator polynomial of the rational transfer function \(b(z)/a(z)\). If \(r\) is a pole of multiplicity \(s_r\), then \(H(z)\) has terms of the form

\[
\frac{r(j)}{1 - p(j)z^{-1}} + \frac{r(j + 1)}{(1 - p(j)z^{-1})^2} + \cdots + \frac{r(j + s_r - 1)}{(1 - p(j)z^{-1})^{s_r}}
\]

The `residuez` function in the Signal Processing Toolbox converts transfer functions to and from the partial fraction expansion form. The “z” on the end of `residuez` stands for z-domain, or discrete domain. `residuez` returns the poles in a column vector \(p\), the residues corresponding to the poles in a column vector \(r\), and any improper part of the original transfer function in a row vector \(k\). `residuez` determines that two poles are the same if the magnitude of their difference is smaller than 0.1 percent of either of the poles’ magnitudes.

Partial fraction expansion arises in signal processing as one method of finding the inverse z-transform of a transfer function. For example, the partial fraction expansion of

\[
H(z) = \frac{-4 + 8z^{-1}}{1 + 6z^{-1} + 8z^{-2}}
\]
is
\[ b = [-4\ 8]; \]
\[ a = [1\ 6\ 8]; \]
\[ [r,p,k] = \text{residuez}(b,a) \]
\[ r = \]
\[ \begin{array}{c}
-12 \\
8 
\end{array} \]
\[ p = \]
\[ \begin{array}{c}
-4 \\
-2 
\end{array} \]
\[ k = \]
\[ [] \]
corresponding to
\[ H(z) = \frac{-12}{1+4z^{-1}} + \frac{8}{1+2z^{-1}} \]

To find the inverse z-transform of \( H(z) \), find the sum of the inverse z-transforms of the two addends of \( H(z) \), giving the causal impulse response
\[ h(n) = -12(-4)^n + 8(-2)^n, \quad n = 0, 1, 2, ... \]

To verify this in MATLAB
\[ \text{imp} = [1\ 0\ 0\ 0\ 0]; \]
\[ \text{resptf} = \text{filter}(b,a,\text{imp}) \]
\[ \text{resptf} = \]
\[ \begin{array}{cccccc}
-4 & 32 & -160 & 704 & -2944 
\end{array} \]
\[ \text{respres} = \text{filter}(r(1),[1 -p(1)],\text{imp}) + \text{filter}(r(2),[1 -p(2)],\text{imp}) \]
\[ \text{respres} = \]
\[ \begin{array}{cccccc}
-4 & 32 & -160 & 704 & -2944 
\end{array} \]

**Second-Order Sections (SOS)**
Any transfer function \( H(z) \) has a second-order sections representation
\[ H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{p_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}} \]

where \( L \) is the number of second-order sections that describe the system.
MATLAB represents the second-order section form of a discrete-time system as an \( L \)-by-6 array \( \text{sos} \). Each row of \( \text{sos} \) contains a single second-order section, where the row elements are the three numerator and three denominator coefficients that describe the second-order section.

\[
\text{sos} = \begin{bmatrix}
  b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\
  b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L}
\end{bmatrix}
\]

There are an uncountable number of ways to represent a filter in second-order section form. Through careful pairing of the pole and zero pairs, ordering of the sections in the cascade, and multiplicative scaling of the sections, it is possible to reduce quantization noise gain and avoid overflow in some fixed-point filter implementations. The functions \text{zp2sos} and \text{ss2sos}, described later in “Linear System Transformations,” perform pole-zero pairing, section scaling, and section ordering.

\textbf{Lattice Structure}

For a discrete \( N \)-th order all-pole or all-zero filter described by the polynomial coefficients \( a(n), n = 1,2,...,N+1 \), there are \( N \) corresponding lattice structure coefficients \( k(n), n = 1,2,...,N \). The parameters \( k(n) \) are also called the reflection
coefficients of the filter. Given these reflection coefficients, you can implement a discrete filter as

For a general pole-zero IIR filter described by polynomial coefficients $a$ and $b$, there are both lattice coefficients $k(n)$ for the denominator $a$ and ladder coefficients $v(n)$ for the numerator $b$. The lattice/ladder filter may be implemented as
The toolbox function `tf2latc` accepts an FIR or IIR filter in polynomial form and returns the corresponding reflection coefficients. An example IIR filter in polynomial form is

```
a = [1.0000  0.6149  0.9899  0.0000  0.0031   -0.0082];
```

This filter's lattice (reflection coefficient) representation is

```
k = tf2latc(a)
k =
0.3090
0.9800
0.0031
0.0081
-0.0082
```

The magnitude of the reflection coefficients provides an easy stability check for a filter. If all the reflection coefficients corresponding to a polynomial have magnitude less than 1, all of that polynomial's roots are inside the unit circle.

The function `latc2tf` calculates the polynomial coefficients for a filter from its lattice (reflection) coefficients. Given the reflection coefficient vector `k` (above), the corresponding polynomial form is

```
a = latc2tf(k)
a =
1.0000  0.6149  0.9899  -0.0000  0.0031  -0.0082
```

The lattice or lattice/ladder coefficients can be used to implement the filter using the function `latcfilt`.

**Convolution Matrix**

In signal processing, convolving two vectors or matrices is equivalent to filtering one of the input operands by the other. This relationship permits the representation of a digital filter as a convolution matrix.

Given any vector, the toolbox `convmtx` function generates a matrix whose inner product with another vector is equivalent to the convolution of the two vectors. The generated matrix represents a digital filter that you can apply to any
vector of appropriate length; the inner dimension of the operands must agree to compute the inner product.

The convolution matrix for a vector $b$, representing the numerator coefficients for a digital filter, is

$$b = [1 \ 2 \ 3]; \ x = \text{randn}(3,1);$$

$$C = \text{convmtx}(b',3)$$

$$C =$$

$$\begin{bmatrix}
1 & 0 & 0 \\
2 & 1 & 0 \\
3 & 2 & 1 \\
0 & 3 & 2 \\
0 & 0 & 3
\end{bmatrix}$$

Two ways to convolve $b$ with $x$ are

$$y_1 = C*x;$$

$$y_2 = \text{conv}(b,x);$$

Type this example into MATLAB; the results for $y_1$ and $y_2$ are equal.

**Continuous-Time System Models**

The continuous-time system models are representational schemes for analog filters. Many of the discrete-time system models described earlier are also appropriate for the representation of continuous-time systems:

- State-space form
- Partial fraction expansion
- Transfer function
- Zero-pole-gain form

It is possible to represent any system of linear time-invariant differential equations as a set of first-order differential equations. In matrix or state-space form, you can express the equations as

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$
where \( u \) is a vector of \( n_u \) inputs, \( x \) is an \( n_x \)-element state vector, and \( y \) is a vector of \( n_y \) outputs. In MATLAB, store \( A, B, C, \) and \( D \) in separate rectangular arrays.

An equivalent representation of the state-space system is the Laplace transform transfer function description

\[
Y(s) = H(s)U(s)
\]

where

\[
H(s) = C(sI - A)^{-1}B + D
\]

For single-input, single-output systems, this form is given by

\[
H(s) = \frac{b(s)}{a(s)} = \frac{b(1)s^{nb} + b(2)s^{nb-1} + \ldots + b(nb + 1)}{a(1)s^{na} + a(2)s^{na-1} + \ldots + a(na + 1)}
\]

Given the coefficients of a Laplace transform transfer function, \texttt{residue} determines the partial fraction expansion of the system. See the description of \texttt{residue} in the MATLAB Language Reference Manual for details.

The factored zero-pole-gain form is

\[
H(s) = \frac{Z(s)}{P(s)} = k\frac{(s-z(1))(s-z(2))\cdots(s-z(n))}{(s-p(1))(s-p(2))\cdots(s-p(n))}
\]

As in the discrete-time case, MATLAB stores polynomial coefficients in row vectors in descending powers of \( s \). MATLAB stores polynomial roots, or zeros and poles, in column vectors.

**Linear System Transformations**

The Signal Processing Toolbox provides a number of functions that convert between the various linear system models; see the reference description in Chapter 6 for a complete description of each. You can use the following chart to find an appropriate transfer function: find the row of the model to convert from on the left side of the chart and the column of the model to convert to on the top of the chart and read the function name(s) at the intersection of the row and column.
Many of the toolbox filter design functions use these functions internally. For example, the \texttt{zp2ss} function converts the poles and zeros of an analog prototype into the state-space form required for creation of a Butterworth, Chebyshev, or elliptic filter. Once in state-space form, the filter design function performs any required frequency transformation, that is, it transforms the initial lowpass design into a bandpass, highpass, or bandstop filter, or a lowpass filter with the desired cutoff frequency. See Chapter 6 and the reference descriptions of the individual filter design functions for more details.
Discrete Fourier Transform

The discrete Fourier transform, or DFT, is the primary tool of digital signal processing. The foundation of the Signal Processing Toolbox is the Fast Fourier Transform (FFT), a method for computing the DFT with reduced execution time. Many of the toolbox functions (including z-domain frequency response, spectrum and cepstrum analysis, and some filter design and implementation functions) incorporate the FFT.

MATLAB provides the functions `fft` and `ifft` to compute the discrete Fourier transform and its inverse, respectively. For the input sequence `x` and its transformed version `X` (the discrete-time Fourier transform at equally spaced frequencies around the unit circle), the two functions implement the relationships:

\[
X(k + 1) = \sum_{n=0}^{N-1} x(n + 1)W_n^{kn}
\]

\[
x(n + 1) = \frac{1}{N} \sum_{k=0}^{N-1} X(k + 1)W_n^{-kn}
\]

In these equations, the series subscripts begin with 1 instead of 0 because of MATLAB’s vector indexing scheme, and

\[
W_N = e^{-j(2\pi \frac{k}{N})}
\]

**NOTE** MATLAB uses a negative `j` for the `fft` function. This is an engineering convention; physics and pure mathematics typically use a positive `j`.

`fft`, with a single input argument `x`, computes the DFT of the input vector or matrix. If `x` is a vector, `fft` computes the DFT of the vector; if `x` is a rectangular array, `fft` computes the DFT of each array column.
For example, create a time vector and signal.
\[
t = (0:1/99:1); \quad \% \text{time vector}
\]
\[
x = \sin(2\pi \times 15 \times t) + \sin(2\pi \times 40 \times t); \quad \% \text{signal}
\]
The DFT of the signal, and the magnitude and phase of the transformed sequence, are then
\[
y = \text{fft}(x); \quad \% \text{Compute DFT of } x.
\]
\[
m = \text{abs}(y); \quad p = \text{unwrap} (\text{angle}(y)); \quad \% \text{mag. and phase}
\]
To plot the magnitude and phase
\[
f = (0: \text{length}(y)-1) \times 99 / \text{length}(y); \quad \% \text{frequency vector}
\]
\[
\text{plot}(f,m)
\]
\[
\text{set(gca,'XTick',[15 40 60 85]);}
\]
\[
\text{plot}(f,p \times 180 / \pi)
\]
\[
\text{set(gca,'XTick',[15 40 60 85]);}
\]

A second argument to \texttt{fft} specifies a number of points \( n \) for the transform, representing DFT length.
\[
y = \text{fft}(x,n);
\]
In this case, \texttt{fft} pads the input sequence with zeros if it is shorter than \( n \), or truncates the sequence if it is longer than \( n \). If \( n \) is not specified, it defaults to the length of the input sequence.
Execution time for `fft` depends on the length `n` of the DFT it performs:

- For any `n` that is a power of two, `fft` uses the high-speed radix-2 algorithm. This results in the fastest execution time. Additionally, the algorithm for power of two `n` is highly optimized for real `x`, providing a 40% speed-up over the complex case.

- For any composite number `n` that is not a power of two, `fft` uses a prime factor algorithm. The speed of this algorithm depends on both the size of `n` and number of prime factors it has. Although 1013 and 1000 are close in magnitude, `fft` transforms a sequence of length 1000 much more quickly than a sequence of length 1013.

- For a prime number `n`, `fft` cannot use an FFT algorithm, and instead performs the slower, computation-intensive DFT directly.

The inverse discrete Fourier transform function `ifft` also accepts an input sequence and, optionally, the number of desired points for the transform. Try the example below; the original sequence `x` and the reconstructed sequence are identical (within rounding error).

```matlab
t = (0:1/255:1);
x = sin(2*pi*120*t);
y = real(ifft(fft(x)));
```

This toolbox also includes functions for the two-dimensional FFT and its inverse, `fft2` and `ifft2`. These functions are useful for two-dimensional signal or image processing; see the reference descriptions in Chapter 6 for details.

It is sometimes convenient to rearrange the output of the `fft` or `fft2` function so the zero frequency component is at the center of the sequence. The MATLAB function `fftshift` moves the zero frequency component to the center of a vector or matrix.
References

Algorithm development for the Signal Processing Toolbox has drawn heavily upon the references listed below. All are recommended to the interested reader who needs to know more about signal processing than is covered in this manual.


Filter Design

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Filter Requirements and Specification

The Signal Processing Toolbox provides functions that support a range of filter design methodologies. This chapter explains how to apply the filter design tools to Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) filter design problems.

The goal of filter design is to perform frequency dependent alteration of a data sequence. A possible requirement might be to remove noise above 30 Hz from a data sequence sampled at 100 Hz. A more rigorous specification might call for a specific amount of passband ripple, stopband attenuation, or transition width. A very precise specification could ask to achieve the performance goals with the minimum filter order, or it could call for an arbitrary magnitude shape, or it might require an FIR filter.

Filter design methods differ primarily in how performance is specified. For "loosely specified" requirements, as in the first case above, a Butterworth IIR filter is often sufficient. To design a fifth-order 30 Hz lowpass Butterworth filter and apply it to the data in vector \( x \),

\[
[b,a] = \text{butter}(5,30/50);
y = \text{filter}(b,a,x);
\]

The second input argument to \( \text{butter} \) indicates the cutoff frequency, normalized to half the sampling frequency (the Nyquist frequency).

Frequency Normalization in the Signal Processing Toolbox

All of the filter design functions operate with normalized frequencies, so they do not require the system sampling rate as an extra input argument. This toolbox uses the convention that unit frequency is the Nyquist frequency, defined as half the sampling frequency. The normalized frequency, therefore, is always in the interval \( 0 \leq f \leq 1 \). For a system with a 1000 Hz sampling frequency, 300 Hz is \( 300/500 = 0.6 \). To convert normalized frequency to angular frequency around the unit circle, multiply by \( \pi \). To convert normalized frequency back to Hertz, multiply by half the sample frequency.
More rigorous filter requirements traditionally include passband ripple ($R_p$, in decibels), stopband attenuation ($R_s$, in decibels), and transition width ($W_s–W_p$, in Hertz).

You can design Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filters that meet this type of performance specification. The toolbox order selection functions estimate the minimum filter order that meets a given set of requirements.

To meet specifications with more rigid constraints like linear phase or arbitrary filter shape, use the FIR and direct IIR filter design routines.
IIR Filter Design

The primary advantage of IIR filters over FIR filters is that they typically meet a given set of specifications with a much lower filter order than a corresponding FIR filter. Although IIR filters have nonlinear phase, data processing within MATLAB is commonly performed “off-line,” that is, the entire data sequence is available prior to filtering. This allows for a noncausal, zero-phase filtering approach (via the `filtfilt` function), which eliminates the nonlinear phase distortion of an IIR filter.

The classical IIR filters, Butterworth, Chebyshev types I and II, elliptic, and Bessel, all approximate the ideal “brickwall” filter in different ways. This toolbox provides functions to create all these types of classical IIR filters in both the analog and digital domains (except Bessel, for which only the analog case is supported), and in lowpass, highpass, bandpass, and bandstop configurations. For most filter types, you can also find the lowest filter order that fits a given filter specification in terms of passband and stopband attenuation, and transition width(s).

The direct filter design function `yulewalk` finds a filter with magnitude response approximating a desired function. This is one way to create a multiband bandpass filter.

You can also use the parametric modeling or system identification functions to design IIR filters. These functions are discussed in the “Parametric Modeling” section of Chapter 4.

The generalized Butterworth design function `maxflat` is discussed in the section “Generalized Butterworth Filter Design” on page 2-14.

The following table summarizes the various filter methods in the toolbox and lists the functions available to implement these methods.
## IIR Filter Design

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Prototyping</td>
<td>Using the poles and zeros of a classical lowpass prototype filter in the continuous (Laplace) domain, obtain a digital filter through frequency transformation and filter discretization.</td>
<td>Complete design functions: [ \text{besself, butter, cheby1, cheby2, ellip} ] Order estimation functions: [ \text{buttord, cheb1ord, cheb2ord, ellipord} ] Lowpass analog prototype functions: [ \text{besselap, buttap, cheb1ap, cheb2ap, ellipap} ] Frequency transformation functions: [ \text{lp2bp, lp2bs, lp2hp, lp2lp} ] Filter discretization functions: [ \text{bilinear, impinvar} ]</td>
</tr>
<tr>
<td>Direct Design</td>
<td>Design digital filter directly in the discrete domain by approximating a piecewise linear magnitude response.</td>
<td>yulewalk</td>
</tr>
<tr>
<td>Parametric Modeling*</td>
<td>Find a digital filter that approximates a prescribed time or frequency domain response.</td>
<td>Time-domain modeling functions: [ \text{lpc, prony, stmcb} ] Frequency-domain modeling functions: [ \text{invfreqs, invfreqz} ]</td>
</tr>
<tr>
<td>Generalized Butterworth Design</td>
<td>Design lowpass Butterworth filters with more zeros than poles.</td>
<td>maxflat</td>
</tr>
</tbody>
</table>

* See the System Identification Toolbox for an extensive collection of parametric modeling tools.
Classical IIR Filter Design Using Analog Prototyping

The principal IIR digital filter design technique this toolbox provides is based on the conversion of classical lowpass analog filters to their digital equivalents. The following sections describe how to design filters and summarize the characteristics of the supported filter types. See “Special Topics in IIR Filter Design” on page 2-37 for detailed steps on the filter design process.

Complete Classical IIR Filter Design

You can easily create a filter of any order with a lowpass, highpass, bandpass, or bandstop configuration using the filter design functions.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Design Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butterworth</td>
<td>[b,a] = butter(n,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[z,p,k] = butter(n,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[A,B,C,D] = butter(n,Wn,options)</td>
</tr>
<tr>
<td>Chebyshev type I</td>
<td>[b,a] = cheby1(n,Rp,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[z,p,k] = cheby1(n,Rp,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[A,B,C,D] = cheby1(n,Rp,Wn,options)</td>
</tr>
<tr>
<td>Chebyshev type II</td>
<td>[b,a] = cheby2(n,Rs,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[z,p,k] = cheby2(n,Rs,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[A,B,C,D] = cheby2(n,Rs,Wn,options)</td>
</tr>
<tr>
<td>Elliptic</td>
<td>[b,a] = ellip(n,Rp,Rs,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[z,p,k] = ellip(n,Rp,Rs,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[A,B,C,D] = ellip(n,Rp,Rs,Wn,options)</td>
</tr>
<tr>
<td>Bessel (analog only)</td>
<td>[b,a] = besself(n,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[z,p,k] = besself(n,Wn,options)</td>
</tr>
<tr>
<td></td>
<td>[A,B,C,D] = besself(n,Wn,options)</td>
</tr>
</tbody>
</table>

By default, each of these functions returns a lowpass filter; you need only specify the desired cutoff frequency Wn in normalized frequency (Nyquist frequency = 1 Hz). For a highpass filter, append the string 'high' to the function’s parameter list. For a bandpass or bandstop filter, specify Wn as a two-element vector containing the passband edge frequencies, appending the string 'stop' for the bandstop configuration.
Here are some example digital filters.

\[ b, a = \text{butter}(5, 0.4); \] % lowpass Butterworth
\[ b, a = \text{cheby1}(4, 1, [0.4, 0.7]); \] % bandpass Chebyshev type I
\[ b, a = \text{cheby2}(6, 60, 0.8, \text{\texttt{'high'}}); \] % highpass Chebyshev type II
\[ b, a = \text{ellip}(3, 1, 60, [0.4, 0.7], \text{\texttt{'stop'}}); \] % bandstop elliptic

To design an analog filter, perhaps for simulation, use a trailing \texttt{'}s\texttt{'} and specify cutoff frequencies in radians/second.

\[ b, a = \text{butter}(5, 0.4, \text{\texttt{'s'}}); \] % analog Butterworth filter

All filter design functions return a filter in the transfer function, zero-pole-gain, or state-space linear system model representation, depending on how many output arguments are present.

\textbf{NOTE} All classical IIR lowpass filters are ill-conditioned for extremely low cut-off frequencies. Therefore, instead of designing a lowpass IIR filter with a very narrow passband, it can be better to design a wider passband and decimate the input signal.

\textbf{Designing IIR Filters to Frequency Domain Specifications}
This toolbox provides order selection functions that calculate the minimum filter order that meets a given set of requirements.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Order Estimation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butterworth</td>
<td>[ n, Wn = \text{buttord}(Wp, Ws, Rp, Rs) ]</td>
</tr>
<tr>
<td>Chebyshev type I</td>
<td>[ n, Wn = \text{cheb1ord}(Wp, Ws, Rp, Rs) ]</td>
</tr>
<tr>
<td>Chebyshev type II</td>
<td>[ n, Wn = \text{cheb2ord}(Wp, Ws, Rp, Rs) ]</td>
</tr>
<tr>
<td>Elliptic</td>
<td>[ n, Wn = \text{ellipord}(Wp, Ws, Rp, Rs) ]</td>
</tr>
</tbody>
</table>

These are useful in conjunction with the filter design functions. Suppose you want a bandpass filter with a passband from 1000 to 2000 Hz, stopbands starting 500 Hz away on either side, a 10 kHz sampling frequency, at most 1 dB
of passband ripple, and at least 60 dB of stopband attenuation. To meet these specifications with the `butter` function:

\[
[n,Wn] = \text{buttord}([1000 \ 2000]/5000,[500 \ 2500]/5000,1,60)
\]

\[
n = 12
\]

\[
Wn = 0.1951 \quad 0.4080
\]

\[
[b,a] = \text{butter}(n,Wn);
\]

An elliptic filter that meets the same requirements is given by

\[
[n,Wn] = \text{ellipord}([1000 \ 2000]/5000,[500 \ 2500]/5000,1,60)
\]

\[
n = 5
\]

\[
Wn = 0.2000 \quad 0.4000
\]

\[
[b,a] = \text{ellip}(n,1,60,Wn);
\]

These functions also work with the other standard band configurations, as well as for analog filters; see Chapter 6 for details.

**Comparison of Classical IIR Filter Types**

The toolbox provides five different types of classical IIR filters, each optimal in some way. This section shows the basic analog prototype form for each and summarizes major characteristics.

**Butterworth Filter**

The Butterworth filter provides the best Taylor Series approximation to the ideal lowpass filter response at \(\Omega = 0\) and \(\Omega = \infty\); for any order \(N\), the magnitude squared response has \(2N-1\) zero derivatives at these locations (maximally flat
at $\Omega = 0$ and $\Omega = \infty$. Response is monotonic overall, decreasing smoothly from $\Omega = 0$ to $\Omega = \infty$. $|H(j\Omega)| = \sqrt{1/2}$ at $\Omega = 1$.

Chebyshev Type I Filter

The Chebyshev type I filter minimizes the absolute difference between the ideal and actual frequency response over the entire passband by incorporating an equal ripple of $R_p$ dB in the passband. Stopband response is maximally flat. The transition from passband to stopband is more rapid than for the Butterworth filter. $|H(j\Omega)| = 10^{R_p/20}$ at $\Omega = 1$. 
**Chebyshev Type II Filter**

The Chebyshev type II filter minimizes the absolute difference between the ideal and actual frequency response over the entire stopband, by incorporating an equal ripple of $Rs$ dB in the stopband. Passband response is maximally flat.

The stopband does not approach zero as quickly as the type I filter (and does not approach zero at all for even-valued $n$). The absence of ripple in the passband, however, is often an important advantage. $|H(j\Omega)| = 10^{-Rs/20}$ at $\Omega = 1$.

**Elliptic Filter**

Elliptic filters are equiripple in both the passband and stopband. They generally meet filter requirements with the lowest order of any supported filter type. Given a filter order $n$, passband ripple $Rp$ in decibels, and stopband ripple $Rs$ dB.

![Frequency Response Graph](image)
Rs in decibels, elliptic filters minimize transition width. \(|H(j\Omega)| = 10^{-Rp/20}\) at \(\Omega = 1\).

**Bessel Filter**

Analog Bessel lowpass filters have maximally flat group delay at zero frequency and retain nearly constant group delay across the entire passband. Filtered signals therefore maintain their waveshapes in the passband frequency range. Frequency mapped and digital Bessel filters, however, do not have this maximally flat property; this toolbox supports only the analog case for the complete Bessel filter design function.
Bessel filters generally require a higher filter order than other filters for satisfactory stopband attenuation. $|H(j\Omega)| < 1/\sqrt{2}$ at $\Omega = 1$ and decreases as $n$ increases.

**NOTE** The lowpass filters shown above were created with the analog prototype functions `besselap`, `buttap`, `cheb1ap`, `cheb2ap`, and `ellipap`. These functions find the zeros, poles, and gain of an order $n$ analog filter of the appropriate type with cutoff frequency of 1 rad/sec. The complete filter design functions (besself, butter, cheby1, cheby2, and ellip) call the prototyping functions as a first step in the design process. See “Special Topics in IIR Filter Design” on page 2-37 for details.

To create similar plots, use $n = 5$ and, as needed, $R_p = 0.5$ and $R_s = 20$. For example, to create the elliptic filter plot:

```matlab
[z, p, k] = ellipap(5, 0.5, 20);
w = logspace(-1, 1, 1000);
h = freqs(k*poly(z), poly(p), w);
semilogx(w, abs(h)), grid
```
Direct IIR Filter Design

This toolbox uses the term *direct methods* to describe techniques for IIR design that find a filter based on specifications in the discrete domain. Unlike the analog prototyping method, direct design methods are not constrained to the standard lowpass, highpass, bandpass, or bandstop configurations. Rather, these functions design filters with an arbitrary, perhaps multiband, frequency response. This section discusses the `yulewalk` function, which is intended specifically for filter design; “Parametric Modeling” in Chapter 4 discusses other methods that may also be considered direct, such as Prony’s method, Linear Prediction, the Steiglitz-McBride method, and inverse frequency design.

`yulewalk` designs recursive IIR digital filters by fitting a specified frequency response. `yulewalk`’s name reflects its method for finding the filter’s denominator coefficients: it finds the inverse FFT of the ideal desired power spectrum and solves the “modified Yule-Walker equations” using the resulting autocorrelation function samples. The statement

```matlab
[b,a] = yulewalk(n,f,m)
```

returns row vectors `b` and `a` containing the `n+1` numerator and denominator coefficients of the order `n` IIR filter whose frequency-magnitude characteristics approximate those given in vectors `f` and `m`. `f` is a vector of frequency points ranging from 0 to 1, where 1 represents the Nyquist frequency. `m` is a vector containing the desired magnitude response at the points in `f`. `f` and `m` can describe any piecewise linear shape magnitude response, including a multiband response. The FIR counterpart of this function is `fir2`, which also designs a filter based on an arbitrary piecewise linear magnitude response. See “FIR Filter Design” on page 2-16 for details.

Note that `yulewalk` does not accept phase information, and no statements are made about the optimality of the resulting filter.
Design a multiband filter with `yulewalk`, and plot the desired and actual frequency response.

```matlab
m = [0 0 1 1 0 0 1 1 0 0];
f = [0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 1];
[b,a] = yulewalk(10,f,m);
[h,w] = freqz(b,a,128);
plot(f,m,w/pi,abs(h))
```

**Generalized Butterworth Filter Design**

The toolbox function `maxflat` enables you to design generalized Butterworth filters, that is, Butterworth filters with differing numbers of zeros and poles. This is desirable in some implementations where poles are more expensive computationally than zeros. `maxflat` is just like the `butter` function, except that it you can specify two orders (one for the numerator and one for the denominator) instead of just one. These filters are maximally flat. This means that the resulting filter is optimal for any numerator and denominator orders, with the maximum number of derivatives at 0 and the Nyquist frequency $\omega = \pi$ both set to 0.
For example, when the two orders are the same, `maxflat` is the same as `butter`.

```matlab
[b,a] = maxflat(3,3,0.25)
```

```
b = 0.0317  0.0951  0.0951  0.0317
a = 1.0000 -1.4590  0.9104 -0.1978
```

```matlab
[b,a] = butter(3,0.25)
```

```
b = 0.0317  0.0951  0.0951  0.0317
a = 1.0000 -1.4590  0.9104 -0.1978
```

However, `maxflat` is more versatile, because you can design a filter with more zeros than poles.

```matlab
[b,a] = maxflat(3,1,0.25)
```

```
b = 0.0950  0.2849  0.2849  0.0950
a = 1.0000  -0.2402
```

The third input to `maxflat` is the half-power frequency, a frequency between 0 and 1 with a desired magnitude response of \( 1/\sqrt{2} \).

You can also design linear phase filters that have the maximally flat property using the `'sym'` option.

```matlab
maxflat(4,'sym',0.3)
```

```
ans = 0.0331  0.2500  0.4337  0.2500  0.0331
```

For complete details of the `maxflat` algorithm, see Selesnick and Burrus [2].
FIR Filter Design

Digital filters with finite-duration impulse response (all-zero, or FIR filters) have both advantages and disadvantages compared to infinite-duration impulse response (IIR) filters.

FIR filters have the following primary advantages:

- They can have exactly linear phase.
- They are always stable.
- The design methods are generally linear.
- They can be realized efficiently in hardware.
- The filter startup transients have finite duration.

The primary disadvantage of FIR filters is that they often require a much higher filter order than IIR filters to achieve a given level of performance. Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filter.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windowing</td>
<td>Apply window to truncated inverse Fourier transform of desired “brickwall” filter</td>
<td>fir1,fir2,kaiserord</td>
</tr>
<tr>
<td>Multiband with Transition Bands</td>
<td>Equiripple or least squares approach over sub-bands of the frequency range</td>
<td>fir1s,remez,remezord</td>
</tr>
<tr>
<td>Constrained Least Squares</td>
<td>Minimize squared integral error over entire frequency range subject to maximum error constraints</td>
<td>fircls,fircls1</td>
</tr>
</tbody>
</table>
Linear Phase Filters

Except for `cremez`, all of the FIR filter design functions design linear phase filters only. The filter coefficients, or “taps,” of such filters obey either an even or odd symmetry relation. Depending on this symmetry, and on whether the order \( n \) of the filter is even or odd, a linear phase filter (stored in length \( n+1 \) vector \( b \)) has certain inherent restrictions on its frequency response.

### Linear Phase Filters

<table>
<thead>
<tr>
<th>Linear Phase Filter Type</th>
<th>Filter Order ( n )</th>
<th>Symmetry of Coefficients</th>
<th>Response ( H(f) ), ( f = 0 )</th>
<th>Response ( H(f) ), ( f = 1 ) (Nyquist)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>Even</td>
<td>even: ( b(k) = b(n + 2 - k) ), ( k = 1, \ldots, n + 1 )</td>
<td>No restriction</td>
<td>No restriction</td>
</tr>
<tr>
<td>Type II</td>
<td>Odd</td>
<td>No restriction</td>
<td>( H(1) = 0 )</td>
<td>( H(1) = 0 )</td>
</tr>
<tr>
<td>Type III</td>
<td>Even</td>
<td>odd: ( b(k) = -b(n + 2 - k) ), ( k = 1, \ldots, n + 1 )</td>
<td>( H(0) = 0 )</td>
<td>( H(1) = 0 )</td>
</tr>
<tr>
<td>Type IV</td>
<td>Odd</td>
<td>No restriction</td>
<td>( H(0) = 0 )</td>
<td>No restriction</td>
</tr>
</tbody>
</table>

The phase delay and group delay of linear phase FIR filters are equal and constant over the frequency band. For an order \( n \) linear phase FIR filter, the group delay is \( n/2 \), and the filtered signal is simply delayed by \( n/2 \) time steps (and the magnitude of its Fourier transform is scaled by the filter’s magnitude response). This property preserves the wave shape of signals in the passband, that is, there is no phase distortion.

The functions `fir1`, `fir2`, `firls`, `remez`, `fircls`, `fircls1`, and `firrcos` all design type I and II linear phase FIR filters by default. Both `firls` and `remez` design type III and IV linear phase FIR filters given a ‘hilbert’ or ‘differentiator’ flag. `cremez` can design any type of linear phase filter, and nonlinear phase filters as well.
NOTE  Because the frequency response of a type II filter is zero at the Nyquist rate ("high" frequency), \texttt{fir1} does not design type II highpass and bandstop filters. For odd-valued \(n\) in these cases, \texttt{fir1} adds 1 to the order and returns a type I filter.

Windowing Method
Consider the ideal, or "brick-wall," digital lowpass filter with a cutoff frequency of \(\omega_0\) rad/sec. This filter has magnitude 1 at all frequencies with magnitude less than \(\omega_0\), and magnitude 0 at frequencies with magnitude between \(\omega_0\) and \(\pi\). Its impulse response sequence \(h(n)\) is

\[
h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} e^{j\omega n} d\omega = \frac{\omega_0}{\pi} \sin(\frac{\omega_0}{\pi} n)
\]

This filter is not implementable since its impulse response is infinite and noncausal. To create a finite-duration impulse response, truncate it by applying a window. By retaining the central section of impulse response in this truncation, you obtain a linear phase FIR filter. For example, a length 51 filter with a lowpass cutoff frequency \(\omega_0\) of 0.4\(\pi\) rad/sec is

\[
b = 0.4\cdot\text{sinc}(0.4\cdot(-25:25));
\]

The window applied here is a simple rectangular or "boxcar" window. By Parseval’s theorem, this is the length 51 filter that best approximates the ideal...
lowpass filter, in the integrated least squares sense. To view its frequency response:

\[
[H,w] = \text{freqz}(b,1,512,2);
\]
\[
\text{plot}(w,\text{abs}(H)), \text{ grid}
\]

Note the ringing and ripples in the response, especially near the band edge. This "Gibbs effect" does not vanish as the filter length increases, but a nonrectangular window reduces its magnitude. Multiplication by a window in
the time domain causes a convolution or smoothing in the frequency domain. Apply a length 51 Hamming window to the filter.

```matlab
b = b.*hamming(51)';
[H,w] = freqz(b,1,512,2);
plot(w,abs(H)), grid
```

As you can see, this greatly reduces the ringing. This improvement is at the expense of transition width (the windowed version takes longer to ramp from passband to stopband) and optimality (the windowed version does not minimize the integrated squared error).

The functions `fir1` and `fir2` are based on this windowing process. Given a filter order and description of an ideal desired filter, these functions return a windowed inverse Fourier transform of that ideal filter. Both use a Hamming window by default, but they accept any window function. See the “Windows” section of Chapter 4 for an overview of windows and their properties.

**Standard Band FIR Filter Design: fir1**

`fir1` implements the classical method of windowed linear phase FIR digital filter design. It resembles the IIR filter design functions in that it is formulated to design filters in standard band configurations: lowpass, bandpass, highpass, and bandstop.
The statements
\[
\begin{align*}
n &= 50; \\
Wn &= 0.4; \\
b &= \text{fir1}(n,Wn);
\end{align*}
\]
create row vector \(b\) containing the coefficients of the order \(n\) Hamming-windowed filter. This is a lowpass, linear phase FIR filter with cutoff frequency \(Wn\). \(Wn\) is a number between 0 and 1, where 1 corresponds to the Nyquist frequency, half the sampling frequency. (Unlike other methods, here \(Wn\) corresponds to the 6 dB point.) For a highpass filter, simply append the string 'high' to the function's parameter list. For a bandpass or bandstop filter, specify \(Wn\) as a two-element vector containing the passband edge frequencies; append the string 'stop' for the bandstop configuration.

\[
b = \text{fir1}(n,Wn,\text{window})
\]
uses the window specified in column vector \(\text{window}\) for the design. The vector \(\text{window}\) must be \(n+1\) elements long. If you do not specify a window, \text{fir1} applies a Hamming window.

**Kaiser Window Order Estimation.** The \text{kaiserord} function estimates the filter order, cutoff frequency, and Kaiser window beta parameter needed to meet a given set of specifications. Given a vector of frequency band edges and a corresponding vector of magnitudes, as well as maximum allowable ripple, \text{kaiserord} returns appropriate input parameters for the \text{fir1} function. For details on \text{kaiserord}, see the reference description in Chapter 6.

**Multiband FIR Filter Design: fir2**
The function \text{fir2} also designs windowed FIR filters, but with an arbitrarily shaped piecewise linear frequency response. This is in contrast to \text{fir1}, which only designs filters in standard lowpass, highpass, bandpass, and bandstop configurations.

The commands
\[
\begin{align*}
n &= 50; \\
f &= [0 \ 0.4 \ 0.5 \ 1]; \\
m &= [1 \ 1 \ 0 \ 0]; \\
b &= \text{fir2}(n,f,m);
\end{align*}
\]
return row vector \(b\) containing the \(n+1\) coefficients of the order \(n\) FIR filter whose frequency-magnitude characteristics match those given by vectors \(f\) and \(m\). \(f\) is a vector of frequency points ranging from 0 to 1, where 1 represents the
Nyquist frequency. \( m \) is a vector containing the desired magnitude response at the points specified in \( f \). (The IIR counterpart of this function is \texttt{yulewalk}, which also designs filters based on arbitrary piecewise linear magnitude responses. See “IIR Filter Design” for details.)

**Multiband FIR Filter Design with Transition Bands**

The \texttt{firls} and \texttt{remez} functions provide a more general means of specifying the ideal desired filter than the \texttt{fir1} and \texttt{fir2} functions. These functions design Hilbert transformers, differentiators, and other filters with odd symmetric coefficients (type III and type IV linear phase). They also let you include transition or “don’t care” regions in which the error is not minimized, and perform band dependent weighting of the minimization.

\texttt{firls} is an extension of the \texttt{fir1} and \texttt{fir2} functions in that it minimizes the integral of the square of the error between the desired frequency response and the actual frequency response.

\texttt{remez} implements the Parks-McClellan algorithm, which uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with optimal fits between the desired and actual frequency responses. The filters are optimal in the sense that they minimize the maximum error between the desired frequency response and the actual frequency response; they are sometimes called minimax filters. Filters designed in this way exhibit an equiripple behavior in their frequency response, and hence are also known as equiripple filters. The Parks-McClellan FIR filter design algorithm is perhaps the most popular and widely used FIR filter design methodology.

The syntax for \texttt{firls} and \texttt{remez} is the same; the only difference is their minimization schemes. The next example shows how filters designed with \texttt{firls} and \texttt{remez} reflect these different schemes.

**Basic Configurations**

The default mode of operation of \texttt{firls} and \texttt{remez} is to design type I or type II linear phase filters, depending on whether the order you desire is even or odd, respectively. A lowpass example with approximate amplitude 1 from 0 to 0.4 Hz, and approximate amplitude 0 from 0.5 to 1.0 Hz is

```matlab
n = 20;            % filter order
f = [0 .4 .5 1];   % frequency band edges
a = [1 1 0 0];     % desired amplitudes
b = remez(n,f,a);
```
From 0.4 to 0.5 Hz, `remez` performs no error minimization; this is a transition band or “don’t care” region. A transition band minimizes the error more in the bands that you do care about, at the expense of a slower transition rate. In this way, these types of filters have an inherent trade-off similar to FIR design by windowing.

To compare least squares to equiripple filter design, use `firls` to create a similar filter.

```matlab
bb = firls(n,f,a);
```

and compare their frequency responses:

```matlab
[H,w]=freqz(b);
[HH,w]=freqz(bb);
plot(w/pi,abs(H),w/pi,abs(HH),'--'), grid
```

You can see that the filter designed with `remez` exhibits equiripple behavior. Also note that the `firls` filter has a better response over most of the passband and stopband, but at the band edges (f = 0.4 and f = 0.5), the response is further away from the ideal than the `remez` filter. This shows that the `remez` filter’s maximum error over the pass- and stopbands is smaller and, in fact, it is the smallest possible for this band edge configuration and filter length.

Think of frequency bands as lines over short frequency intervals. `remez` and `firls` use this scheme to represent any piecewise linear desired function with
any transition bands. firls and remez design lowpass, highpass, bandpass, and bandstop filters; a bandpass example is

```
f = [0 0.3 0.4 0.7 0.8 1]; % band edges in pairs
a = [0 0 1 1 0 0]; % bandpass filter amplitude
```

Technically, these \( f \) and \( a \) vectors define five bands:

- Two stopbands, from 0.0 to 0.3 and from 0.8 to 1.0
- A passband from 0.4 to 0.7
- Two transition bands, from 0.3 to 0.4 and from 0.7 to 0.8

Example highpass and bandstop filters are

```
f = [0 0.7 0.8 1]; % band edges in pairs
a = [0 0 1 1]; % highpass filter amplitude
```

```
f = [0 0.3 0.4 0.5 0.8 1]; % band edges in pairs
a = [1 1 0 0 1 1]; % bandstop filter amplitude
```

An example multiband bandpass filter is

```
f = [0 0.1 0.15 0.25 0.3 0.4 0.45 0.55 0.6 0.7 0.75 0.85 0.9 1];
a = [1 1 0 0 1 1 0 0 1 1 0 0 1 1];
```

Another possibility is a filter that has as a transition region the line connecting the passband with the stopband; this can help control “runaway” magnitude response in wide transition regions:

```
f = [0 0.4 0.42 0.48 0.5 1];
a = [1 1 0.8 0.2 0 0]; % passband, linear transition, stopband
```

**The Weight Vector**

Both firls and remez allow you to place more or less emphasis on minimizing the error in certain frequency bands relative to others. To do this, specify a weight vector following the frequency and amplitude vectors. An example
A lowpass equiripple filter with 10 times less ripple in the stopband than the passband is

\[
\begin{align*}
n &= 20; \quad \text{% filter order} \\
f &= [0 \ 0.4 \ 0.5 \ 1]; \quad \text{% frequency band edges} \\
a &= [1 \ 1 \ 0 \ 0]; \quad \text{% desired amplitudes} \\
w &= [1 \ 10]; \quad \text{% weight vector}
\end{align*}
\]

\[
b = \text{remez}(n,f,a,w);
\]

A legal weight vector is always half the length of the \(f\) and \(a\) vectors; there must be exactly one weight per band.

**Anti-Symmetric Filters / Hilbert Transformers**

When called with a trailing 'h' or 'Hilbert' option, \texttt{remez} and \texttt{firls} design FIR filters with odd symmetry, that is, type III (for even order) or type IV (for odd order) linear phase filters. An ideal Hilbert transformer has this anti-symmetry property and an amplitude of 1 across the entire frequency range. Try the following approximate Hilbert transformers:

\[
b = \text{remez}(21,[0.05 \ 1],[1 \ 1],'h'); \quad \text{% highpass Hilbert} \\
bb = \text{remez}(20,[0.05 \ 0.95],[1 \ 1],'h'); \quad \text{% bandpass Hilbert}
\]

You can find the delayed Hilbert transform of a signal \(x\) by passing it through these filters:

\[
\begin{align*}
Fs &= 1000; \quad \text{% sampling frequency} \\
t &= (0:1/Fs:2)'; \quad \text{% two second time vector} \\
x &= \sin(2\pi t \times 300); \quad \text{% 300 Hz sine wave example signal} \\
xh &= \text{filter}(bb,1,x); \quad \text{% Hilbert transform of } x
\end{align*}
\]
The analytic signal corresponding to $x$ is the complex signal that has $x$ as its real part and the Hilbert transform of $x$ as its imaginary part. For this FIR method (an alternative to the `hilbert` function), you must delay $x$ by half the filter order to create the analytic signal.

$$xd = [\text{zeros}(10,1); x(1:length(x)-10)]; \text{ % delay } 10 \text{ samples}$$

$$xa = xd + j*xh; \text{ % analytic signal}$$

This method does not work directly for filters of odd order, which require a noninteger delay. In this case, the `hilbert` function, described in the “Specialized Transforms” section in Chapter 4, estimates the analytic signal. Alternatively, use the `resample` function to delay the signal by a noninteger number of samples.

**Differentiators**

Differentiation of a signal in the time domain is equivalent to multiplication of the signal’s Fourier transform by an imaginary ramp function. That is, to differentiate a signal, pass it through a filter that has a response $H(w) = jw$.

Approximate the ideal differentiator (with a delay) using `remez` or `firls` with a `'d'` or `'differentiator'` option.

$$b = \text{remez}(21,[0 1],[0 \text{ pi*Fs}],'d');$$

To obtain the correct derivative, scale by $\text{pi*Fs}$ rad/sec, where $\text{Fs}$ is the sampling frequency in Hertz. For a type III filter, the differentiation band should stop short of the Nyquist frequency, and the amplitude vector must reflect that change to ensure the correct slope.

$$bb = \text{remez}(20,[0 0.9],[0 0.9*\text{pi*Fs}],'d');$$

In the `'d'` mode, `remez` weights the error by $1/w$ in nonzero amplitude bands to minimize the maximum relative error. `firls` weights the error by $(1/w)^2$ in nonzero amplitude bands in the `'d'` mode.
The magnitude response plots for the differentiators shown above are

![Differentiator, odd order](image1)

![Differentiator, even order](image2)

**Constrained Least Squares FIR Filter Design**

The Constrained Least Squares (CLS) FIR filter design functions implement a technique that enables you to design FIR filters without explicitly defining the transition bands for the magnitude response. The ability to omit the specification of transition bands is useful in several situations. For example, it may not be clear where a rigidly defined transition band should appear if noise and signal information appear together in the same frequency band. Similarly, it may make sense to omit the specification of transition bands if they appear only to control the results of Gibbs phenomena that appear in the filter’s response. See Selesnick, Lang, and Burrus [2] for discussion of this method.

Instead of defining passbands, stopbands, and transition regions, the CLS method accepts a cutoff frequency (for the highpass, lowpass, bandpass, or bandstop cases), or passband and stopband edges (for multiband cases), for the desired response. In this way, the CLS method defines transition regions implicitly, rather than explicitly.

The key feature of the CLS method is that it enables you to define upper and lower thresholds that contain the maximum allowable ripple in the magnitude response. Given this constraint, the technique applies the least square error minimization technique over the frequency range of the filter’s response, instead of over specific bands. The error minimization includes any areas of discontinuity in the ideal, “brick wall” response. An additional benefit is that
the technique enables you to specify arbitrarily small peaks resulting from Gibbs' phenomena.

There are two toolbox functions that implement this design technique.

<table>
<thead>
<tr>
<th>Description</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constrained least square multiband FIR filter design.</td>
<td>fircls</td>
</tr>
<tr>
<td>Constrained least square filter design for lowpass and highpass linear phase filters</td>
<td>fircls1</td>
</tr>
</tbody>
</table>

For details on the calling syntax for these functions, see their reference descriptions in Chapter 6.

**Basic Lowpass and Highpass CLS Filter Design**

The most basic of the CLS design functions, `fircls1`, uses this technique to design lowpass and highpass FIR filters. As an example, consider designing a filter with order 61 impulse response and cutoff frequency of 0.3 (normalized). Further, define the upper and lower bounds that constrain the design process as:

- Maximum passband deviation from 1 (passband ripple) of 0.02.
- Maximum stopband deviation from 0 (stopband ripple) of 0.008.

![Diagram](attachment:image.png)
To approach this design problem using `fircls1`

```matlab
n = 61;
wo = 0.3;
dp = 0.02;
ds = 0.008;
h = fircls1(n,wo,dp,ds,'plot');
```

**Multiband CLS Filter Design**

`fircls` uses the same technique to design FIR filters with a desired piecewise constant magnitude response. In this case, you can specify a vector of band edges and a corresponding vector of band amplitudes. In addition, you can specify the maximum amount of ripple for each band.

For example, assume the specifications for a filter call for:

- From 0 to 0.3 (normalized): amplitude 0, upper bound 0.005, lower bound -0.005
- From 0.3 to 0.5: amplitude 0.5, upper bound 0.51, lower bound 0.49
- From 0.5 to 0.7: amplitude 0, upper bound 0.03, lower bound -0.03
- From 0.7 to 0.9: amplitude 1, upper bound 1.02, lower bound 0.98
- From 0.9 to 1: amplitude 0, upper bound 0.05, lower bound -0.05
Design a CLS filter with impulse response order 129 that meets these specifications.

\[ n = 129; \]
\[ f = [0 \ 0.3 \ 0.5 \ 0.7 \ 0.9 \ 1]; \]
\[ a = [0 \ 0.5 \ 0 \ 1 \ 0]; \]
\[ up = [0.005 \ 0.51 \ 0.03 \ 1.02 \ 0.05]; \]
\[ lo = [-0.005 \ 0.49 \ -0.03 \ 0.98 \ -0.05]; \]
\[ h = \text{fircls}(n,f,a,up,lo,'plot'); \]

**Weighted CLS Filter Design**

Weighted CLS filter design lets you design lowpass or highpass FIR filters with relative weighting of the error minimization in each band. The `fircls1` function enables you to specify the passband and stopband edges for the least squares weighting function, as well as a constant \( k \) that specifies the ratio of the stopband to passband weighting.

For example, consider specifications that call for an FIR filter with impulse response order of 55 and cutoff frequency of 0.3 (normalized). Also assume maximum allowable passband ripple of 0.02 and maximum allowable stopband ripple of 0.004. In addition, add weighting requirements:
• Passband edge for the weight function of 0.28 (normalized)
• Stopband edge for the weight function of 0.32
• Weight error minimization 10 times as much in the stopband as in the passband

To approach this using fircls1:

```matlab
n = 55;
wo = 0.3;
dp = 0.02;
ds = 0.004;
wp = 0.28;
ws = 0.32;
k = 10;
h = fircls1(n,wo,dp,ds,wp,ws,k,'plot');
```

Arbitrary-Response Filter Design

The `cremez` filter design function provides a tool for designing FIR filters with arbitrary complex responses. It differs from the other filter design functions in how the frequency response of the filter is specified: it accepts the name of a function which returns the filter response calculated over a grid of frequencies. This capability makes `cremez` a highly versatile and powerful technique for filter design.
This design technique may be used to produce nonlinear-phase FIR filters, asymmetric frequency-response filters (with complex coefficients), or more symmetric filters with custom frequency responses.

The design algorithm optimizes the Chebyshev (or minimax) error using an extended Remez-exchange algorithm for an initial estimate. If this exchange method fails to obtain the optimal filter, the algorithm switches to an ascent-descent algorithm that takes over to finish the convergence to the optimal solution.

For details on the calling syntax for `cremez`, see the reference description in Chapter 6.

**Multiband Filter Design**

Consider a multiband filter with the following special frequency-domain characteristics:

<table>
<thead>
<tr>
<th>Band</th>
<th>Amplitude</th>
<th>Optimization Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-1 -0.5]</td>
<td>[5 1]</td>
<td>1</td>
</tr>
<tr>
<td>[-0.4 +0.3]</td>
<td>[2 2]</td>
<td>10</td>
</tr>
<tr>
<td>[+0.4 +0.8]</td>
<td>[2 1]</td>
<td>5</td>
</tr>
</tbody>
</table>

A linear-phase multiband filter may be designed using the predefined frequency-response function `multiband`, as follows.

```matlab
b = cremez(38, [-1 -0.5 -0.4 0.3 0.4 0.8], ...         
            {'multiband', [5 1 2 2 2 1]}, [1 10 5]);
```

For the specific case of a multiband filter, we can use a shorthand filter design notation similar to the syntax for `remez`:

```matlab
b = cremez(38, [-1 -0.5 -0.4 0.3 0.4 0.8], ...         
            [5 1 2 2 2 1], [1 10 5]);
```

As with `remez`, a vector of band edges is passed to `cremez`. This vector defines the frequency bands over which optimization is performed; note that there are two transition bands, from -0.5 to -0.4 and from 0.3 to 0.4.
In either case, the frequency response is obtained and plotted using linear scale.

\[
[h, w] = \text{freqz}(b, 1, 512, 'whole');
\]
\[
\text{plot}(w/\pi - 1, \text{fftshift}(|h|));
\]

Note that the frequency response has been calculated over the entire normalized frequency range \([-1, 1]\) by passing the option ‘whole’ to \text{freqz}. In order to plot the negative frequency information in a natural way, the response has been “wrapped,” just as FFT data is, using \text{fftshift}.
The filter response for this multiband filter is complex, which is expected because of the asymmetry in the frequency domain. The filter response is

Filter Design with Reduced Delay
Consider the design of a 62-tap lowpass filter with a half-Nyquist cutoff. If we specify a negative offset value to the `lowpass` filter design function, the group delay offset for the design is significantly less than that obtained for a standard linear-phase design. This filter design may be computed as follows.

```matlab
b =remez(61, [0 0.5 0.55 1], {'lowpass', -16});
```
The resulting magnitude response is

\[
[h, w] = \text{freqz}(b, 1, 512, 'whole');
\text{plot}(w/pi-1, \text{fftshift}(|h|));
\]

The group delay of the filter reveals that the offset has been reduced from $N/2 = 30.5$ to $N/2 - 16 = 14.5$. Now, however, the group delay is no longer flat in the passband region (plotted over the normalized frequency range 0 to 0.5 for clarity).
If we compare this nonlinear-phase filter to a linear-phase filter that has exactly 14.5 samples of group delay, the resulting filter is of order $2 \times 14.5$ or 29. Using $b = \text{remez}(29, [0 \ 0.5 \ 0.55 \ 1], \text{''lowpass''})$, the passband and stopband ripple is much greater for the order 29 filter. These comparisons can assist you in deciding which filter is more appropriate for a specific application.
Special Topics in IIR Filter Design

The classic IIR filter design technique finds an analog lowpass filter with cutoff frequency of 1, translates this “prototype” filter to the desired band configuration, then transforms the filter to the digital domain. The toolbox provides functions for each step of this process.

<table>
<thead>
<tr>
<th>Classical IIR Filter Design</th>
<th>Frequency Transformation</th>
<th>Discretization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Lowpass Prototype Creation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>buttap</td>
<td>lp2lp</td>
<td>bilinear</td>
</tr>
<tr>
<td>cheb1ap</td>
<td>lp2hp</td>
<td>impinvar</td>
</tr>
<tr>
<td>besselap</td>
<td>lp2bp</td>
<td></td>
</tr>
<tr>
<td>cheb2ap</td>
<td>lp2bs</td>
<td></td>
</tr>
<tr>
<td>ellipap</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Complete Design                      |                          |                |
| butter                               | cheby1                   |                |
| cheb1                               | cheby2                   |                |
| ellip                                | besself                  |                |

<table>
<thead>
<tr>
<th>Minimum Order Computation for Classical IIR Filter Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>butterord</td>
</tr>
<tr>
<td>cheb1ord</td>
</tr>
<tr>
<td>cheb2ord</td>
</tr>
<tr>
<td>ellipord</td>
</tr>
</tbody>
</table>

The butter, cheby1, cheby2, and ellip functions are sufficient for many design problems, and the lower level functions are generally not needed. But if you do have an application where you need to transform the band edges of an analog filter, or discretize a rational transfer function, this section describes tools to do so.
Analog Prototype Design

This toolbox provides a number of functions to create lowpass analog prototype filters with cutoff frequency of 1, the first step in the classical approach to IIR filter design. The table below summarizes the analog prototype design functions for each supported filter type; plots for each type are shown in the “IIR Filter Design” section above.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Analog Prototype Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bessel</td>
<td>([z, p, k] = \text{besselap}(n))</td>
</tr>
<tr>
<td>Butterworth</td>
<td>([z, p, k] = \text{butterap}(n))</td>
</tr>
<tr>
<td>Chebyshev type I</td>
<td>([z, p, k] = \text{cheb1ap}(n,Rp))</td>
</tr>
<tr>
<td>Chebyshev type II</td>
<td>([z, p, k] = \text{cheb2ap}(n,Rs))</td>
</tr>
<tr>
<td>Elliptic</td>
<td>([z, p, k] = \text{ellipap}(n,Rp,Rs))</td>
</tr>
</tbody>
</table>

Frequency Transformation

The second step in the analog prototyping design technique is the frequency transformation of a lowpass prototype. The toolbox provides a set of functions to transform analog lowpass prototypes (with cutoff frequency of 1 rad/sec) into bandpass, highpass, bandstop, and lowpass filters of the desired cutoff frequency.
As shown, all of the frequency transformation functions can accept two linear system models: transfer function and state-space form. For the bandpass and bandstop cases

\[
\omega_0 = \sqrt[\lambda]{\omega_1 \omega_2}
\]

and

\[
B_\omega = \omega_2 - \omega_1
\]

where \( \omega_1 \) is the lower band edge and \( \omega_2 \) is the upper band edge.

The frequency transformation functions perform frequency variable substitution. In the case of \( \text{lp2bp} \) and \( \text{lp2bs} \), this is a second-order substitution, so the output filter is twice the order of the input. For \( \text{lp2lp} \) and \( \text{lp2hp} \), the output filter is the same order as the input.

<table>
<thead>
<tr>
<th>Freq. Transformation</th>
<th>Transformation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowpass to lowpass</td>
<td>([\text{numt}, \text{dent}] = \text{lp2lp}(\text{num}, \text{den}, \omega_0)) [\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{lp2lp}(A, B, C, D, \omega_0)</td>
</tr>
<tr>
<td>Lowpass to highpass</td>
<td>([\text{numt}, \text{dent}] = \text{lp2hp}(\text{num}, \text{den}, \omega_0)) [\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{lp2hp}(A, B, C, D, \omega_0)</td>
</tr>
<tr>
<td>Lowpass to bandpass</td>
<td>([\text{numt}, \text{dent}] = \text{lp2bp}(\text{num}, \text{den}, \omega_0, B_\omega)) [\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{lp2bp}(A, B, C, D, \omega_0, B_\omega)</td>
</tr>
<tr>
<td>Lowpass to bandstop</td>
<td>([\text{numt}, \text{dent}] = \text{lp2bs}(\text{num}, \text{den}, \omega_0, B_\omega)) [\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{lp2bs}(A, B, C, D, \omega_0, B_\omega)</td>
</tr>
</tbody>
</table>
To begin designing an order 10 bandpass Chebyshev type I filter with a value of 3 dB for passband ripple

\[ [z, p, k] = \text{cheb1ap}(5, 3); \]

\( z, p, \) and \( k \) contain the poles, zeros, and gain of a lowpass analog filter with cutoff frequency \( \Omega_c \) equal to 1 rad/sec. Use the \text{lp2bp} function to transform this lowpass prototype to a bandpass analog filter with band edges \( W_1 = \pi/5 \) and \( W_2 = \pi \). First, convert the filter to state-space form so the \text{lp2bp} function can accept it.

\[ [A, B, C, D] = \text{zp2ss}(z, p, k); \quad \% \text{ Convert to state-space form.} \]

Now, find the bandwidth and center frequency, and call \text{lp2bp}.

\[ u1 = 0.1*2*\pi; \quad u2 = 0.5*2*\pi; \quad \% \text{ in radians per second} \]
\[ Bw = u2-u1; \]
\[ Wo = \sqrt{u1*u2}; \]
\[ [At, Bt, Ct, Dt] = \text{lp2bp}(A, B, C, D, Wo, Bw); \]
Finally, calculate the frequency response and plot its magnitude.

```matlab
[b,a] = ss2tf(At,Bt,Ct,Dt); % Convert to TF form.
w = linspace(.01,1,500)*2*pi; % Generate frequency vector.
h = freqs(b,a,w); % Compute frequency response.
semilogy(w/2/pi,abs(h)), grid % Plot log magnitude vs. freq.
```

![Frequency Response Plot](image)

**Filter Discretization**

The third step in the analog prototyping technique is the transformation of the filter to the discrete-time domain. The toolbox provides two methods for this: the impulse invariant and bilinear transformations. The filter design functions
butter, cheby1, cheby2, and ellip use the bilinear transformation for discretization in this step.

<table>
<thead>
<tr>
<th>Analog to Digital Transformation</th>
<th>Transformation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse invariance</td>
<td>[numd, dend] = impinvar(num, den, Fs)</td>
</tr>
<tr>
<td>Bilinear transform</td>
<td>[zd, pd, kd] = bilinear(z, p, k, Fs, Fp)</td>
</tr>
<tr>
<td></td>
<td>[numd, dend] = bilinear(num, den, Fs, Fp)</td>
</tr>
<tr>
<td></td>
<td>[Ad, Bd, Cd, Dd] = bilinear(At, Bt, Ct, Dt, Fs, Fp)</td>
</tr>
</tbody>
</table>

**Impulse Invariance**

The toolbox function `impinvar` creates a digital filter whose impulse response is the samples of the continuous impulse response of an analog filter. This function works only on filters in transfer function form. For best results, the analog filter should have negligible frequency content above half the sampling frequency, because such high frequency content is aliased into lower bands upon sampling. Impulse invariance works for some lowpass and bandpass filters, but is not appropriate for highpass and bandstop filters.
Design a Chebyshev type I filter and plot its frequency response.

```matlab
[bz,az] = impinvar(b,a,2);
freqz(bz,az)
```

![Magnitude Response](image1)

![Phase Response](image2)

Impulse invariance retains the cutoff frequencies of 0.1 Hz and 0.5 Hz.

**Bilinear Transformation**

The bilinear transformation is a nonlinear mapping of the continuous domain to the discrete domain; it maps the s-plane into the z-plane by

\[ H(z) = H(s)|_{s = \frac{z - 1}{z + 1}} \]

Bilinear transformation maps the \( j\Omega \) axis of the continuous domain to the unit circle of the discrete domain according to

\[ \omega = 2\tan^{-1}\left(\frac{\Omega}{k}\right) \]

The toolbox function `bilinear` implements this operation, where the frequency warping constant \( k \) is equal to twice the sampling frequency \( (2\times F_s) \) by default and equal to \( 2\pi F_p / \tan(\pi F_p / F_s) \) if you give `bilinear` a trailing argument that represents a “match” frequency \( F_p \). If a match frequency \( F_p \) (in Hertz) is
present, bilinear maps the frequency \( \Omega = 2\pi f_p \) (in radians/second) to the same frequency in the discrete domain, normalized to the sampling rate: \( \omega = 2\pi f_p / f_s \) (also in radians/second).

The bilinear function can perform this transformation on three different linear system representations: zero-pole-gain, transfer function, and state-space form. Try calling \texttt{bilinear} with the state-space matrices that describe the Chebyshev type I filter from the previous section, using a sampling frequency of 2 Hz, and retaining the lower band edge of 0.1 Hz.

\[
[Ad, Bd, Cd, Dd] = \texttt{bilinear}(At, Bt, Ct, Dt, 2, 0.1);
\]

The frequency response of the resulting digital filter is

\[
[bz, az] = \texttt{ss2tf}(Ad, Bd, Cd, Dd); \% \text{convert to TF}
\]

\[
\texttt{freqz(bz, az)}
\]

The lower band edge is at 0.1 Hz as expected. Notice, however, that the upper band edge is slightly less than 0.5 Hz, although in the analog domain it was exactly 0.5 Hz. This illustrates the nonlinear nature of the bilinear transformation. To counteract this nonlinearity, it is necessary to create analog domain filters with “prewarped” band edges, which map to the correct locations.
upon bilinear transformation. Here the prewarped frequencies $u_1$ and $u_2$ generate $B_w$ and $W_0$ for the $lp2bp$ function.

```matlab
Fs = 2; % sampling frequency (Hertz)
u1 = 2*Fs*tan(.1*(2*pi/Fs)/2); % lower band edge (radians/second)
u2 = 2*Fs*tan(.5*(2*pi/Fs)/2); % upper band edge (radians/second)
Bw = u2-u1; % bandwidth
Wo = sqrt(u1*u2); % center frequency
[At,Bt,Ct,Dt] = lp2bp(A,B,C,D,Wo,Bw);
```

A digital bandpass filter with correct band edges 0.1 and 0.5 times the Nyquist frequency is

```matlab
[Ad,Bd,Cd,Dd] = bilinear(At,Bt,Ct,Dt,Fs);
```

The example bandpass filters from the last two sections could also be created in one statement using the complete IIR design function `cheby1`. For instance, an analog version of the example Chebyshev filter is

```matlab
[b,a] = cheby1(5,3,[0.1 0.5]*2*pi,'s');
```

Note that the band edges are in radians/second for analog filters, whereas for the digital case, frequency is normalized (the Nyquist frequency is equal to 1 Hz).

```matlab
[bz,az] = cheby1(5,3,[0.1 0.5]);
```

All of the complete design functions call `bilinear` internally. They prewarp the band edges as needed to obtain the correct digital filter. See Chapter 6 for more on these functions.
References


Statistical Signal Processing

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Correlation and Covariance

The Signal Processing Toolbox provides tools for estimating important functions of random signals. In particular, there are tools to estimate correlation and covariance sequences and spectral density functions of discrete signals. This chapter explains the correlation and covariance functions and discusses the mathematically related functions for estimating the power spectrum.

The functions `xcorr` and `xcov` estimate the cross-correlation and cross-covariance sequences of random processes. They also handle autocorrelation and autocovariance as special cases.

The true cross-correlation sequence is a statistical quantity defined as

\[ \gamma_{xy}(m) = E \{ x_n y^*_{n+m} \} \]

where \( x_n \) and \( y_n \) are stationary random processes, \( -\infty < n < \infty \), and \( E \{ \} \) is the expected value operator. The cross-covariance sequence is the mean-removed cross-correlation sequence

\[ C_{xy}(m) = E \{ (x_n - \mu_x)(y^*_{n+m} - \mu^*_y) \} \]

or, in terms of the cross-correlation,

\[ C_{xy}(m) = \gamma_{xy}(m) - \mu_x \mu^*_y \]

In practice, you must estimate these sequences, because it is possible to access only a finite segment of the infinite-length random process. A common estimate based on \( N \) samples of \( x_n \) and \( y_n \) is the deterministic cross-correlation sequence (also called the time-ambiguity function)

\[
\hat{R}_{xy}(m) = \begin{cases} 
\sum_{n=0}^{N-|m|} x_n y^*_{n+m} & m \geq 0 \\
\hat{R}^*_{yx}(-m) & m < 0 
\end{cases}
\]
where we assume for this discussion that \( x_n \) and \( y_n \) are indexed from 0 to \( N-1 \),
and \( \hat{R}_{yx}(-m) \) from -(N-1) to N-1. The \texttt{xcorr} function evaluates this sum with
an efficient FFT-based algorithm, given inputs \( x_n \) and \( y_n \) stored in length \( N \)
_vectors \( x \) and \( y \). Its operation is equivalent to convolution with one of the two
subsequences reversed in time.

For example,

\[
x = [1 1 1 1 1]';
y = x;
xyc = xcorr(x,y)
\]

\[
xyc =
\begin{array}{c}
1.0000 \\
2.0000 \\
3.0000 \\
4.0000 \\
5.0000 \\
4.0000 \\
3.0000 \\
2.0000 \\
1.0000
\end{array}
\]

Notice that the resulting sequence is twice the length of the input sequence
minus 1. Thus, the \( N \)th element is the correlation at lag 0. Also notice the
triangular pulse of the output that results when convolving two square pulses.

The \texttt{xcov} function estimates autocovariance and cross-covariance sequences.
This function has the same options and evaluates the same sum as \texttt{xcorr}, but
first removes the means of \( x \) and \( y \).

**Bias and Normalization**

An estimate of a quantity is biased if its expected value is not equal to the
quantity it estimates. The expected value of the output of \texttt{xcorr} is

\[
E \{ \hat{R}_{xy}(m) \} = \sum_{n=0}^{N-|m|-1} E \{ x_n y^{*}_{n+m} \} = (N-|m|) \gamma_{xy}(m)
\]
xcorr provides the unbiased estimate, dividing by $N-|m|$, when you specify an ‘unbiased’ flag after the input sequences.

$$\text{xcorr}(x,y,'\text{unbiased}')$$

Although this estimate is unbiased, the end points (near $-(N-1)$ and $N-1$) suffer from large variance because xcorr computes them using only a few data points. A possible trade-off is to simply divide by $N$ using the ‘biased’ flag.

$$\text{xcorr}(x,y,'\text{biased}')$$

With this scheme, only the sample of the correlation at 0 lag (the Nth output element) is unbiased. This estimate is often more desirable than the unbiased one because it avoids random large variations at the end points of the correlation sequence.

xcorr provides one other normalization scheme. The syntax

$$\text{xcorr}(x,y,'\text{coeff}')$$

divides the output by $\text{norm}(x) \times \text{norm}(y)$ so that, for autocorrelations, the sample at 0 lag is 1.

**Multiple Channels**

For a multichannel signal, xcorr and xcov estimate the autocorrelation and cross-correlation and covariance sequences for all of the channels at once. If $S$ is an $M$-by-$N$ signal matrix representing $N$ channels in its columns, xcorr($S$) returns a $(2M-1)$-by-$N^2$ matrix with the autocorrelations and cross-correlations of the channels of $S$ in its $N^2$ columns. If $S$ is a 3-channel signal

$$S = [s1 \ s2 \ s3]$$

then the result of xcorr($S$) is organized as

$$R = [Rs1s1 \ Rs1s2 \ Rs1s3 \ Rs2s1 \ Rs2s2 \ Rs2s3 \ Rs3s1 \ Rs3s2 \ Rs3s3]$$

Two related functions, cov and corrcoef, are available in the standard MATLAB environment. They estimate covariance and normalized covariance respectively between the different channels at lag 0 and arrange them in a square matrix.
Spectral Analysis

Spectral analysis seeks to describe the frequency content of a signal, random process, or system, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise.

The power spectral density (PSD) of a stationary random process $x_n$ is related mathematically to the correlation sequence by the discrete-time Fourier transform,

$$P_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \gamma_{xx}(m)e^{-j\omega m}$$

This function of frequency has the property that its integral over a frequency band is equal to the power of the signal $x_n$ in that band.

The PSD is a special case of the cross spectral density (CSD) function, defined between two signals $x_n$ and $y_n$ as

$$P_{xy}(\omega) = \sum_{m=-\infty}^{\infty} \gamma_{xy}(m)e^{-j\omega m}$$

As is the case for the correlation and covariance sequences, the toolbox estimates the PSD and CSD because signal lengths are finite.

The various methods of PSD estimation can be identified as parametric or nonparametric. One technique offered in the Signal Processing Toolbox is the popular nonparametric scheme developed by Welch [5]. This is complemented by more modern nonparametric techniques such as the multitaper method (MTM) and the multiple signal classification (MUSIC) or eigenvector (EV) method, which is well suited for line spectra (data made up of sinusoids). The Yule-Walker autoregressive (AR) method is a parametric method that estimates the autocorrelation function to solve for the AR model parameters. The Burg method is another parametric spectral estimation method that minimizes the forward and backward linear prediction errors while satisfying the Levinson-Durbin recursion. These methods are listed in the table below with the corresponding toolbox function name. The number below each method name indicates the page that describes the method in greater detail. See
“Parametric Modeling” in Chapter 4 for details about \texttt{lp} and other parametric estimation functions.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
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**Welch’s Method**

One way of estimating the power spectrum of a process is to simply find the discrete-time Fourier transform of the samples of the process (usually done on
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a grid with an FFT) and take the magnitude squared of the result. An example 1001-element signal \( x_n \), which consists of two sinusoids plus noise, is given by

\[
\begin{align*}
Fs &= 1000; & \text{% sampling frequency} \\
\text{t} &= 0:1/Fs:1; & \text{% one second worth of samples} \\
x_n &= \sin(2\pi50\cdot t) + 2\sin(2\pi120\cdot t) + \text{.randn(size(t))};
\end{align*}
\]

A crude estimate of the PSD of \( x_n \) is

\[
P_{xx} = \text{abs(fft}(x_n,1024))^{\cdot 2/1001};
\]

This estimate is called the periodogram. Scale the magnitude squared of the FFT by the square of the norm of the data window applied to the signal (in this case, a length 1001 rectangular window) to ensure that the estimate is asymptotically unbiased. That is, as the number of samples increases, the expected value of the periodogram approaches the true PSD.

The problem with the periodogram estimate is that its variance is large (on the order of the PSD squared) and does not decrease as the number of samples increases. The following two examples show this; as FFT length increases, the periodogram does not become smoother.

\[
P_{xx\_short} = \text{abs(fft}(x_n,256))^{\cdot 2/256};
\]

\[
\begin{align*}
\text{plot}((0:255)/256*Fs,10\log10(P_{xx\_short})) \\
\text{plot}((0:1023)/1024*Fs,10\log10(P_{xx}))
\end{align*}
\]

Reduce the variance of the PSD estimate by breaking the signal into nonoverlapping sections and averaging the periodograms of these sections.

\[
P_{xx} = (\text{abs(fft}(x_n(1:256))^{\cdot 2} + \text{abs(fft}(x_n(257:512))^{\cdot 2} + \ldots \text{abs(fft}(x_n(513:768))^{\cdot 2}) / (256*3);
\]

\[
\begin{align*}
\text{plot}((0:255)/256*Fs,10\log10(P_{xx}))
\end{align*}
\]
This averaged estimate has one third the variance of the length 256 periodogram shown earlier. The more sections you average, the lower the variance of the result. However, the signal length limits the number of sections.
possible (to three sections of length 256 in the previous example). To obtain more sections, break the signal into overlapping sections.

\[
P_{xx} = (\text{abs}(\text{fft}(x_n(1:256))).^2 + \text{abs}(\text{fft}(x_n(129:384))).^2 + \ldots \\
\quad \text{abs}(\text{fft}(x_n(257:512))).^2 + \text{abs}(\text{fft}(x_n(385:640))).^2 + \ldots \\
\quad \text{abs}(\text{fft}(x_n(513:768))).^2 + \ldots \\
\quad \text{abs}(\text{fft}(x_n(641:896))).^2 ) / (256*6);
\]

plot((0:255)/256*Fs,10*log10(P_{xx}))

In this case the sections are statistically dependent, resulting in higher variance; thus there is a trade-off between the number of sections and the overlap rate.

Another way to improve the periodogram estimate is to apply a nonrectangular data window to the sections before computing the periodogram, resulting in a modified periodogram. This reduces the effect of section dependence due to overlap, because the window is tapered to 0 on the edges. Also, a nonrectangular window diminishes the side-lobe interference or “spectral leakage” while increasing the width of spectral peaks. With a suitable window (such as Hamming, Hanning, or Kaiser), overlap rates of about half the section length have been found to lower the variance of the estimate significantly.
The application of a Hanning window results in

\[
w = \text{hanning}(256)';
\]

\[
Pxx = ( \text{abs(fft}(w.*xn(1:256))).^2 + ... \text{abs(fft}(w.*xn(129:384))).^2 + ... \text{abs(fft}(w.*xn(257:512))).^2 + ... \text{abs(fft}(w.*xn(385:640))).^2 + ... \text{abs(fft}(w.*xn(513:768))).^2 + ... \text{abs(fft}(w.*xn(641:896))).^2 ) / (\text{norm}(w)^2*6);
\]

\[
\text{plot}((0:255)/256*Fs,10\log_{10}(Pxx))
\]

Notice in this plot that the spectral peaks have widened, and the noise floor, or level of the noise, seems to be the flattest of any estimate so far. This method of averaged, modified periodograms is Welch's method of PSD estimation.

The functions \texttt{pwelch} and \texttt{csd} provide control over all the parameters discussed so far (FFT length, window, and amount of overlap) in computing the PSD and CSD of signals using Welch's method.

For a more detailed discussion of Welch's method of PSD estimation, see Kay [1] and Welch [5].

\textbf{Power Spectral Density Function}

The \texttt{pwelch} function averages and scales the modified periodograms of sections of a signal. Simply specify the parameters that control the algorithm as arguments to the function.

An estimate for the PSD of a sequence \texttt{xn} using \texttt{pwelch}'s default FFT length (256), window (Hanning of length 256), and overlap samples (none) is

\[
Pxx = \text{pwelch}(xn);
\]
Pxx has units of power per unit frequency interval. For example, if the original sequence \( x_n \) has units of volts, \( Pxx \) has units of Watts/Hz.

To recreate the last example accurately, specify 128 as the number of samples to overlap:

\[
\begin{align*}
\text{nfft} &= 256; \quad \% \text{length of FFT} \\
\text{window} &= \text{hanning}(256); \quad \% \text{window function} \\
\text{noverlap} &= 128; \quad \% \text{number of samples overlap} \\
\text{Pxx} &= \text{pwelch}(x_n, \text{nfft}, \text{Fs}, \text{window}, \text{noverlap});
\end{align*}
\]

\( Pxx \) is scaled by the reciprocal of the sampling frequency, \( 1/\text{Fs} \). \text{pwelch} without any outputs generates a plot of the PSD over the frequency range \([0, \text{Fs}/2)\):

\[
\text{pwelch}(x_n, \text{nfft}, \text{Fs}, \text{window}, \text{noverlap})
\]

If you want to plot the PSD yourself, obtain the frequency vector through an additional output argument.

\[
\begin{align*}
[\text{Pxx}, f] &= \text{pwelch}(x_n, \text{nfft}, \text{Fs}, 256, \text{noverlap}); \\
\text{plot}(f, 10*\log10(\text{Pxx}))
\end{align*}
\]

All the spectral estimation functions allow you to specify an empty matrix, [], in place of an input argument to use the default value of that argument. In the command above, because the values for \text{nfft} and \text{window} are actually the same as the defaults, you could replace them both with the empty matrix.

\[
[\text{Pxx}, f] = \text{pwelch}(x_n, [], \text{Fs}, [], \text{noverlap});
\]

If you want to plot the PSD yourself, obtain the frequency vector through an additional output argument.
Since the signal $x_n$ is real, \texttt{pwelch} returns only the frequencies from 0 through the Nyquist frequency. In contrast, the earlier FFT example generated PSD estimates ranging from 0 through $F_s$.

**Bias and Normalization in Welch’s Method**

In studying the output of \texttt{pwelch} shown earlier, several revealing characteristics about the signal $x_n$ are evident. The noise floor is flat at 0 decibels (dB), implying white noise of variance 1. Furthermore, the “signal” part of $x_n$ is concentrated in two peaks at 50 and 120 Hz. The relation of the peak heights is meaningful. For instance, the 50 Hz peak is 6 dB below the 120 Hz peak, verifying that the higher frequency sinusoid has twice the magnitude as the lower ($10^{6/20} = 2.0$). Unlike the relative heights, the actual height of the peaks does not tell us much about the original amplitude of the sinusoids without additional analysis.

To obtain useful information about the peak amplitudes of the underlying sinusoids, note that the expected value of the estimated PSD is

$$
E\{\hat{P}_{xx}(\omega)\} = \frac{1}{2\pi\|w\|^2} \pi \int_{-\pi}^{\pi} P_{xx}(\theta)|W(\omega-\theta)|d\theta
$$

Because the expected value is not equal to the true PSD, the estimate is biased. This quantity is the convolution of the true PSD with the squared magnitude of the window’s discrete-time Fourier transform $W(\omega)$, scaled by the squared norm of the window. The scaling factor is the sum of the squares of the window function:

$$
\|w\|^2 = \sum w(n)^2
$$

This says that if $P_{xx}(\omega)$ has a peak of height 1 at a particular frequency $\omega_0$, the estimate will have approximate height $|W(0)|^2/\|w\|^2$ at that frequency, provided the window $W(\omega)$ is narrow with respect to the spacing between the peak and other spectral features. So, to obtain an estimate which on average reflects the height of the original peaks, multiply the result of \texttt{pwelch} by
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\[ \text{norm}(w)^2 / \text{sum}(w)^2 \]
where \( w \) is the window vector. This scaling is independent of window length and shape. For example:

```matlab
w1 = hanning(256); w2 = hanning(500);
[Pxx1,f1] = pwelch(xn,256,Fs,w1,128);
Pxx2,f2] = pwelch(xn,1024,Fs,w2,250);
```

```matlab
subplot(2,1,1)
plot(f1,10*log10(Pxx1*norm(w1)^2/sum(w1)^2))
axis([0 500 –70 –20]); grid
subplot(2,1,2)
plot(f2,10*log10(Pxx2*norm(w2)^2/sum(w2)^2))
axis([0 500 –70 –20]); grid
```

In both plots, which show the spectrum at positive frequencies only (the negative frequencies are the same), the higher frequency peak has a value of 0 dB, and the lower frequency peak is at -6 dB. The 120 Hz sinusoid height of 0 dB corresponds to a squared amplitude of 1. This results from the sinusoid of amplitude 2 having complex exponential components of amplitude 1 at both positive and negative frequency. Similarly, the 50 Hz sinusoid has both positive and negative frequency components with squared amplitude of \( (\frac{1}{2})^2 = \frac{1}{4} \), or \( 10 \log_{10}(\frac{1}{4}) = -6 \) dB, as shown in the plot. Also, note that the second plot reflects a slightly lower noise floor, which is the result of a longer window length.
Cross-Spectral Density Function
To estimate the cross-spectral density of two equal length signals \( x \) and \( y \) using Welch’s method, the \texttt{csd} function forms the periodogram as the product of the FFT of \( x \) and the conjugate of the FFT of \( y \). Unlike the real-valued PSD, the CSD is a complex function. \texttt{csd} handles the sectioning and windowing of \( x \) and \( y \) in the same way as the \texttt{pwelch} function.

\[
P_{xy} = \text{csd}(x, y, \text{nfft}, \text{Fs}, \text{window}, \text{noverlap})
\]

Confidence Intervals
Both the \texttt{pwelch} and \texttt{csd} functions can compute confidence intervals. Simply provide an input argument \( p \), which specifies the percentage of the confidence interval.

\[
[P_{xx}, P_{xxc}, f] = \text{pwelch}(x, \text{nfft}, \text{Fs}, \text{window}, \text{noverlap}, p)
\]
\[
[P_{xy}, P_{xy c}, f] = \text{csd}(x, y, \text{nfft}, \text{Fs}, \text{window}, \text{noverlap}, p)
\]

\( p \) must be a scalar between 0 and 1. The functions assume chi-squared distributed periodograms of nonoverlapping sections in computing the confidence intervals. (This assumption is valid when the signal is a Gaussian distributed random process.) Provided these assumptions are correct, there is a \( p \times 100\% \) probability that the confidence interval

\[
[P_{xx} - P_{xxc}(; 1), P_{xx} + P_{xxc}(; 2)]
\]

covers the true PSD. If the sections overlap, the confidence interval is not reliable and the functions display a warning message.

Transfer Function Estimate
One application of Welch’s method is nonparametric system identification. Assume that \( H \) is a linear, time invariant system, and \( x(n) \) and \( y(n) \) are the input to and output of \( H \), respectively. Then the PSD of \( x(n) \) is related to the CSD of \( x(n) \) and \( y(n) \) by

\[
P_{xy}(\omega) = H(\omega)P_{xx}(\omega)
\]

An estimate of the transfer function between \( x(n) \) and \( y(n) \) is

\[
\hat{H}(\omega) = \frac{\hat{P}_{xy}(\omega)}{\hat{P}_{xx}(\omega)}
\]
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This method estimates both magnitude and phase information. The `tfe` function uses Welch’s method to compute the CSD and PSD and then forms their quotient for the transfer function estimate. Use `tfe` the same way that you use the `csd` function.

Filter the signal \( x_n \) with an FIR filter, then plot the actual magnitude response and the estimated response.

\[
h = \text{ones}(1,10)/10; \quad \text{moving average filter}
\]

\[
y_n = \text{filter}(h,1,xn);
\]

\[
[H, f] = \text{tfe}(x_n, y_n, 256, Fs, 256, 128, 'none');
\]

\[
H = \text{freqz}(h,1,f,Fs);
\]

\[
\text{subplot}(2,1,1); \quad \text{plot}(f,\text{abs}(H));
\]

\[
\text{title('Actual Transfer Function Magnitude')};
\]

\[
\text{subplot}(2,1,2); \quad \text{plot}(f,\text{abs}(HEST));
\]

\[
\text{title('Transfer Function Magnitude Estimate')};
\]

\[
\text{xlabel('Frequency (Hz)')};
\]

Coherence Function

The magnitude-squared coherence between two signals \( x(n) \) and \( y(n) \) is

\[
C_{xy}(\omega) = \frac{|P_{xy}(\omega)|^2}{P_{xx}(\omega)P_{yy}(\omega)}
\]
This quotient is a real number between 0 and 1 that measures the correlation between \( x(n) \) and \( y(n) \) at the frequency \( \omega \).

The `cohere` function takes sequences \( x \) and \( y \), computes their PSDs and CSD, and returns the quotient of the magnitude squared of the CSD and the product of the PSDs. Its options and operation are similar to the `csd` and `tfe` functions.

The coherence function of \( x_n \) and the filter output \( y_n \) versus frequency is

```matlab
cohere(xn,yn,256,Fs,256,128,'none')
```

If the input sequence length, window length, and overlap are such that `cohere` operates on only a single record, the function returns all ones.

**Multitaper Method**

The multitaper method (MTM) uses orthogonal windows (or “tapers”) to obtain approximately independent estimates of the power spectrum and then combines them to yield an estimate. This estimate exhibits more degrees of freedom and allows for easier quantification of the bias and variance trade-offs, compared to conventional periodogram methods. Many conventional spectral estimates use a single taper (or “window”), with some irretrievable loss of information at the beginning and the end of the series. In the multitaper method, additional tapers are used to recover some of the lost information.

This brief discussion of the multitaper method provides an intuitive look at the algorithm to assist in determining when to use it. For a more detailed and thorough explanation, see Percival and Walden [3].

The simple parameter for the multitaper method is the time-bandwidth product, \( NW \). This parameter is a “resolution” parameter directly related to the number of tapers used to compute the spectrum. There are always \( 2 \times NW - 1 \) tapers used to form the estimate. This means that, as \( NW \) increases, there are
more estimates of the power spectrum, and the variance of the estimate decreases. However, the bandwidth of each taper is also proportional to $NW$, so as $NW$ increases, each estimate exhibits more spectral leakage (i.e., wider peaks) and the overall spectral estimate is more biased. For each data set, there is usually a value for $NW$ that allows an optimal trade-off between bias and variance.

Using \texttt{pmtm} on the data from the previous section, $xn$, yields

```matlab
Fs = 1000;
t = 0:1/Fs:1;
randn('seed',0)

xn = sin(2*pi*50*t) + 2*sin(2*pi*120*t) + randn(size(t));

[P,f] = pmtm(xn,4,1024,Fs);
plot(f,10*log10(P)) % plot in decibels
axis([30 150 -20 30])
```

![Spectral Analysis](image)
By lowering the time-bandwidth product, the peaks become narrower.

\[
[P1,f] = 
\text{pmtm}(x_n,3/2,1024,Fs);
\]
\[
\text{plot}(f,10\times\log10(P1)) \quad \% \text{plot in decibels}
\]
\[
\text{axis([30 150 -20 30])}
\]

Note that the area under the peaks remains about the same, as can be seen when both are plotted together on a linear scale.

\[
\text{plot}(f,[P \ P1])
\]
\[
\text{axis([30 150 0 400])}
\]
This conservation of total power is verifiable numerically.

\[
\text{sum}(P)
\]

\[
\text{ans} = 1.8447\times10^3
\]

\[
\text{sum}(P1)
\]

\[
\text{ans} = 1.8699\times10^3
\]

Note that total power is only approximately conserved in this case. This is because the adaptive weighting procedure that is used to minimize leakage does not strictly conserve total power.

This method is more expensive computationally than Welch’s method, because of the cost of computing the discrete prolate spheroidal sequences (DPSSs, also known as Slepian sequences). For long data series (10,000 points or more), it is useful to compute the DPSSs once and save them in a MAT-file. The M-files \text{dpssave}, \text{dpssload}, \text{dpssdir}, and \text{dpssclear} are provided, to keep a database of saved DPSSs in the MAT-file \text{dpss.mat}.

**Yule-Walker AR Method**

The Yule-Walker AR method is an autoregressive technique for spectral density estimation (see Marple [2], Chapter 7, and Proakis[4], Section 12.3.2). This method solves for the AR model parameters by the autocorrelation method.

The Yule-Walker AR estimate is obtained by solution of the normal equations.

\[
\begin{bmatrix}
  r(1) & r(2)^* & \cdots & r(n)^* \\
  r(2) & r(1) & \cdots & r(n-1)^* \\
  \vdots & \vdots & \ddots & \vdots \\
  r(n) & \cdots & r(2) & r(1)
\end{bmatrix}
\begin{bmatrix}
  a(2) \\
  a(3) \\
  \vdots \\
  a(n+1)
\end{bmatrix}
= 
\begin{bmatrix}
  -r(2) \\
  -r(3) \\
  \vdots \\
  -r(n+1)
\end{bmatrix}
\]

Here, \(a = [1 \ a(2) \ \ldots \ a(n+1)]\) is a vector of autoregressive coefficients, the elements of vector \(r = [r(1) \ r(2) \ \ldots \ r(n+1)]\) are correlations, and the left-hand side autocorrelation matrix is Hermitian Toeplitz and positive definite.
The spectral density estimate is
\[ P_{YuleAR}(f) = \frac{1}{|a^H \mathbf{e}(f)|^2} \]
where \( \mathbf{e}(f) \) is a complex sinusoid.

The toolbox function `pyulear` implements the Yule-Walker AR method.

For example, compare the spectrum of a speech signal using Welch's method and the Yule-Walker AR method:

```matlab
load mtlb
[P1,f] = pwelch(mtlb,1024,Fs,256);
[P2,f] = pyulear(mtlb,14,1024,Fs);  % 14th order model
plot(f,10*log10(P1),':',f,10*log10(P2)); grid
ylabel('Magnitude (dB)'); xlabel('Frequency (Hz)');
legend('Welch','Yule–Walker AR')
```

The solid Yule-Walker AR spectrum is smoother than the periodogram because of the simple underlying all-pole model.

**Burg Method**

The Burg method for AR spectral estimation is based on minimizing the forward and backward prediction errors while satisfying the Levinson-Durbin recursion (see Marple[2], Chapter 7, and Proakis[4], Section 12.3.3). In contrast to other AR estimation methods, the Burg method avoids calculating
the autocorrelation function, and instead estimates the reflection coefficients directly.

The primary advantages of the Burg method are resolving closely spaced sinusoids in signals with low noise levels, and estimating short data records, in which case the AR power spectral density estimates are very close to the true values. In addition, the Burg method ensures a stable AR model and is computationally efficient.

The accuracy of the Burg method is lower for high-order models, long data records, and high signal-to-noise ratios (which can cause line splitting, or the generation of extraneous peaks in the spectrum estimate). The spectral density estimate computed by the Burg method is also susceptible to frequency shifts (relative to the true frequency) resulting from the initial phase of noisy sinusoidal signals. This effect is magnified when analyzing short data sequences.

The toolbox function \texttt{pburg} implements the Burg method. Compare the spectrum of the speech signal generated by both the Burg method and the Yule-Walker AR method. They are very similar for large signal lengths.

```matlab
load mtlb
[P1,f] = pburg(mtlb(1:512),14,1024,Fs); % 14th order model
[P2,f] = pyulear(mtlb(1:512),14,1024,Fs); % 14th order model
plot(f,10*log10(P1),':',f,10*log10(P2)); grid
ylabel('Magnitude (dB)'); xlabel('Frequency (Hz)');
legend('Burg','Yule–Walker AR')
```

![Graph comparing Burg and Yule–Walker AR methods](image-url)
Compare the spectrum of a noisy signal computed using the Burg method and the Welch method.

\[
\begin{align*}
Fs &= 1000; \\
t &= 0:1/Fs:1; \\
xn &= \sin(2\pi*50*t) + 2*\sin(2\pi*120*t) + \text{randn(size(t))}; \\
[P1,f] &= \text{pwelch}(xn,1024,Fs); \\
[P2,f] &= \text{pburg}(xn,17,1024,Fs); \quad \% 17\text{th order model} \\
\text{plot}(f,10*\log10(P1),':',f,10*\log10(P2)), \text{grid} \\
\text{ylabel('Magnitude (dB)'}) ; \text{xlabel('Frequency (Hz)'}) ; \\
\text{axis([0 200 -50 0])} \\
\text{legend('Welch','Burg')} \\
\end{align*}
\]

Note that, as the model order for the Burg method is reduced, a frequency shift due to the initial phase of the sinusoids will become apparent.

**Covariance and Modified Covariance Methods**

The covariance method for AR spectral estimation is based on minimizing the forward prediction error. The modified covariance method is based on minimizing the forward and backward prediction errors. The toolbox functions \text{pcov} and \text{pmcov} implement the respective methods.
Compare the spectrum of the speech signal generated by both the covariance method and the modified covariance method. They are nearly identical, even for a short signal length.

load mtlb
[P1,f] = pcov(mtlb(1:64),14,1024,Fs);% 14th order model
[P2,f] = pmcov(mtlb(1:64),14,1024,Fs);% 14th order model
plot(f,10*log10(P1),':',f,10*log10(P2)); grid
ylabel('Magnitude (dB)'); xlabel('Frequency (Hz)');
legend('Covariance','Modified Covariance')

MUSIC and Eigenvector Analysis Methods
The `pmusic` function provides two related spectral analysis methods:

• The multiple signal classification method (MUSIC) developed by Schmidt
• The eigenvector (EV) method developed by Johnson

See Marple [2] (pgs. 373-378) for a summary of these methods.

Both of these methods are frequency estimator techniques based on eigenanalysis of the autocorrelation matrix. This type of spectral analysis categorizes the information in a correlation or data matrix, assigning information to either a signal subspace or a noise subspace.
**Eigenanalysis Overview**

Consider a number of complex sinusoids embedded in white noise. You can write the autocorrelation matrix \( R \) for this system as the sum of the signal autocorrelation matrix (\( S \)) and the noise autocorrelation matrix (\( W \)).

\[
R = S + W
\]

There is a close relationship between the eigenvectors of the signal autocorrelation matrix and the signal and noise subspaces. The eigenvectors \( v \) of \( S \) span the same signal subspace as the signal vectors. If the system contains \( M \) complex sinusoids and the order of the autocorrelation matrix is \( p \), eigenvectors \( v_{M+1} \) through \( v_{p+1} \) span the noise subspace of the autocorrelation matrix.

**Frequency Estimator Functions.** To generate their frequency estimates, eigenanalysis methods calculate functions of the vectors in the signal and noise subspaces. Both the MUSIC and EV techniques choose a function that theoretically goes to infinity at one of the sinusoidal frequencies in the input signal. Using digital technology, the resulting estimate has sharp peaks at the frequencies of interest; this means that there won't be infinity values in the vectors.

The MUSIC estimate is given by the formula

\[
P_{\text{music}}(f) = \frac{1}{N} \sum_{k=p+1}^{N} e^{H}(f) v_k v_k^H e(f) = \frac{1}{N} \sum_{k=p+1}^{N} |v_k^H e(f)|^2
\]

where \( N \) is the size of the eigenvectors and \( e(f) \) is a complex sinusoid vector:

\[
e(f) = [1 \ \exp(2\pi f) \ \exp(2\pi f \cdot 2) \ \exp(2\pi f \cdot 4) \ \ldots \ \exp(2\pi f \cdot (n - 1))]^H
\]

\( v \) represents the eigenvectors of the input signal’s correlation matrix; \( v_k \) is the \( k^{\text{th}} \) eigenvector. \( H \) is the conjugate transpose operator. The eigenvectors used in the sum correspond to the smallest eigenvalues and span the noise subspace (\( p \) is the size of the signal subspace).

The expression \( v_k^H e(f) \) is equivalent to a Fourier transform (the vector \( e(f) \) consists of complex exponentials). This form is useful for numeric computation because the FFT can be computed for each \( v_k \) and then the squared magnitudes can be summed.
The EV method weights the summation by the eigenvalues of the correlation matrix:

\[ p_{ev}(f) = \frac{1}{N} \left( \sum_{k=p+1}^{N} \frac{|v_k^H e(f)|^2}{\lambda_k} \right) \]

The `pmusic` function in this toolbox uses the `svd` (singular value decomposition) function in the signal case and the `eig` function for analyzing the correlation matrix and assigning eigenvectors to the signal or noise subspaces. When `svd` is used, `pmusic` never computes the correlation matrix explicitly, but the singular values are the eigenvalues.

**Controlling Subspace Thresholds**

To provide user control over the assignments of eigenvectors to the signal and noise subspaces, the `pmusic` function accepts a threshold argument `thresh`. `thresh` is a two-element vector where the first element is the number of eigenvectors spanning the signal subspace and the second element is a threshold test:

- If `thresh(2) ≤ 1`, then `thresh(1)` specifies the number of eigenvectors spanning the signal subspace. In this case the values of `thresh(1)` must be in the range \([0, N]\), where \(N\) is:
  - The column length of \(xR\) if \(xR\) is a data matrix
  - The matrix size if \(xR\) is a correlation matrix
  - The window length if \(xR\) is a signal vector
- If `thresh(1) ≥ N`, then `thresh(2)` is a value greater than or equal to 1 that specifies the absolute threshold for splitting the eigenvalues between the signal and noise subspaces. That is, if a given eigenvalue is less than or equal to the product `thresh(2)\min\{\lambda_k\}`, then the given eigenvector is assigned to the noise subspace.
- If `thresh(1) < N` and `thresh(2) ≥ 1`, `thresh(1)` still specifies the maximum number of eigenvectors in the signal subspace. However, the threshold test specified by `thresh(2)` can also assign eigenvectors to the noise subspace.
- If `thresh(1) ≥ N` and `thresh(2) < 1`, there are no noise eigenvectors. This is an invalid case and `pmusic` generates an error.
For complete details on using the \texttt{thresh} parameter, see the reference description of \texttt{pmusic} in Chapter 6.
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Windows

In both digital filter design and power spectrum estimation, the choice of a windowing function can play an important role in determining the quality of overall results. The main role of the window is to damp out the effects of the Gibbs phenomenon that results from truncation of an infinite series.

The toolbox window functions are shown in the table below.

<table>
<thead>
<tr>
<th>Window Function</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bartlett window</td>
<td>bartlett</td>
</tr>
<tr>
<td>Blackman window</td>
<td>blackman</td>
</tr>
<tr>
<td>Rectangular window</td>
<td>boxcar</td>
</tr>
<tr>
<td>Chebyshev window</td>
<td>chebwin</td>
</tr>
<tr>
<td>Hamming window</td>
<td>hamming</td>
</tr>
<tr>
<td>Hanning window</td>
<td>hanning</td>
</tr>
<tr>
<td>Kaiser window</td>
<td>kaiser</td>
</tr>
<tr>
<td>Triangular window</td>
<td>triang</td>
</tr>
</tbody>
</table>

**Basic Shapes**

The basic window is the rectangular window, a vector of ones of the appropriate length. A rectangular window of length 50 is

```matlab
n = 50;
w = boxcar(n);
```

This toolbox stores windows in column vectors by convention, so an equivalent expression is

```matlab
w = ones(50,1);
```

The Bartlett (or triangular) window is the convolution of two rectangular windows. The functions bartlett and triang compute similar triangular windows, with three important differences. The bartlett function always
returns a window with two zeros on the ends of the sequence, so that for \( n \) odd, the center section of \( \text{bartlett}(n+2) \) is equivalent to \( \text{triang}(n) \):

\[
\text{bartlett}(7)
\]

\[
\begin{array}{c}
0 \\
0.3333 \\
0.6667 \\
1.0000 \\
0.6667 \\
0.3333 \\
0
\end{array}
\]

\[
\text{triang}(5)
\]

\[
\begin{array}{c}
0.3333 \\
0.6667 \\
1.0000 \\
0.6667 \\
0.3333 \\
\end{array}
\]

For \( n \) even, \( \text{bartlett} \) is still the convolution of two rectangular sequences. There is no standard definition for the triangular window for \( n \) even; the slopes of the line segments of \( \text{triang} \)'s result are slightly steeper than those of \( \text{bartlett} \)'s in this case:

\[
w = \text{bartlett}(8);
[w(2:7) \ \text{triang}(6)]
\]

\[
\begin{array}{cc}
0.2857 & 0.1667 \\
0.5714 & 0.5000 \\
0.8571 & 0.8333 \\
0.8571 & 0.8333 \\
0.5714 & 0.5000 \\
0.2857 & 0.1667 \\
\end{array}
\]

The final difference between the Bartlett and triangular windows is evident in the Fourier transforms of these functions. The Fourier transform of a Bartlett
window is negative for \( n \) even. The Fourier transform of a triangular window, however, is always nonnegative.

This difference can be important when choosing a window for some spectral estimation techniques, such as the Blackman-Tukey method. Blackman-Tukey forms the spectral estimate by calculating the Fourier transform of the autocorrelation sequence. The resulting estimate might be negative at some frequencies if the window’s Fourier transform is negative (see Kay [1], pg. 80).

**Generalized Cosine Windows**

Blackman, Hamming, Hanning, and rectangular windows are all special cases of the generalized cosine window. These windows are combinations of sinusoidal sequences with frequencies \( 0, 2\pi/(N-1), \) and \( 4\pi/(N-1), \) where \( N \) is the window length. One way to generate them is

\[
\text{ind} = (0:n-1)' \times 2\pi/(n-1);
\]

\[
w = A - B\cos(\text{ind}) + C\cos(2\times\text{ind});
\]

where \( A, B, \) and \( C \) are constants you define. The concept behind these windows is that by summing the individual terms to form the window, the low frequency peaks in the frequency domain combine in such a way as to decrease sidelobe height. This has the side effect of increasing the mainlobe width.

The Hamming and Hanning windows are two-term generalized cosine windows, given by \( A = 0.54, B = 0.46 \) for Hamming and \( A = 0.5, B = 0.5 \) for Hanning (\( C = 0 \) in both cases). The hamming and hanning functions, respectively, compute these windows.

Note that the definition of the generalized cosine window shown in the earlier MATLAB code yields zeros at samples 1 and \( n \) for \( A = 0.5 \) and \( B = 0.5 \). To eliminate these zeros on the edges of the window, hanning uses a cosine of frequency \( 2\pi/(N+1) \) instead of \( 2\pi/(N-1) \).

The Blackman window is a popular three-term window, given by \( A = 0.42, B = 0.5, C = 0.08 \). The blackman function computes this window.

**Kaiser Window**

The Kaiser window is an approximation to the prolate-spheroidal window, for which the ratio of the mainlobe energy to the sidelobe energy is maximized. For a Kaiser window of a particular length, the parameter \( \beta \) controls the sidelobe height. For a given \( \beta \), the sidelobe height is fixed with respect to window length.
The statement \texttt{kaiser(n,beta)} computes a length \textit{n} Kaiser window with parameter \textit{beta}.

Examples of Kaiser windows with length 50 and various values for the \textit{beta} parameter are

\begin{verbatim}
    n = 50;
    w1 = kaiser(n,1);
    w2 = kaiser(n,4);
    w3 = kaiser(n,9);
    [W1,f] = freqz(w1/sum(w1),1,512,2);
    [W2,f] = freqz(w2/sum(w2),1,512,2);
    [W3,f] = freqz(w3/sum(w3),1,512,2);
    plot(f,20*log10(abs([W1 W2 W3])))
\end{verbatim}
As $\beta$ increases, the sidelobe height decreases and the mainlobe width increases. To see how the sidelobe height stays the same for a fixed $\beta$ parameter as the length is varied, try

\[
\begin{align*}
  w_1 &= \text{kaiser}(50, 4); \\
  w_2 &= \text{kaiser}(20, 4); \\
  w_3 &= \text{kaiser}(101, 4); \\
  [W1, f] &= \text{freqz}(w1/\text{sum}(w1), 1, 512, 2); \\
  [W2, f] &= \text{freqz}(w2/\text{sum}(w2), 1, 512, 2); \\
  [W3, f] &= \text{freqz}(w3/\text{sum}(w3), 1, 512, 2); \\
  \text{plot}(f, 20*\log10(\text{abs}([W1 W2 W3])))
\end{align*}
\]
Kaiser Windows in FIR Design

There are two design formulas that can help you design FIR filters to meet a set of filter specifications using a Kaiser window. To achieve a sidelobe height of $-\alpha$ dB, the beta parameter is

$$
\beta = \begin{cases} 
0.1102(\alpha - 8.7), & \alpha > 50 \\
0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\
0, & \alpha < 21 
\end{cases}
$$

For a transition width of $\Delta \omega$ rad/sec, use the length

$$
n = \frac{\alpha - 8}{2.285\Delta \omega} + 1
$$

Filters designed using these heuristics will meet the specifications approximately, but you should verify this. To design a lowpass filter with cutoff frequency $0.5\pi$ rad/sec, transition width $0.2\pi$ rad/sec, and 40 dB of attenuation in the stopband, try

```matlab
[n,wn,beta] = kaiserord([0.4 0.6]*pi,[1 0],[0.01 0.01],2*pi);
h = fir1(n,wn,kaiser(n+1,beta),'noscale');
```

The kaiserord function estimates the filter order, cutoff frequency, and Kaiser window beta parameter needed to meet a given set of frequency domain specifications.
The ripple in the passband is roughly the same as the ripple in the stopband. As you can see from the frequency response, this filter nearly meets the specifications.

```matlab
[H,f] = freqz(h,1,512,2);
plot(f,20*log10(abs(H))), grid
```

For details on `kaiserord`, see the reference description in Chapter 6.
Chebyshev Window

The Chebyshev window minimizes the mainlobe width, given a particular sidelobe height. It is characterized by an equiripple behavior, that is, its sidelobes all have the same height. The `chebwin` function, with length and sidelobe height parameters, computes a Chebyshev window.

```matlab
n = 51;
Rs = 40;  % sidelobe height in decibels
w = chebwin(n,Rs);
stem(w)
```

As shown in the plot, the Chebyshev window has large spikes at its outer samples.

Plot the frequency response to see the equiripples at -40 dB.

```matlab
[W,f] = freqz(w,1,512,2);
plot(f,20*log10(abs(W)/sum(w))), grid
```

Parametric Modeling

Parametric modeling techniques find the parameters for a mathematical model describing a signal, system, or process. These techniques use known information about the system to determine the model. Applications for parametric modeling include speech and music synthesis, data compression, high-resolution spectral estimation, communications, manufacturing, and simulation.

The toolbox parametric modeling functions operate with the rational transfer function model. Given appropriate information about an unknown system (impulse or frequency response data, or input and output sequences), these functions find the coefficients of a linear system that models the system.

One important application of the parametric modeling functions is in the design of filters that have a prescribed time or frequency response. These functions provide a data-oriented alternative to the IIR and FIR filter design functions discussed in Chapter 2.

Here is a summary of the parametric modeling functions in this toolbox. Note that the System Identification Toolbox provides a more extensive collection of parametric modeling functions.
Because `yulewalk` is geared explicitly toward ARMA filter design, it is discussed in Chapter 2. `pburg` and `pyulear` are discussed in Chapter 3 along with the other (nonparametric) spectral estimation methods.

**Time-Domain Based Modeling**

The `lpc`, `prony`, and `stmcb` functions find the coefficients of a digital rational transfer function that approximates a given time-domain impulse response. The algorithms differ in complexity and accuracy of the resulting model.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Functions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td><code>arburg</code></td>
<td>Generate all-pole filter coefficients that model an input data sequence using the Levinson-Durbin algorithm.</td>
</tr>
<tr>
<td></td>
<td><code>arcov</code></td>
<td>Generate all-pole filter coefficients that model an input data sequence by minimizing the forward prediction error.</td>
</tr>
<tr>
<td></td>
<td><code>armcov</code></td>
<td>Generate all-pole filter coefficients that model an input data sequence by minimizing the forward and backward prediction errors.</td>
</tr>
<tr>
<td></td>
<td><code>aryule</code></td>
<td>Generate all-pole filter coefficients that model an input data sequence using an estimate of the autocorrelation function.</td>
</tr>
<tr>
<td></td>
<td><code>lpc</code>, <code>levinson</code></td>
<td>Linear Predictive Coding. Generate all-pole recursive filter whose impulse response matches a given sequence.</td>
</tr>
<tr>
<td></td>
<td><code>prony</code></td>
<td>Generate IIR filter whose impulse response matches a given sequence.</td>
</tr>
<tr>
<td></td>
<td><code>stmcb</code></td>
<td>Find IIR filter whose output, given a specified input sequence, matches a given output sequence.</td>
</tr>
<tr>
<td>Frequency</td>
<td><code>invfreqz</code>, <code>invfreqs</code></td>
<td>Generate digital or analog filter coefficients given complex frequency response data.</td>
</tr>
</tbody>
</table>
Linear Prediction

Linear prediction modeling assumes that each output sample of a signal, \( x(k) \), is a linear combination of the past \( n \) outputs (that is, it can be “linearly predicted” from these outputs), and that the coefficients are constant from sample to sample.

\[
x(k) = -a(2)x(k-1) - a(3)x(k-2) - \cdots - a(n+1)x(k-n)
\]

An \( n \)th-order all-pole model of a signal \( x \) is

\[
a = \text{lpc}(x,n)
\]

To illustrate \text{lpc}, create a sample signal that is the impulse response of an all-pole filter with additive white noise.

\[
\text{randn('seed',0)}
\]

\[
x = \text{impz}(1,[1 0.1 0.1 0.1 0.1],10) + \text{randn}(10,1)/10;
\]

The coefficients for a fourth-order all-pole filter that models the system are

\[
a = \text{lpc}(x,4)
\]

\[
a = \\
1.0000 \\ 0.0395 \\ 0.0338 \\ 0.0668 \\ 0.1264
\]

\text{lpc} first calls \text{xcorr} to find a biased estimate of the correlation function of \( x \), and then uses the Levinson-Durbin recursion, implemented in the \text{levinson} function, to find the model coefficients \( a \). The Levinson-Durbin recursion is a fast algorithm for solving a system of symmetric Toeplitz linear equations. \text{lpc}'s entire algorithm for \( n = 4 \) is

\[
r = \text{xcorr}(x);
\]

\[
r(1:\text{length}(x)-1) = [] ; \% \text{remove corr. at negative lags}
\]

\[
a = \text{levinson}(r,4)
\]

\[
a = \\
1.0000 \\ 0.0395 \\ 0.0338 \\ 0.0668 \\ 0.1264
\]
You could form the linear prediction coefficients with other assumptions by passing a different correlation estimate to `levinson`, such as the unbiased correlation estimate.

\[
r = \text{xcorr}(x, \text{'unbiased'});
r(1:length(x)-1) = []; \% remove corr. at negative lags
a = \text{levinson}(r,4)
\]

\[
a =
\begin{bmatrix}
1.0000 & 0.0554 & 0.0462 & 0.0974 & 0.2115
\end{bmatrix}
\]

**Prony's Method (ARMA Modeling)**

The `prony` function models a signal using a specified number of poles and zeros. Given a sequence \( x \) and numerator and denominator orders \( nb \) and \( na \), respectively, the statement

\[
[b,a] = \text{prony}(x,nb,na)
\]

finds the numerator and denominator coefficients of an IIR filter whose impulse response approximates the sequence \( x \).

`prony` implements the method described in [3] Parks and Burrus (pgs. 226-228). This method uses a variation of the covariance method of AR modeling to find the denominator coefficients \( a \), and then finds the numerator coefficients \( b \) for which the resulting filter's impulse response matches exactly the first \( nb + 1 \) samples of \( x \). The filter is not necessarily stable, but it can potentially recover the coefficients exactly if the data sequence is truly an autoregressive moving average (ARMA) process of the correct order.

**NOTE** The functions `prony` and `stmcb` (described next) are more accurately described as ARX models in system identification terminology. ARMA modeling assumes noise only at the inputs, while ARX assumes an external input. `prony` and `stmcb` know the input signal: it is an impulse for `prony` and is arbitrary for `stmcb`. 
A model for the test sequence \( x \) (from the earlier \( \text{lpc} \) example) using a third-order IIR filter is

\[
[b, a] = \text{prony}(x, 3, 3)
\]

\[
b = \\
1.1165 \quad -0.2181 \quad -0.6084 \quad 0.5369
\]

\[
a = \\
1.0000 \quad -0.1619 \quad -0.4765 \quad 0.4940
\]

The \( \text{impz} \) command shows how well this filter’s impulse response matches the original sequence.

```matlab
format long
[x impz(b,a,10)]
```

\[
\text{ans} = \\
1.11649535105007 \quad 1.11649535105007 \\
-0.03731609173676 \quad -0.03731609173676 \\
-0.08249198453223 \quad -0.08249198453223 \\
-0.04583930972315 \quad -0.04583930972315 \\
-0.14255125351637 \quad -0.02829072973977 \\
0.20400424807471 \quad 0.01433198229497 \\
0.02685697779814 \quad 0.01148698991026 \\
0.18956307836948 \quad 0.02266475846451 \\
0.02717716288172 \quad 0.00206242734272 \\
0.08057060786906 \quad 0.00545783754743
\]

Notice that the first four samples match exactly. For an example of exact recovery, recover the coefficients of a Butterworth filter from its impulse response.

```matlab
[b,a] = \text{butter}(4,.2);
\text{h} = \text{impz}(b,a,26);
[bb,aa] = \text{prony}(h,4,4);
```

Try this example; you’ll see that \( bb \) and \( aa \) match the original filter coefficients to within a tolerance of \( 10^{-13} \).
**Steiglitz-McBride Method (ARMA Modeling)**

`stmcb` determines the coefficients for the system $b(z)/a(z)$ given an approximate impulse response $x$, as well as the desired number of zeros and poles. This function identifies an unknown system based on both input and output sequences that describe the system’s behavior, or just the impulse response of the system. In its default mode, `stmcb` works like `prony`

```
[b,a] = stmcb(x,3,3)
```

```
b =
   1.1165   -0.6213   -0.8365    1.3331

a =
   1.0000   -0.5401   -0.6109    1.1298
```

`stmcb` also finds systems that match given input and output sequences

```
y = filter(1,[1 1],x);  % Create an output signal.
[b,a] = stmcb(y,x,0,1)
```

```
b =
   1

a =
   1     1
```

In this example, `stmcb` correctly identifies the system used to create $y$ from $x$.

The Steiglitz-McBride method is a fast iterative algorithm that solves for the numerator and denominator coefficients simultaneously in an attempt to minimize the signal error between the filter output and the given output signal. This algorithm usually converges rapidly, but might not converge if the model order is too large. As for `prony`, `stmcb`’s resulting filter is not necessarily stable due to its exact modeling approach.

`stmcb` provides control over several important algorithmic parameters; modify these parameters if you are having trouble modeling the data. To change the number of iterations from the default of five and provide an initial estimate for the denominator coefficients

```
n = 10;            % number of iterations
a = lpc(x,3);      % initial estimates for denominator
[b,a] = stmcb(x,3,3,n,a);
```

In this example, `stmcb` correctly identifies the system used to create $y$ from $x$. 

The Steiglitz-McBride method is a fast iterative algorithm that solves for the numerator and denominator coefficients simultaneously in an attempt to minimize the signal error between the filter output and the given output signal. This algorithm usually converges rapidly, but might not converge if the model order is too large. As for `prony`, `stmcb`’s resulting filter is not necessarily stable due to its exact modeling approach.

`stmcb` provides control over several important algorithmic parameters; modify these parameters if you are having trouble modeling the data. To change the number of iterations from the default of five and provide an initial estimate for the denominator coefficients

```
n = 10;            % number of iterations
a = lpc(x,3);      % initial estimates for denominator
[b,a] = stmcb(x,3,3,n,a);
```
The function uses an all-pole model created with \texttt{prony} as an initial estimate when you do not provide one of your own.

To compare the functions \texttt{lpc}, \texttt{prony}, and \texttt{stmcb}, compute the signal error in each case

\[
\begin{align*}
\texttt{a1} &= \texttt{lpc}(x,3); \\
\texttt{[b2,a2]} &= \texttt{prony}(x,3,3); \\
\texttt{[b3,a3]} &= \texttt{stmcb}(x,3,3); \\
\texttt{[ x–impz(1,a1,10) x–impz(b2,a2,10) x–impz(b3,a3,10) ]}
\end{align*}
\]

\[
\texttt{ans} =
\begin{bmatrix}
0.1165 & 0 & 0 \\
-0.0058 & 0 & -0.0190 \\
-0.0535 & 0.0000 & 0.0818 \\
0.0151 & -0.0000 & -0.0176 \\
-0.1473 & -0.1143 & -0.0476 \\
0.2005 & 0.1897 & 0.0889 \\
0.0233 & 0.0154 & -0.0103 \\
0.1901 & 0.1669 & -0.0093 \\
0.0275 & 0.0251 & 0.0294 \\
0.0808 & 0.0751 & 0.0022
\end{bmatrix}
\]

\[
\texttt{sum(ans.^2)} =
\begin{bmatrix}
0.1226 & 0.0834 & 0.0182
\end{bmatrix}
\]

In comparing modeling capabilities for a given order IIR model, the last result shows that for this example, \texttt{stmcb} performs best, followed by \texttt{prony}, then \texttt{lpc}. This relative performance is typical of the modeling functions.

**Frequency-Domain Based Modeling**

The \texttt{invfreqs} and \texttt{invfreqz} functions implement the inverse operations of \texttt{freqs} and \texttt{freqz}; they find an analog or digital transfer function of a specified order that matches a given complex frequency response. Though the following examples demonstrate \texttt{invfreqz}, the discussion also applies to \texttt{invfreqs}.
To recover the original filter coefficients from the frequency response of a simple digital filter

\[
[b, a] = \text{butter}(4, .4) \quad \% \text{design Butterworth lowpass}
\]

\[
b = \\
0.0466 \quad 0.1863 \quad 0.2795 \quad 0.1863 \quad 0.0466
\]

\[
a = \\
1.0000 \quad -0.7821 \quad 0.6800 \quad -0.1827 \quad 0.0301
\]

\[
[h, w] = \text{freqz}(b, a, 64); \quad \% \text{compute frequency resp.}
\]

\[
[bb, aa] = \text{invfreqz}(h, w, 4, 4) \quad \% \text{model: nb = 4, na = 4}
\]

\[
bb = \\
0.0466 \quad 0.1863 \quad 0.2795 \quad 0.1863 \quad 0.0466
\]

\[
aa = \\
1.0000 \quad -0.7821 \quad 0.6800 \quad -0.1827 \quad 0.0301
\]

The vector of frequencies \( w \) has the units in rads/sample, and the frequencies need not be equally spaced. \text{invfreqz} finds a filter of any order to fit the frequency data; a third-order example is

\[
[bb, aa] = \text{invfreqz}(h, w, 3, 3) \quad \% \text{find third-order IIR}
\]

\[
bb = \\
0.0464 \quad 0.1785 \quad 0.2446 \quad 0.1276
\]

\[
aa = \\
1.0000 \quad -0.9502 \quad 0.7382 \quad -0.2006
\]

Both \text{invfreqs} and \text{invfreqz} design filters with real coefficients; for a data point at positive frequency \( f \), the functions fit the frequency response at both \( f \) and \(-f\).
By default, `invfreqz` uses an equation error method to identify the best model from the data. This finds $b$ and $a$ in

$$
\min_{b,a} \sum_{k=1}^{n} w(k)|h(k)A(w(k)) - B(w(k))|^2
$$

by creating a system of linear equations and solving them with MATLAB's \ operator. Here $A(w(k))$ and $B(w(k))$ are the Fourier transforms of the polynomials $a$ and $b$ respectively at the frequency $w(k)$, and $n$ is the number of frequency points (the length of $h$ and $w$). $w(k)$ weights the error relative to the error at different frequencies. The syntax

```
invfreqz(h,w,nb,na,wt)
```

includes a weighting vector. In this mode, the filter resulting from `invfreqz` is not guaranteed to be stable.

`invfreqz` provides a superior (“output-error”) algorithm that solves the direct problem of minimizing the weighted sum of the squared error between the actual frequency response points and the desired response

$$
\min_{b,a} \sum_{k=1}^{n} w(k)|h(k) - \frac{B(w(k))}{A(w(k))}|^2
$$

To use this algorithm, specify a parameter for the iteration count after the weight vector parameter

```
wt = ones(size(w)); % create unity weighting vector
[bbb,aaa] = invfreqz(h,w,3,3,wt,30) % 30 iterations
```

```
bbb =

0.0464  0.1829  0.2572  0.1549

aaa =

1.0000  -0.8664  0.6630  -0.1614
```
The resulting filter is always stable. Graphically compare the results of the first and second algorithms to the original Butterworth filter.

```
[H1,w1] = freqz(b,a,'magnitude','linear','phase','no');
[H2,w2] = freqz(bb,aa,'magnitude','linear','phase','no');
[H3,w3] = freqz(bbb,aaa,'magnitude','linear','phase','no');
plot(w1/pi,abs(H1),w2/pi,abs(H2),'--',w3/pi,abs(H3),':')
legend('Original','First Estimate','Second Estimate');
grid on
```

To verify the superiority of the fit numerically

```
sum(abs(h–freqz(bb,aa,w)).^2)   % total error, algorithm 1
ans =
0.0200
```

```
sum(abs(h–freqz(bbb,aaa,w)).^2) % total error, algorithm 2
ans =
0.0096
```
Resampling

The toolbox provides a number of functions that resample a signal at a higher or lower rate.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resample at new rate</td>
<td>resample</td>
</tr>
<tr>
<td>Decimation</td>
<td>decimate</td>
</tr>
<tr>
<td>Interpolation</td>
<td>interp</td>
</tr>
<tr>
<td>Apply FIR filter with resampling</td>
<td>upfirdn</td>
</tr>
<tr>
<td>Cubic spline interpolation</td>
<td>spline</td>
</tr>
<tr>
<td>Other 1-D interpolation</td>
<td>interp1</td>
</tr>
</tbody>
</table>

The `resample` function changes the sampling rate for a sequence to any rate that is a ratio of two integers. The basic syntax for `resample` is

\[
y = \text{resample}(x,p,q)
\]

where the function resamples the sequence \( x \) at \( p/q \) times the original sampling rate. The length of the result \( y \) is \( p/q \) times the length of \( x \).

One resampling application is the conversion of digitized audio signals from one sampling rate to another, such as from 48 kHz (the Digital Audio Tape standard) to 44.1 kHz (the Compact Disc standard). In the next example, the sampling rates are different but the idea is the same.
The example file contains a length 4001 vector of speech sampled at 7418 Hz.

```matlab
clear
load mtlb
whos
```

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Bytes</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fs</td>
<td>1x1</td>
<td>8</td>
<td>double array</td>
</tr>
<tr>
<td>mtlb</td>
<td>4001x1</td>
<td>32008</td>
<td>double array</td>
</tr>
</tbody>
</table>

Grand total is 4002 elements using 32016 bytes

```matlab
Fs
Fs =
    7418
```

To play this speech signal on a workstation that can only play sound at 8192 Hz, use the `rat` function to find integers \( p \) and \( q \) that yield the correct resampling factor.

```matlab
[p,q] = rat(8192/Fs,.0001)
p =
    127
q =
    115
```

Since \( p/q*Fs = 8192.05 \) Hz, the tolerance of 0.0001 is acceptable; to resample the signal at very close to 8192 Hz.

```matlab
y = resample(mtlb,p,q);
```

`resample` applies a lowpass filter to the input sequence to prevent aliasing during resampling. It designs this filter using the `firls` function with a Kaiser window. The syntax

```matlab
resample(x,p,q,l,beta)
```

tools the filter's length and the beta parameter of the Kaiser window. Alternatively, use the function `intfilt` to design an interpolation filter \( b \) and use it with

```matlab
resample(x,p,q,b)
```
The decimate and interp functions do the same thing as resample with \( p = 1 \) and \( q = 1 \), respectively. These functions provide different anti-alias filtering options, and they incur a slight signal delay due to filtering. The interp function is significantly less efficient than the resample function with \( q = 1 \).

The toolbox also contains a function, upfirdn, that applies an FIR filter to an input sequence and outputs the filtered sequence at a different sample rate than its original rate. See “Multirate Filter Bank Implementation” on page 1-19 and the reference description of upfirdn in Chapter 6 for more details.

The standard MATLAB environment contains a function, spline, that works with irregularly spaced data. The MATLAB function interp1 performs interpolation, or table lookup, using various methods including linear and cubic interpolation. See the online MATLAB Function Reference for information on spline and interp1.
Cepstrum Analysis

Cepstrum analysis is a nonlinear signal processing technique with a variety of applications in areas such as speech and image processing. The Signal Processing Toolbox provides three functions for cepstrum analysis.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex cepstrum</td>
<td>cceps</td>
</tr>
<tr>
<td>Real cepstrum</td>
<td>rceps</td>
</tr>
<tr>
<td>Inverse complex cepstrum</td>
<td>icceps</td>
</tr>
</tbody>
</table>

The complex cepstrum for a sequence $x$ is calculated by finding the complex natural logarithm of the Fourier transform of $x$, then the inverse Fourier transform of the resulting sequence.

$$
\hat{x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log|X(e^{j\omega})|e^{j\omega n} d\omega
$$

The toolbox function `cceps` performs this operation, estimating the complex cepstrum for an input sequence. It returns a real sequence the same size as the input sequence

$$
xhat = \text{cceps}(x)
$$

The complex cepstrum transformation is central to the theory and application of homomorphic systems, that is, systems that obey certain general rules of superposition. See [2] Oppenheim and Schafer for a discussion of the complex cepstrum and homomorphic transformations, with details on speech processing applications.

Try using `cceps` in an echo detection application. First, create a 45 Hz sine wave sampled at 100 Hz:

```matlab
  t = 0:0.01:1.27;
  s1 = sin(2*pi*45*t);
```
Add an echo of the signal, with half the amplitude, 0.2 seconds after the beginning of the signal.

\[ s2 = s1 + 0.5 \times [\text{zeros}(1,20) \ s1(1:108)]; \]

The complex cepstrum of this new signal is

\[ c = \text{cceps}(s2); \]
\[ \text{plot}(t,c) \]

Note that the complex cepstrum shows a peak at 0.2 seconds, indicating the echo.

The real cepstrum of a signal \( x \), sometimes called simply the cepstrum, is calculated by determining the natural logarithm of magnitude of the Fourier transform of \( x \), then obtaining the inverse Fourier transform of the resulting sequence

\[ c_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log|X(e^{i\omega})|e^{i\omega n}d\omega \]
The toolbox function `rceps` performs this operation, returning the real cepstrum for a sequence `x`. The returned sequence is a real-valued vector the same size as the input vector.

\[ y = rceps(x) \]

By definition, you cannot reconstruct the original sequence from its real cepstrum transformation, as the real cepstrum is based only on the magnitude of the Fourier transform for the sequence (see [2]). The `rceps` function, however, can reconstruct a minimum-phase version of the original sequence by applying a windowing function in the cepstral domain. To obtain both the real cepstrum and the minimum phase reconstruction for a sequence, use

\[ [y, ym] = rceps(x) \]

where `y` is the real cepstrum and `ym` is the minimum phase reconstruction of `x`.

**Inverse Complex Cepstrum**

To invert the complex cepstrum, use the `icceps` function. Inversion is complicated by the fact that the `cceps` function performs a data dependent phase modification so that the unwrapped phase of its input is continuous at zero frequency. The phase modification is equivalent to an integer delay. This delay term is returned by `ccceps` if you ask for a second output. For example,

```matlab
x = 1:10;
[xh, nd] = cceps(x)
```

\[ xh = \]

Columns 1 through 7
\[
2.2428\quad -0.0420\quad -0.0210\quad 0.0045\quad 0.0366\quad 0.0788\quad 0.1386
\]

Columns 8 through 10
\[
0.2327\quad 0.4114\quad 0.9249
\]

\[ nd = \]

\[ 1 \]
To invert the complex cepstrum, use `icceps` with the original delay parameter.

```matlab
icceps(xh,nd)
```

```
ans =
Columns 1 through 7
     1.0000     2.0000     3.0000     4.0000     5.0000     6.0000     7.0000
Columns 8 through 10
     8.0000     9.0000    10.0000
```

**NOTE**  With any modification of the complex cepstrum, the original delay term may no longer be valid. Use the `icceps` function with care.
FFT-Based Time-Frequency Analysis

The Signal Processing Toolbox provides a function, `specgram`, that returns the time-dependent Fourier transform for a sequence, or displays this information as a spectrogram. The time-dependent Fourier transform is the discrete-time Fourier transform for a sequence, computed using a sliding window. This form of the Fourier transform, also known as the short-time Fourier transform (STFT), has numerous applications in speech, sonar, and radar processing. The spectrogram of a sequence is the magnitude of the time-dependent Fourier transform versus time.

To display the spectrogram of a linear FM signal

```matlab
Fs = 10000;
t = 0:1/Fs:2;
x = vco(sawtooth(2*pi*t,.75),[0.1 0.4]*Fs,Fs);
specgram(x,512,Fs,kaiser(256,5),220)
```

Note that the spectrogram display is an image, not a plot.
Median Filtering

The function `medfilt1` implements one-dimensional median filtering, a nonlinear technique that applies a sliding window to a sequence. The median filter replaces the center value in the window with the median value of all the points within the window [4]. In computing this median, `medfilt1` assumes zeros beyond the input points.

When the number of elements $n$ in the window is even, `medfilt1` sorts the numbers, then takes the average of the $((n-1)/2)$ and $((n+1)/2)$ elements.

Two simple examples with fourth- and third-order median filters are

```
medfilt1([4 3 5 2 8 9 1],4)
ans =
    1.5000    3.5000    3.5000    4.0000    6.5000    5.0000    4.5000

medfilt1([4 3 5 2 8 9 1],3)
ans =
    3     4     3     5     8     8     1
```

See the Image Processing Toolbox User’s Guide for information on two-dimensional median filtering.
The toolbox provides three functions for communications simulation.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>modulate</td>
</tr>
<tr>
<td>Demodulation</td>
<td>demod</td>
</tr>
<tr>
<td>Voltage controlled oscillation</td>
<td>vco</td>
</tr>
</tbody>
</table>

Modulation varies the amplitude, phase, or frequency of a carrier signal with reference to a message signal. The `modulate` function modulates a message signal with a specified modulation method.

The basic syntax for the `modulate` function is

\[ y = \text{modulate}(x,Fc,Fs,'method',opt) \]

where:

- \( x \) is the message signal.
- \( F_c \) is the carrier frequency.
- \( F_s \) is the sampling frequency.
- \( method \) is a flag for the desired modulation method.
- \( opt \) is any additional argument that the method requires. (Not all modulation methods require an option argument.)

The table below summarizes the modulation methods provided; see Chapter 6 for complete details on each.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>amdsb–sc or am</td>
<td>Amplitude modulation, double side-band, suppressed carrier</td>
</tr>
<tr>
<td>amdsb–tc</td>
<td>Amplitude modulation, double side-band, transmitted carrier</td>
</tr>
</tbody>
</table>
If the input \( x \) is an array rather than a vector, \texttt{modulate} modulates each column of the array.

To obtain the time vector that \texttt{modulate} uses to compute the modulated signal, specify a second output parameter.

\[
[y,t] = \texttt{modulate}(x,Fc,Fs,'method',opt)
\]

The \texttt{demod} function performs demodulation, that is, it obtains the original message signal from the modulated signal.

The syntax for \texttt{demod} is

\[
x = \texttt{demod}(y,Fc,Fs,'method',opt)
\]

demod uses any of the methods shown for \texttt{modulate}, but the syntax for quadrature amplitude demodulation requires two output parameters.

\[
[X1,X2] = \texttt{demod}(y,Fc,Fs,'qam')
\]

If the input \( y \) is an array, \texttt{demod} demodulates all columns.

Try modulating and demodulating a signal. A 50 Hz sine wave sampled at 1000 Hz is

\[
t = (0:1/1000:2);
\]
\[
x = \sin(2\pi*50*t);
\]
With a carrier frequency of 200 Hz, the modulated and demodulated versions of this signal are
\[
y = \text{modulate}(x,200,1000,'am');
\]
\[
z = \text{demod}(y,200,1000,'am');
\]

To plot portions of the original, modulated, and demodulated signal
\[
\text{figure; plot}(t(1:150),x(1:150)); \text{title}('Original Signal');
\]
\[
\text{figure; plot}(t(1:150),y(1:150)); \text{title}('Modulated Signal');
\]
\[
\text{figure; plot}(t(1:150),z(1:150)); \text{title}('Demodulated Signal');
\]

The voltage controlled oscillator function \texttt{vco} creates a signal that oscillates at a frequency determined by the input vector. The basic syntax for \texttt{vco} is
\[
y = \text{vco}(x, F_c, F_s)
\]
where \( F_c \) is the carrier frequency and \( F_s \) is the sampling frequency.

To scale the frequency modulation range
\[
y = \text{vco}(x, [F_{min} \; F_{max}], F_s)
\]
In this case, \texttt{vco} scales the frequency modulation range so values of \( x \) on the interval \([-1 \ 1]\) map to oscillations of frequency on \([F_{\text{min}} \ F_{\text{max}}]\).

If the input \( x \) is an array, \texttt{vco} produces an array whose columns oscillate according to the columns of \( x \).

See “FFT-Based Time-Frequency Analysis” on page 4-27 for an example using the \texttt{vco} function.
Deconvolution

Deconvolution, or polynomial division, is the inverse operation of convolution. Deconvolution is useful in recovering the input to a known filter, given the filtered output. This method is very sensitive to noise in the coefficients, however, so use caution in applying it.

The syntax for deconv is

\[
[q, r] = \text{deconv}(b, a)
\]

where \(b\) is the polynomial dividend, \(a\) is the divisor, \(q\) is the quotient, and \(r\) is the remainder.

To try deconv, first convolve two simple vectors \(a\) and \(b\) (see Chapter 1 for a description of the convolution function)

\[
a = [1 2 3];
b = [4 5 6];
c = \text{conv}(a, b)
\]

\[
c =
\begin{array}{cccc}
4 & 13 & 28 & 27 & 18
\end{array}
\]

Now use deconv to deconvolve \(b\) from \(c\)

\[
[q, r] = \text{deconv}(c, a)
\]

\[
q =
\begin{array}{cccc}
4 & 5 & 6
\end{array}
\]

\[
r =
\begin{array}{cccc}
0 & 0 & 0 & 0 & 0
\end{array}
\]

See the System Identification Toolbox User’s Guide for advanced applications of signal deconvolution.
Specialized Transforms

In addition to the discrete Fourier transform (DFT) described in Chapter 1, the Signal Processing Toolbox and the MATLAB environment together provide the following transform functions:

- The chirp z-transform (CZT), useful in evaluating the z-transform along contours other than the unit circle. The chirp z-transform is also more efficient than the DFT algorithm for the computation of prime-length transforms, and it is useful in computing a subset of the DFT for a sequence.
- The discrete cosine transform (DCT), closely related to the DFT. The DCT’s energy compaction properties are useful for applications like signal coding.
- The Hilbert transform, which facilitates the formation of the analytic signal. The analytic signal is useful in the area of communications, particularly in bandpass signal processing.

Chirp z-Transform

The chirp z-transform, or CZT, computes the z-transform along spiral contours in the z-plane for an input sequence. Unlike the DFT, the CZT is not constrained to operate along the unit circle, but can evaluate the z-transform along contours described by

\[ z_i = AW^{-l}, \quad l = 0, \ldots, M - 1 \]

where A is the complex starting point, W is a complex scalar describing the complex ratio between points on the contour, and M is the length of the transform.
One possible spiral is

```matlab
A = 0.8*exp(j*pi/6);
W = 0.995*exp(-j*pi*.05);
M = 91;
z = A*(W.^(-(0:M-1)));
zplane([],z.')
```

czt(x,M,W,A) computes the z-transform of x on these points.

An interesting and useful spiral set is m evenly spaced samples around the unit circle, parameterized by \( A = 1 \) and \( W = \exp(-j\pi/M) \). The z-transform on this contour is simply the DFT, obtained by

\[ y = \text{czt}(x) \]

czt is faster than the \texttt{fft} function for computing the DFT of sequences with certain odd lengths, particularly long prime-length sequences. (Try comparing the execution time for the \texttt{fft} and \texttt{czt} functions for a sequence of length 1013.)
**Discrete Cosine Transform**

The toolbox function `dct` computes the unitary discrete cosine transform, or DCT, for an input vector or matrix. Mathematically, the unitary DCT of an input sequence $x$ is

$$y(k) = \sum_{n=1}^{N} w(n)x(n)\cos\frac{(2n-1)(k-1)}{2N}, \quad k = 1, ..., N$$

where

$$w(n) = \begin{cases} \frac{1}{\sqrt{N}}, & n = 1 \\ \frac{2}{\sqrt{N}}, & 2 \leq n \leq N \end{cases}$$

The DCT is closely related to the discrete Fourier transform; the DFT is actually one step in the computation of the DCT for a sequence. The DCT, however, has better energy compaction properties, with just a few of the transform coefficients representing the majority of the energy in the sequence. The energy compaction properties of the DCT make it useful in applications such as data communications.

The function `idct` computes the inverse DCT for an input sequence, reconstructing a signal from a complete or partial set of DCT coefficients. The inverse discrete cosine transform is

$$x(n) = \sum_{k=1}^{N} w(k)y(k)\cos\frac{(2n-1)(k-1)}{2N}, \quad n = 1, ..., N$$

where

$$w(k) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 1 \\ \frac{2}{\sqrt{N}}, & 2 \leq k \leq N \end{cases}$$
Because of the energy compaction mentioned above, it is possible to reconstruct a signal from only a fraction of its DCT coefficients. For example, generate a 10 Hz sinusoidal sequence, sampled at 1000 Hz.

```matlab
t = (0:1/999:1);
x = sin(2*pi*25*t);
```

Compute the DCT of this sequence and reconstruct the signal using only those components with value greater than 53 (12 of the original 1000 DCT coefficients).

```matlab
y = dct(x);               % compute DCT
y2 = find(abs(y) < 53);   % use 12 coefs.
y(y2) = zeros(size(y2));  % zero out points < 53
z = idct(y);              % reconstruct signal using inverse DCT
```

Plot the original and reconstructed sequences.

```matlab
plot(t,x)
plot(t,z), axis([0 1 –1 1])
```

![Original Signal](image1)

![Reconstructed Signal](image2)
One measure of the accuracy of the reconstruction is
\[
\frac{\text{norm}(x-z)}{\text{norm}(x)}
\]
that is, the norm of the difference between the original and reconstructed signals, divided by the norm of the original signal. In this case, the relative error of reconstruction is 0.1778. The reconstructed signal retains approximately 82% of the energy in the original signal.

**Hilbert Transform**

The toolbox function `hilbert` computes the Hilbert transform for a real input sequence `x` and returns a complex result of the same length
\[
y = \text{hilbert}(x)
\]
where the real part of `y` is the original real data and the imaginary part is the actual Hilbert transform. `y` is sometimes called the analytic signal, in reference to the continuous-time analytic signal. A key property of the discrete-time analytic signal is that its `z`-transform is 0 on the lower half of the unit circle. Many applications of the analytic signal are related to this property; for example, the analytic signal is useful in avoiding aliasing effects for bandpass sampling operations. The magnitude of the analytic signal is the complex envelope of the original signal.
The Hilbert transform is related to the actual data by a 90° phase shift; sines become cosines and vice versa. To plot a portion of data (solid line) and its Hilbert transform (dotted line):

```matlab
t = (0:1/1023:1);
x = sin(2*pi*60*t);
y = hilbert(x);
plot(t(1:50),real(y(1:50))), hold on
plot(t(1:50),imag(y(1:50)),':'), hold off
```

The analytic signal is useful in calculating instantaneous attributes of a time series, the attributes of the series at any point in time. The instantaneous amplitude of the input sequence is the amplitude of the analytic signal. The instantaneous phase angle of the input sequence is the (unwrapped) angle of the analytic signal; the instantaneous frequency is the time rate of change of the instantaneous phase angle. You can calculate the instantaneous frequency using `diff`, as described in the online MATLAB Function Reference.
References


## Interactive Tools

- **SPTool: An Interactive Signal Processing Environment**  .  5-2
- **Using SPTool**  .  5-3
- **Using the Signal Browser: Interactive Signal Analysis**  .  5-43
- **Using the Filter Designer: Interactive Filter Design**  .  5-59
- **Using the Filter Viewer: Interactive Filter Analysis**  .  5-84
- **Using the Spectrum Viewer: Interactive PSD Analysis**  .  5-97
- **Example: Generation of Bandlimited Noise**  .  5-113
SPTool: An Interactive Signal Processing Environment

The Signal Processing Toolbox includes an interactive graphical user interface (GUI), called SPTool, for performing digital signal processing tasks. SPTool provides an easy-to-use interface to many of the most important toolbox functions. With it, you can use the mouse and on-screen controls to import, view, measure, and print digital signals; design, view, and implement digital filters; and analyze the frequency content of signals.

This chapter describes how to use the different components of SPTool. Where appropriate, we point you to other areas of the manual that describe how to perform similar tasks by calling functions from the command line or from M-files.

The section “Example: Generation of Bandlimited Noise” at the end of this chapter describes how to use this graphical environment for a complete filter design and analysis task.

Overview

SPTool is a graphical environment for analyzing and manipulating digital signals, filters, and spectra. It is the starting point for using the interactive signal processing environment. In SPTool, you can import signals, filters, and spectra either from the workspace or as MAT-files. Through SPTool, you access four additional GUI tools that provide an integrated environment for signal browsing, filter design, analysis, and implementation. The four components of the interactive signal processing environment include:

- The Signal Browser, which provides a graphical view of the signal objects currently selected in SPTool and enables you to display, measure, analyze, and print these signals interactively
- The Filter Designer, which enables you to create and edit lowpass, highpass, bandpass, and bandstop FIR and IIR digital filters of various lengths and types using the filter design functions of the Signal Processing Toolbox
- The Filter Viewer, which enables you to view various characteristics of a filter that you've imported or designed, including its magnitude and phase responses, group delay, zero-pole plot, and impulse and step responses
- The Spectrum Viewer, which enables you to create, view, modify, and print spectra interactively, and to perform graphical analysis of frequency domain data using a variety of common methods of spectral estimation
Using SPTool

SPTool is the data management tool for the interactive GUI environment of the Signal Processing Toolbox. Using SPTool you can:

• Load a saved session
• Import a signal, filter, or spectrum
• Duplicate or clear a signal, filter, or spectrum
• Change the name of a signal, filter, or spectrum
• Change the sampling frequency of a signal or filter
• Activate the Signal Browser, Filter Viewer, Filter Designer, or Spectrum Viewer
• Save a session
• Use the Window menu to change to any open MATLAB figure window

Opening SPTool

Open SPTool from the MATLAB command window by typing

```
sptool
```

and pressing Enter.

Quick Start

Once SPTool is open, you can import data from the workspace or a file. You can then view it in the Signal Browser or generate its spectrum in the Spectrum Viewer.

To get started right away, work through the following example. Then continue through this chapter to learn the details of using SPTool and its component tools.

Or, you can skip the example, read through the rest of this section (from “Basic SPTool Functions” to “Using the Signal Browser: Interactive Signal Analysis”), and then work through the example.

Example: Importing Signal Data from a MAT-File

This example uses the sample file mtlb.mat, which is in the toolbox/signal/signal directory.
1 Select **Import** from the **File** menu.

The **Import to SPTool** window is displayed.

![Import to SPTool window](image)

2 Make sure that **Signal** is displayed in the **Import As** pop-up menu.

3 Press the **From Disk** radio button.

   You can either enter a MAT-file name or press **Browse** to open the file dialog box and select a MAT-file.

4 To get started, type the file name `mtlb` and press **Tab** or **Enter**. Note that SPTool adds the `.mat` extension automatically.

   The data from the file you selected is displayed in the **File Contents** list. In this example, `mtlb` is the signal data and `Fs` is the sampling frequency.

   Notice that `sig1` is displayed in the **Name** field. This is the default name of the imported signal.

5 Click on `mtlb` to select it, and then press the arrow at the left of the **Data** field. `mtlb` is transferred to the **Data** field.

6 Click on `Fs` to select it, and then press the arrow at the left of the **Sampling Frequency** field.

   `Fs` is transferred to the **Sampling Frequency** field.
7 Press OK.

The signal has been imported into SPTool with the name sig1.

You can look at this signal in the Signal Browser by pressing the View button under Signals.

You can look at the frequency content of the signal in the Spectrum Viewer by pressing the Create button under Spectra and then pressing Apply in the Spectrum Viewer.

**Basic SPTool Functions**

When you first open SPTool, it contains a collection of default signals, filters, and spectra. You can import additional signals, filters, and spectra into SPTool, and you can also design filters using the Filter Designer and create spectra using the Spectrum Viewer. You can save and export data from SPTool and customize many properties of the SPTool environment. The following figure shows the SPTool window and its File and Help menus, which are described below.
File Menu

Open Session. Select **Open Session...** from the **File** menu to load a saved session file. An SPTool session is saved in a file with an `.spt` extension.

Import. Select **Import...** from the **File** menu to import a signal, filter, or spectrum into SPTool from either the workspace or from a file. You can import variables from any MAT-file into SPTool. See “Importing Signals, Filters, and Spectra” on page 5-7 and “Example: Importing Signal Data from a MAT-File” on page 5-3 for more information.

Export. Select **Export...** from the **File** menu to export signals, filters, and spectra to the MATLAB workspace as structure variables. See “Saving Signal Data” on page 5-57, “Saving Filter Data” on page 5-79, and “Saving Spectrum Data” on page 5-110 for complete information.

Save Session. Select **Save Session** and **Save Session As...** from the **File** menu to save the current session. **Save Session** overwrites the existing session file. **Save Session As...** saves the current session with a name you specify. An SPTool session is saved in a MAT-file with an `.spt` extension.

Preferences. Select **Preferences...** from the **File** menu to customize preferences for the behavior of all the Signal Processing GUI tools. See “Customizing Preferences” on page 5-20 for a complete discussion.

Close. Select **Close** from the **File** menu to close SPTool and all other active Signal Processing GUI tools. SPTool prompts you to save if you have not recently saved the current session.

When you close SPTool, all signal and filter customization and ruler information set in any of the GUI tools are lost. Settings you changed and saved using the **Preferences** dialog box in SPTool are used the next time you open SPTool.

Help Menu

Overview... Select **Overview...** from the **File** menu to get general help on SPTool and the Signal Processing Toolbox GUI environment. This also gives you access to the MATLAB Help Desk.
Context Sensitive... Select **Context Sensitive...** from the **File** menu for help on a specific part of SPTool. When you click on **Context Sensitive...**, the mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in SPTool, including menu items, to find out what it is and how to use it.

Importing Signals, Filters, and Spectra

You can import a signal, filter, or spectrum into SPTool from either the workspace or from a file.

Select **Import...** from the **File** menu to open the **Import to SPTool** window:

[Diagram of Import to SPTool window]

Loading Variables from the MATLAB Workspace

To import variables from the MATLAB workspace, first list the workspace variables in the **Workspace Contents** list. Then select the variables to be imported into SPTool:
1 Click the **From Workspace** radio button.

The contents of the MATLAB workspace are displayed in the **Workspace Contents** list.

2 You can now import one or more variables from the **Workspace Contents** list into SPTool. See “Importing Workspace Contents and File Contents” below.

**Loading Variables from Disk**

To import variables from a MAT-file on disk, first list the file's variables in the **File Contents** list. Then select the variables to be imported into SPTool:

1 Click the **From Disk** radio button.

2 Type the name of the file you want to import into the **MAT-file Name** field and press **Tab** or **Enter**.

or

Press **Browse**, and then find and select the file you want to import using the File Search window. Press **OK**.

The data from the file you selected is displayed in the **File Contents** list.

3 You can now import one or more variables from the **File Contents** list into SPTool (see below).

**Importing Workspace Contents and File Contents**

Once you’ve loaded the contents of the workspace or a file into the **Workspace Contents** or **File Contents** list, you can select one or more variables from the list to import into SPTool. You can import a variable as a signal, a filter, or a spectrum. You can also import a variable whose value represents a sampling frequency or other design parameter.

Depending on whether you’re importing a signal, a filter, or a spectrum, you can customize different parameters before you import the data into SPTool. In each case, however, the general procedure for specifying a variable or a value is the same. In the following example, the selected variable is being imported as a signal. See “Importing a Signal” on page 5-10, “Importing a Filter” on page
5-11, and “Importing a Spectrum” on page 5-13 for details on customizing variables that are imported into SPTool.

1 Click on a variable name in the **Workspace Contents** list or the **File Contents** list to select it.

   If the variable is not a saved data object from SPTool, select the appropriate data type (**Signal**, **Filter**, or **Spectrum**) from the **Import As** pop-up menu and type a name into the **Name** field.

   ![Import As Menu](image)

   If the variable is a saved data object from SPTool, its name is displayed in the **Name** field, and its type (**Signal**, **Filter**, or **Spectrum**) is automatically selected in the **Import As** pop-up menu.

2 Press the arrow at the left of the **Data** field. The selected variable is transferred to the **Data** field.

   **NOTE** You can also type a variable name into the **Data** field directly.

3 To change the sampling frequency of the variable you’re importing, you can either:

   a Click on a variable in the **Workspace Contents** list or **File Contents** list whose value is a sampling frequency, and then press the arrow at the left of the **Sampling Frequency** field.

      The selected variable is transferred to the **Sampling Frequency** field.

   or

   b Type a value or variable name in the **Sampling Frequency** field.
4 Press OK.

The signal is imported into SPTool with the specified name and sampling frequency.

5 To import another variable, select Import... again, press the From Workspace or From File radio button, and repeat steps 1 through 4 for each variable that you want to load into SPTool.

NOTE When you’re importing from the workspace, you can specify either a variable or a value for each data field. When you’re importing from a disk, you can only specify variables.

Importing a Signal. When you import a signal, you specify:

• A variable name for the signal data (or the signal data values) in the Data field
• A variable or a value for the signal’s sampling frequency in the Sampling Frequency field
Importing a Filter. When you import a filter, first select the appropriate filter form from the **Form** pop-up menu.

Each filter form requires different variables.

- **For Transfer Function**, you specify the filter by its transfer function representation:

  \[
  H(z) = \frac{B(z)}{A(z)} = \frac{b_1 + b_2 z^{-1} + \cdots + b_{m+1} z^{-m}}{a_1 + a_2 z^{-1} + \cdots + a_{n+1} z^{-n}}
  \]

  - The **Numerator** field specifies a variable name or value for the numerator coefficient vector \( b \), which contains \( m+1 \) coefficients in descending powers of \( z \).
  - The **Denominator** field specifies a variable name or value for the denominator coefficient vector \( a \), which contains \( n+1 \) coefficients in descending powers of \( z \).

- **For State Space**, you specify the filter by its state-space representation:

  \[
  \dot{x} = Ax + Bu \\
  y = Cx + Du
  \]

  The **A-Matrix**, **B-Matrix**, **C-Matrix**, and **D-Matrix** fields specify a variable name or a value for each matrix in this system.

- **For Zeros, Poles, Gain**, you specify the filter by its zero-pole-gain representation:
Interactive Tools

\[ H(z) = \frac{Z(z)}{P(z)} = \frac{k(z-z(1))(z-z(2)) \cdots (z-z(m))}{(z-p(1))(z-p(2)) \cdots (z-p(n))} \]

- The **Zeros** field specifies a variable name or value for the zeros vector \( z \), which contains the locations of \( m \) zeros.
- The **Poles** field specifies a variable name or value for the zeros vector \( p \), which contains the locations of \( n \) poles.
- The **Gain** field specifies a variable name or value for the gain \( k \).

• For **2nd Order Sections** you specify the filter by its second-order section representation:

\[ H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}} \]

The **SOS Matrix** field specifies a variable name or a value for the \( L \)-by-6 SOS matrix

\[ \text{sos} = \begin{bmatrix} b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L} \end{bmatrix} \]

whose rows contain the numerator and denominator coefficients \( b_{ik} \) and \( a_{ik} \) of the second-order sections of \( H(z) \):

• For every filter, you specify:

A variable name or a value for the filter's sampling frequency in the **Sampling Frequency** field.

\[ \text{sos} \]
Importing a Spectrum. When you import a spectrum, you specify:

- A variable name or a value for the power spectral density (PSD) vector in the PSD field
- A variable name or a value for the frequency vector in the Freq. Vector field

The PSD values in the PSD vector correspond to the frequencies contained in the Freq. Vector vector; the two vectors must have the same length.

Working with Signals, Filters, and Spectra

When a signal, filter, or spectrum is imported into SPTool or created in SPTool, it is displayed in the appropriate list box, as shown below. Using the Edit menu functions and SPTool buttons, you can edit various properties of the data in SPTool and invoke all of SPTool's digital signal processing functions.
Component Lists in SPTool

Each signal, filter, and spectrum in SPTool is displayed in the appropriate Signals list, Filters list, or Spectra list.

- Signals are displayed with the signal type [vector] or [array]:
  - A vector signal ([vector]) has one column of data.
  - An array signal ([array]) has more than one column of data.
- Filters are displayed with the filter type [design] or [imported]:
  - A designed filter ([design]) is a filter that was created using the Filter Designer. This type of filter can also be edited in the Filter Designer.
  - An imported filter ([imported]) is a filter that was imported from the MATLAB workspace or a file, or one that was edited in the Pole/Zero Editor.
• Spectra are displayed with the spectrum type [auto]:
  
  An auto-spectrum ([auto]) is a spectrum whose source is a single signal, as opposed to the cross-spectrum of two channels of data. spectrum[auto] is the only spectrum type in SPTool.

Selecting Data Objects in SPTool

Each signal, filter, and spectrum in SPTool is one data object. A data object is selected when it is highlighted. When you first import or create a data object, it is selected.

The Signals list shows all vector and array signals in the current SPTool session.

The Filters list shows all designed and imported filters in the current SPTool session.

The Spectra list shows all spectra in the current SPTool session.

You can select a single data object in a list, a range of data objects in a list, or multiple separate data objects in a list. You can also have data objects simultaneously selected in different lists:

• To select a single unselected data object, click on it. All other data objects in that list box become unselected.

• To add or remove a range of data objects, Shift-click on the data objects at the top and bottom of the section of the list that you want to add.

• To add a single data object to a selection or remove a single data object from a multiple selection, Ctrl-click on the object. You can also use the right mouse button.

Editing Data Objects in SPTool

The Edit menu entries are available only when there is at least one selected data object (signal, filter, or spectrum) in SPTool. Use the Edit menu to duplicate and clear objects in SPTool and to edit object names and change sampling frequencies.

A signal, filter, or spectrum must be selected to be edited. When you click on an Edit menu entry, all selected data objects are displayed in a pop-up menu.
To edit an SPTool object:

1. Select a signal, filter, or spectrum.
2. Select the appropriate **Edit** menu function.
   
   The pop-up menu shows the names of all selected data objects.
3. Drag to choose a specific signal, filter, or spectrum for editing.

**Duplicate.** Use **Duplicate** from the **Edit** menu to make a copy of the selected signal, filter, or spectrum in SPTool.

Select **Duplicate** and drag to choose the signal, filter, or spectrum you want to copy. When you select a data object to duplicate it, a new data object of the same type is automatically created. The new data object is named as a copy of the selected data object. It is placed at the bottom of the list and is selected. You can change its name using **Name...** from the **Edit** menu.

**Clear.** Use **Clear** from the **Edit** menu to delete the selected signal, filter, or spectrum from SPTool.

Select **Clear** and drag to choose the signal, filter, or spectrum you want to remove. The data object is deleted from the current SPTool session.

**Name.** Use **Name...** from the **Edit** menu to give the selected signal, filter, or spectrum a new, unique name.

1. Select **Name...** and drag to choose the signal, filter, or spectrum you want to rename.
   
   The **Name Change** dialog box is displayed.

2. Type in the new name and press **OK**.

**Sampling Frequency.** Use **Sampling Frequency...** from the **Edit** menu to supply a sampling frequency for a selected signal or filter. The sampling frequency may be a number, such as 1, 0.001, or 1/5000, or a valid MATLAB expression including workspace variables, such as \( F_s \), \( 1/T_s \), or \( \cos(0.1\pi) \).
1 Select **Sampling Frequency**... and drag to choose the signal or filter you want to change.

The **Sampling Frequency**... dialog box is displayed.

2 Type in the value, variable name, or expression and press **OK**.

### Viewing a Signal

Use the Signal **View** button to make the Signal Browser active and view one or more imported signals. The Signal Browser provides tools for graphical analysis of the selected signal(s).

Select one or more signals from the **Signals** list and press the **View** button in the signal panel. The Signal Browser displays the selected signal(s). See “Using the Signal Browser: Interactive Signal Analysis” on page 5-43 for a full description of Signal Browser functions and operations.

### Viewing a Filter

Use the **View** button in the **Filters** panel to make the Filter Viewer active and view imported filters or filters designed/edited in the Filter Designer. The Filter Viewer provides tools for analyzing filters; you can investigate the magnitude response, phase, group delay, zeros and poles, and impulse and step responses of the selected filters.

Select one or more filters from the **Filters** list and press the **View** button in the filter panel. The Filter Viewer displays the selected filters. See “Using the Filter Viewer: Interactive Filter Analysis” on page 5-84 for a full description of Filter Viewer functions and operations.

### Designing a Filter

**New Design.** Use the Filter **New Design** button to make the Filter Designer active and design a filter. The Filter Designer lets you create FIR and IIR digital filters of various lengths and types using the filter design functions in the Signal Processing Toolbox.

Press the **New Design** button in the **Filter** panel. The Filter Designer displays a filter created with the default settings and assigns it a default name of filt\(n\), where \(n\) is a unique sequential identifying digit. Once the default filter is created, you can use it as is or edit it with the Filter Designer. You can rename it using **Name...** from the **Edit** menu.
**Edit Design.** Use the **Filter** panel’s **Edit Design** button to make the Filter Designer active and edit a filter. Imported filters always open in the Pole/Zero Editor.

Select one or more filters from the **Filters** list and press the **Edit Design** button in the filter panel. The Filter Designer displays the first of the selected filters. The other selected filters can be edited, one at a time, by selecting them from the **Filter** pop-up menu. See “Using the Filter Designer: Interactive Filter Design” on page 5-59 for a full description of Filter Designer functions and operations.

**Applying a Filter**

Use the **Apply Filter** button to apply a filter to a selected signal. This creates a new signal:

1. Select one signal from the **Signals** list and one filter from the **Filters** list and press the **Apply Filter** button in the filter panel.

   The **Apply Filter** dialog box is displayed.

   Drag to select the algorithm you want to use.

   Type the new signal name here.

2. Select the filtering algorithm from the **Algorithm** pop-up menu, type the name for the new signal in the **Output Signal** field, and press **OK**.

   The available filter algorithms are:
   - Transposed direct-form II (uses the `filter` function)
   - Zero-phase IIR (uses the `filtfilt` function)
   - FFT-based FIR (uses the `fftfilt` function)

   The selected filter is applied to the selected input signal and the new output signal is listed in the **Signals** list.
Creating a Spectrum
Use the Create button to activate the Spectrum Viewer and generate a default spectrum of a selected signal. Once you’ve generated a spectrum, you can view it in a variety of ways, measure it, and modify it in the Spectrum Viewer.

1 Select one signal from the Signals list and press the Create button in the Spectra panel.

The Spectrum Viewer is activated and a spectrum object with default parameters is created in the Spectra panel. The PSD data is not computed or displayed yet. The newly created spectrum object has a default name of specn, where n is a unique sequential identifying digit. Once the spectrum object is created, you can use it with the default settings, or continue to edit it in the Spectrum Viewer. You can rename it using Name... from the Edit menu.

2 Modify the spectrum parameters (e.g., Method and Nfft) as desired.

3 Press Apply in the Spectrum Viewer to compute the spectrum data. This button is enabled when the spectrum has just been created or when you have changed one or more parameters in the Spectrum Viewer.

The updated spectrum is displayed in the Spectrum Viewer. See “Using the Spectrum Viewer: Interactive PSD Analysis” on page 5-97 for a full description of Spectrum Viewer parameters and displays.

Viewing a Spectrum
Select one or more spectra from the Spectra list and press View in the Spectra panel. The Spectrum Viewer displays the selected spectrum or spectra.

Updating a Spectrum
Use the Update button to update the selected spectrum so that it reflects the data in the currently selected signal.

1 Select one signal from the Signals list and one spectrum from the Spectra list and press Update in the Spectra panel.

The spectral data from the current spectrum is removed from the Spectrum Viewer. The Spectrum Viewer is activated. No spectrum is displayed yet.
2 Press **Apply** in the Spectrum Viewer to compute the spectrum and complete the update.

The spectrum is regenerated with the same parameters but using the data in the currently selected signal. This feature is useful when you have altered a signal (by filtering it, for example), and want to update the existing spectrum to reflect the change.

**Customizing Preferences**

Use **Preferences** from the **File** menu to customize displays and certain parameters for SPTool and its four component tools. The new settings are saved on disk and are used when you restart MATLAB.

In the **Preferences** panels, you can:

- Select colors and markers for rulers, and set the initial ruler style
- Select color and line style sequence for displayed signals
- Configure axis labels, and enable/disable rulers, panner, and mouse zoom in the Signal Browser
- Configure axis parameters, and enable/disable rulers and mouse zoom in the Spectrum Viewer
- Configure filter and axis parameters and enable/disable mouse zoom in the Filter Viewer
- Configure tiling preferences in the Filter Viewer
- Specify FFT length, and enable/disable mouse zoom and grid in the Filter Designer
- Enable/disable use of a default session file
- Configure filters for export to the Control System Toolbox (for users of that product)
- Enable/disable search for plug-ins at start-up

When you first select **Preferences**..., the **Preferences** dialog box displays the **Rulers** panel. You can change the settings for rulers, or choose on any of the other settings categories to customize other settings.

Click once on a settings category to select it.
The following sections describe all of the settings you can modify. The illustrations show the default settings for each category. For additional information on preference settings, use the Help... button at the bottom of the Preferences dialog box.

Ruler Settings

The Rulers preferences apply to the rulers in the Signal Browser, Spectrum Viewer, and Filter Viewer. These preferences also apply to printouts from the Signal Browser and Spectrum Viewer.

Ruler Color. Specifies the color of the rulers. Color is specified as a string (e.g., 'r') or RGB triple (e.g., [1 0 0]).

Ruler Marker. Specifies the marker used in the track and slope rulers.

Marker Size. Specifies the size of the ruler marker in points. This can be any positive value.

Initial Type. Specifies the type of ruler (horizontal, vertical, track, or slope) that is selected when you first open the Signal Browser, Spectrum Viewer, or Filter Viewer.
Color Settings

The **Colors** preferences apply to signals displayed in the Signal Browser, Spectrum Viewer, and Filter Viewer. These preferences also apply to printouts from the Signal Browser and Spectrum Viewer.

**Color Order.** Specifies the color order to cycle through for data plotted in the Signal Browser, Spectrum Viewer, and Filter Viewer. The default is the axis color order.

Type in a new value or value string to change the color order. Color is specified as a string (e.g., 'r') or RGB triple (e.g., [1 0 0]), an n-by-3 matrix of n colors, or an n-by-1 cell array of such objects.

**Line Style Order.** Specifies the line styles to cycle through for data plotted in the Signal Browser, Spectrum Viewer, and Filter Viewer. The default is the axis line style order.

Type in a new string value (e.g., ' - - ' ) or an array of strings (e.g., {' - ',' - - ',' : '}) to change the line style order.
Signal Browser Settings

The Signal Browser preferences let you set optional x-axis and y-axis labels, enable and disable the display of the rulers and the panner, and toggle zoom persistence.

Except for zoom persistence, these preferences also apply to printing from the Signal Browser. See “Printing Signal Data” on page 5-54 for details on printing from the Signal Browser.

X Label, Y Label. Type in a string for the x-axis label and the y-axis label in the Signal Browser. The default is Time for the x-axis.

Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Signal Browser. See “Ruler Controls” on page 5-33 for details on using the rulers in the Signal Browser.

Panner. Click in the check box to display (checked) or hide (unchecked) the panner in the Signal Browser. See “Panner Display” on page 5-52 for details on using the panner in the Signal Browser.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Signal Browser. See “Zoom Controls” on page 5-31 for details on zoom controls in the Signal Browser.
Spectrum Viewer Settings

The Spectrum Viewer preferences let you set axis parameters, enable and disable the display of the rulers, and toggle zoom persistence.

Except for zoom persistence, these preferences also apply to printing from the Spectrum Viewer. See “Printing Spectrum Data” on page 5-107 for details on printing from the Spectrum Viewer.

Magnitude Axis Scaling. Specifies the scaling units for the magnitude (y) axis in the Spectrum Viewer. Scaling units can be decibels or linear.

Frequency Axis Scaling. Specifies the scaling units for the frequency (x) axis in the Spectrum Viewer. Scaling units can be linear or log.

Frequency Axis Range. Specifies the numerical range for the frequency (x) axis in the Spectrum Viewer. Scaling options are $[0,Fs/2]$, $[0,Fs]$, or $[-Fs/2,Fs/2]$.

Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Spectrum Viewer. See “Ruler Controls” on page 5-33 for details on using the rulers in the Spectrum Viewer.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Spectrum Viewer. See “Zoom Controls” on page 5-31 for details on zoom controls in the Spectrum Viewer.
Filter Viewer Settings

The **Filter Viewer** preferences let you set key filter plot configuration parameters and toggle zoom persistence.

**FFT Length.** Specifies the number of points at which the frequency response is computed (for the magnitude, phase, and group delay plots).

**Time Response Length.** Specifies the time response length (in samples) for the impulse and step response plots. An empty value, [], for an FIR filter indicates that the complete response length will be shown; an empty value for an IIR filter indicates that the response length will be automatically determined using the `impz` function.

**Magnitude Axis Scaling.** Specifies the scaling units for the magnitude (y) axis in the Filter Viewer. Scaling units can be **linear**, **log**, or **decibels**.

**Phase Units.** Specifies the phase units for the phase response plot. Phase units can be **degrees** or **radians**.

**Frequency Axis Scaling.** Specifies the scaling units for the frequency (x) axis in the Filter Viewer. Scaling units can be **linear** or **log**.

**Frequency Axis Range.** Specifies the numerical range for the frequency (x) axis in the Filter Viewer. Scaling options are [0,Fs/2], [0,Fs], or [-Fs/2,Fs/2].
Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Filter Viewer. See “Ruler Controls” on page 5-33 for details on using the rulers in the Filter Viewer.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Filter Viewer. See “Zoom Controls” on page 5-31 for details on zoom controls in the Filter Viewer.

Filter Viewer Tiling Settings
The Filter Viewer-Tiling preferences let you change the way the Filter Viewer displays the analysis plots.

Click the radio button to select how the plots are tiled in the display area. Options are 2-by-3 Grid, 3-by-2 Grid, Vertical (6-by-1 Grid), and Horizontal (1-by-6 Grid).

This specifies how the plots are arranged when all six plot options are turned on. When fewer options are turned on, the plots are displayed as symmetrically as possible.
Filter Designer Settings

The Filter Designer preferences let you set key filter configuration and plot parameters.

**FFT Length.** Specifies the number of points used to calculate a filter’s frequency response.

**Display grid lines.** Turns plot grid lines on (checked) or off (unchecked).

**Auto Design – initial value.** Specifies the default setting for the Auto Design check box in the Filter Designer. When the Filter Designer is first launched, the Auto Design check box will have the same setting (checked or unchecked) as the Auto Design – initial value check box here.

**Stay in Zoom-mode After Zoom.** Turns persistent zooming on and off, as described in “Zoom Controls” on page 5-31.
Default Session Setting

The Default Session preference lets you specify whether or not to load a default session file when SPTool is launched.

**Default Session**. Enables (checked) or disables (unchecked) loading of a default session file, which must have the name `startup.spt`, when starting SPTool. An SPTool session file is a MAT-file (see Using MATLAB) with a `.spt` extension.

The Default Session preference only affects the session contents when you start SPTool. You can load a new session file after starting SPTool using the Open Session option in the SPTool File menu. You can also save a session file at any time using the Save Session option in the File menu. If you close SPTool without having recently saved a session file, you will be prompted to do so. (See “File Menu” on page 5-6.)

**NOTE** Changes that you make in the SPTool preference panels are saved in a separate MAT-file, which is reloaded the next time you open SPTool (see “Saving and Discarding Changes to Preferences Settings” on page 5-30). The Default Session preference does not affect these preference settings.
Exporting Components Setting

The **Exporting Components** preference lets you specify whether or not to export filters created in SPTool as TF objects for the Control System Toolbox.

---

Export Filters as TF objects. Enables (checked) or disables (unchecked) exporting of SPTool filters as TF (transfer function) objects, for use in the Control System Toolbox (see the Control System Toolbox User's Guide).

---

**NOTE** This preference category only appears in the SPTool **Preferences** dialog box if you have installed the Control System Toolbox.
Plug-Ins Setting

The **Plug-Ins** preference lets you search for plug-ins when SPTool is started up.

![Preferences for SPTool window](image)

**Search for Plug-Ins at start-up.** Enables (checked) or disables (unchecked) searching for installed plug-ins.

A plug-in is an extension to SPTool. Plug-ins include customized add-on panels and new buttons in the panels in SPTool, new spectral methods in the Spectrum Viewer, and new SPTool preferences. You can also plug one or more toolboxes into SPTool.

You need to use this setting only when you have installed extensions or have other toolboxes plugged into SPTool.

To use SPTool with extensions, check **Search for Plug-ins at start-up**, close SPTool, and restart it.

**Saving and Discarding Changes to Preferences Settings**

The buttons at the bottom of the **Preferences** panels let you save or discard any changes you have made, or return to the default settings.

![Buttons at the bottom of the Preferences panels](image)

**Factory Settings.** Restores the preferences in the current panel to their original settings; that is, the settings at the time the Signal Processing Toolbox was first installed.
**Revert Panel.** Cancels changes in the current panel only. Settings in the current panel revert to the previous settings.

**Cancel.** Cancels changes in all preference categories and closes the Preferences dialog box. Settings in all panels revert to their previous state.

**OK.** Applies changes in all Preferences panels and closes the Preferences dialog box. Settings in all panels are saved in a MAT-file called sigprefs.mat. If sigprefs.mat does not exist, either on the current MATLAB path or in the current directory, you are prompted for a location to save the file. The saved settings are used the next time you open SPTool.

### Controls for Viewing and Measuring

The GUI tools share common controls for viewing and measuring signals. These controls are described in this section. Not all tools use all of the viewing and measuring controls; specific details about the tools and procedures for viewing and measuring are described in the section on each tool.

### Zoom Controls

The GUI tools share a common set of zoom control buttons. The Signal Browser and Spectrum Viewer use the same set of common zoom control buttons, which affect both on-screen and printed images.

![Zoom Controls](image)

The Filter Designer has one additional viewing button, the **Pass Band** button.

![Pass Band Button](image)

The Filter Viewer has a subset of the zoom control buttons.

![Filter Viewer Zoom Controls](image)

Each button works the same way in every GUI tool in which it occurs. In ordinary use, you press a button once to zoom in or out of the signal display.
**Zoom In-X, Zoom Out-X, Zoom In-Y, and Zoom Out-Y.** Click once to perform one zoom operation (in or out) on the x- or y-axis. Each zoom operation changes the axes limits by a factor of two on the specified axis, about the center of the displayed signal. You can press repeatedly on one or more buttons to continue to change the scale in one or both axes.

When you zoom in the x-axis (horizontal scaling), the y limits (vertical scaling) of the main axes are not changed. Similarly, when you zoom in the y-axis, the x limits of the main axes are not changed.

**Full View.** Click once to restore the displayed signal to its full range in both axes.

**Mouse Zoom.** Click once to activate zoom mode. The cursor changes to a crosshair. You can either zoom in without specifying a zoom window, or you can use a zoom rectangle to select a specific zoom window. In either case, the x- and y-axis are automatically adjusted to display the selected signal:

- To zoom in without specifying a zoom window, click on the plot. The position of the crosshair is the center of a zoom operation that halves both the x- and y-axis limits.
- To use a zoom rectangle, click where you want the rectangle to begin, drag the mouse diagonally to where you want it to end, and release the mouse button.
- To get out of mouse zoom mode without zooming in or out, press the Mouse Zoom button again.

**Zoom Persistence.** Mouse zooming can either be one-time or persistent:

- One-time zooming is activated when you press the Mouse Zoom button. It automatically turns itself off after you click in the display area and the zoom operation occurs. This is the default for all the tools.
- Persistent zooming is also activated by pressing the Mouse Zoom button. It does not turn off after you click in the display area and a zoom operation occurs; you can continue to click and zoom without resetting the Mouse Zoom button.

You can change whether zooming is one-time or persistent by selecting Preferences... from the File menu and toggling Stay in Zoom-mode After Zoom in the Preferences panels for the Signal Browser, the Spectrum Viewer, the Filter Designer, and the Filter Viewer.
When **Stay in Zoom-mode After Zoom** is selected, zooming is persistent. To turn off mouse zooming when **Stay in Zoom-mode After Zoom** is selected, press the **Mouse Zoom** button.

**Passband Zoom (Filter Designer).** Click once to zoom in on the passband of the response.

Both the x- and y-limits of the main axes are changed so that the passband fills the main axes.

There is no stopband zoom button. To zoom the stopband, use standard **Mouse Zoom**, centering the crosshair on the area of the stopband you want to view. If you are in passband zoom, first press **Full View** to return to the standard view.

**Ruler Controls**

The Signal Browser, Filter Viewer, and Spectrum Viewer share a common set of ruler controls. Use the rulers to make measurements on the signals or spectra in the main axes (display) area. The ruler controls give you a variety of ways to read and control the values of the rulers in the main axes. With the rulers you can measure such information as the vertical and horizontal distance between features in a signal or spectrum, the dimensions of peaks and valleys, and slope information.

In the following discussion, the Signal Browser is shown. The ruler controls include the **Selection** controls at the top right of the window and the buttons and edit boxes in the **Rulers** panel. The controls in the Filter Viewer and Spectrum Viewer work the same way. The controls in the Signal Browser and the Spectrum Viewer also affect images printed from these tools.

In the Filter Viewer, the rulers appear on only one subplot at a time. You can choose which subplot the rulers appear on by selecting the subplot from the pop-up menu at the top of the **Rulers** panel, or by clicking on a line in the plot that you want to measure. If a subplot is not currently visible when you select its name from the pop-up menu, the Filter Viewer creates the subplot and places the rulers in it.
Selecting a Line to Measure. When there is only one signal displayed, the displayed signal is automatically selected and is measured when you use the rulers. When there is more than one signal displayed, only one signal (line) may be selected and measured at a time.

When a signal (line) is selected, you can use the ruler controls (**Vertical**, **Horizontal**, **Track**, or **Slope**) and the **Peaks** and/or **Valleys** controls on the selected line. The label of the selected signal (line) is displayed in the **Selection** pop-up menu.

There are two ways to select a signal (line):

- Click on the **Selection** pop-up menu and drag to select the line to measure.
  
  All signals that are currently selected in SPTool are listed. Vector signals in the Signal Browser, spectra in the Spectrum Viewer, and filters in the Filter Viewer are listed as single variables; in the Signal Browser, each column of a two-dimensional signal matrix is listed as a separate variable.

- Move the mouse pointer over any point in the line you want to select and click on it.
The label of the signal, including the column number if the line is one column of a matrix, is displayed in the Selection pop-up menu.

The line selection display changes to the color and pattern of the selected signal, spectrum, or filter.

**Line Selection Pop-Up Menu.** Use to select a line (vector signal, array column, filter, or spectrum) to measure.

Click the Selection pop-up menu and drag to select the line.

**Line Selection Display.** The line color and style of the selected signal are displayed.

**Color... Button.** Use to edit the line style or display color of the selected line.

Click on the Color... button at the top right of the window to display the Edit Line pop-up menu, which is shown on the left. The label of the selected line is displayed in the Label field.

- Click the Line Style pop-up menu and drag to select a line style, as shown on the left.
- Click a radio button to select a color. If you select Other, you can type a color value in the Enter colorspec box; the color value can be a string (e.g., ‘r’) or an RGB triple (e.g., [1 0 0]).
- Click OK to apply the line style and color you selected.

**Find Ruler Buttons.** Use the find ruler buttons to bring one or both rulers into the viewing area of the main axes. When both rulers are within the signal display (main axes) area, the Find Ruler buttons, at the top right of the main axes area, are not displayed.

If the rulers are not within the signal display area, both Find Ruler buttons are displayed, as shown on the right.

If one ruler is within the signal display area, the button for the other ruler is displayed, as shown on the right.

Click a Find Ruler button to bring the specified ruler into the display area.

When a ruler is visible, you can click on it and drag it to make a measurement on the selected signal. See “Making Signal Measurements” on page 5-37 for details on manipulating the rulers and measuring the selected signal.
Ruler Control Buttons. Use the ruler control buttons to select the type of measurement you want to make: **Vertical**, **Horizontal**, **Track**, or **Slope**. The default setting is **Track**.

Click a ruler control button to select it. The **Rulers** panel changes depending on which ruler control is selected. See "Making Signal Measurements" on page 5-37 for details on the four kinds of measurements you can make and the parameters for each one.

**Rulers Panel and Edit Boxes.** The **Rulers** panel changes depending on which ruler control is selected: **Vertical**, **Horizontal**, **Track**, or **Slope**. It shows the parameters for the selected ruler control. Depending on which ruler control is selected, the following fields are displayed: $x_1$, $y_1$, $x_2$, $y_2$, $dx$, $dy$, $m$. The picture on the left shows the **Rulers** panel when **Slope** is selected.

When you press a ruler control button, rulers are displayed superimposed on the signal(s) in the main axes display area. The rulers are either vertical (for **Vertical**, **Track**, and **Slope**) or horizontal (for **Horizontal**). For **Track** and **Slope**, ruler markers are also displayed. The rulers and ruler markers are associated with the currently selected signal. The following picture shows the rulers and ruler markers that are displayed when **Slope** is selected.

To position a ruler, you can click and drag on it. When you drag a ruler, the parameters in the **Rulers** panel change to reflect the measurements on the selected signal.

You can also position a ruler by specifying parameters in the edit boxes in the **Rulers** panel. The parameters are either the $x_1$ and $x_2$ values or the $y_1$ and $y_2$ values, depending on which ruler control is selected.
Type the value or variable for the ruler parameter in the $x_1$ and $x_2$ boxes or the $y_1$ and $y_2$ boxes. See “Making Signal Measurements” on page 5-37 for details on manipulating the rulers and the parameters you can measure with each one.

Peaks and Valleys. Use these buttons to show or hide the local maxima and/or local minima of the currently selected signal, filter response, or spectrum. Only peaks or valleys, or both peaks and valleys may be displayed.

- Click Peaks to toggle showing (down) or hiding (up) the maxima of the signal.
- Click Valleys to toggle showing (down) or hiding (up) the minima of the signal.

In track and slope mode (see “Making Signal Measurements” on page 5-37), the rulers are constrained to the peaks or valleys. In horizontal and vertical mode, the peaks and valleys are only visual and do not affect the behavior of the rulers.

Save Rulers... Button. Once you’ve set up and made a certain set of measurements, you may find it useful to save them for future reference. Use the Save Rulers... button to save a structure in the MATLAB workspace with the fields $x_1$, $y_1$, $x_2$, $y_2$, $dx$, $dy$, $m$, peaks, and valleys. Undefined values are set to NaN.

1. Click Save Rulers... to save the current measurements as a variable in the workspace.

   The Save Rulers dialog box is displayed.

2. Type a variable name in the edit field and press OK.

Making Signal Measurements

Use the rulers to make measurements on a selected line, which is a vector or a column of a matrix in the Signal Browser, a filter response in the Filter Viewer, or a spectrum in the Spectrum Viewer. To make a measurement:

1. Select a line as described in “Selecting a Line to Measure” on page 5-34.

2. Apply a ruler to the display as described in “Ruler Control Buttons” on page 5-36.
3 Position a ruler where you want it in the main axes area by clicking and dragging it:

- Move the mouse over the ruler (1 or 2) that you want to drag.
  
  The hand cursor is displayed when you're over a ruler, with the ruler number inside it: ![Hand Cursor]

- Click and drag the ruler to where you want it on the signal.

  Depending on which ruler control is selected, you can drag the ruler to the right and left (Vertical, Track, and Slope) or up and down (Horizontal).

As you drag a ruler, the Rulers panel shows the current position of both rulers. Depending on which ruler control is selected, the following fields are displayed: \( x_1, y_1, x_2, y_2, dx, dy, m \). These fields will also be displayed in printouts from the Signal Browser, unless you suppress them. See “Printing Signal Data” on page 5-54 and “Printing Spectrum Data” on page 5-107 for details on printing from these tools.

You can also position a ruler by typing its \( x_1 \) and \( x_2 \) or \( y_1 \) and \( y_2 \) values in the Rulers panel, as described on page 5-36.

**Ruler Controls: Vertical.** There are two vertical rulers, called ruler 1 and ruler 2. When vertical rulers are in use, the measurements displayed in the Rulers panel...
panel are $x_1$ (the position of ruler 1 on the x-axis), $x_2$ (the position of ruler 2 on the x-axis), and $dx$ (the value of $x_2-x_1$).

Press **Vertical** to put the rulers in vertical mode.

In vertical mode, you may change the x-values of the rulers (that is, their horizontal position). As the $x_1$ and $x_2$ values change, the value of $dx$ changes automatically.

Change the $x_1$ and $x_2$ values in one of two ways:

- Drag the rulers to the left and the right with the mouse
- Enter their values in the $x_1$ and $x_2$ edit boxes in the **Rulers** panel

**Ruler Controls: Horizontal.** There are two horizontal rulers, called ruler 1 and ruler 2. When horizontal rulers are in use, the measurements displayed in the
Rulers panel are $y_1$ (the position of ruler 1 on the y-axis), $y_2$ (the position of ruler 2 on the y-axis), and $dy$ (the value of $y_2 - y_1$).

Press **Horizontal** to put the rulers in horizontal mode.

In horizontal mode, you may change the y-values of the rulers (that is, their vertical position). As the $y_1$ and $y_2$ values change, the value of $dy$ changes automatically.

Change the $y_1$ and $y_2$ values in one of two ways:

- Dragging the rulers up and down with the mouse
- Entering their values in the $y_1$ and $y_2$ edit boxes in the Rulers panel

Ruler Controls: Track. There are two vertical rulers, called ruler 1 and ruler 2, with a marker on each that shows the y-values of the signal at the x-values of the rulers. When track rulers are in use, the measurements displayed in the Rulers panel are $x_1$ (the position of ruler 1 on the x-axis), $y_1$ (the position of...
ruler 1 on the y-axis), \(x_2\) (the position of ruler 2 on the x-axis), \(y_2\) (the position of ruler 2 on the y-axis), \(dx\) (the value of \(x_2 - x_1\)), and \(dy\) (the value of \(y_2 - y_1\)).

Press **Track** to put the rulers in track mode.

You can change the track marker in the **Rulers** panel from the **Preferences** dialog box; see “Ruler Settings” on page 5-21.

In track mode, you may change the x-values of the rulers (that is, their horizontal position). As the \(x_1\) and \(x_2\) values change, the values of \(y_1\), \(y_2\), \(dx\), and \(dy\) change automatically.

Change the \(x_1\) and \(x_2\) values in one of two ways:

- Dragging the rulers to the left and the right with the mouse
- Entering their values in the \(x_1\) and \(x_2\) edit boxes in the **Rulers** panel

**Ruler Controls: Slope.** There are two vertical rulers, called ruler 1 and ruler 2, with the slope line passing through the y-axis intersections of the two vertical rulers and the signal. The rulers also track the signal with markers on each ruler that shows the y-values of the signal at the x-values of the rulers. The line connecting \((x_1, y_1)\) and \((x_2, y_2)\) is included in the main plot, so you can approximate derivatives and slopes of the signal.
When slope rulers are in use, the measurements displayed in the Rulers panel are $x_1$ (the position of ruler 1 on the x-axis), $y_1$ (the position of ruler 1 on the y-axis), $x_2$ (the position of ruler 2 on the x-axis), $y_2$ (the position of ruler 2 on the y-axis), $dx$ (the value of $x_2 - x_1$), $dy$ (the value of $y_2 - y_1$), and $m$ (equal to $dy/dx$, the slope of the line between $x_1$ and $x_2$).

Press **Slope** to put the rulers in slope mode.

In slope mode, you may change the x-values of the rulers (that is, their horizontal position). As the $x_1$ and $x_2$ values change, the values of $dy$ and $m$ change automatically.

Change the $x_1$ and $x_2$ values in one of two ways:

- Dragging the rulers to the left and the right with the mouse
- Entering their values in the $x_1$ and $x_2$ edit boxes in the Rulers panel
Using the Signal Browser: Interactive Signal Analysis

The Signal Browser tool is an interactive signal exploration environment. It provides a graphical view of the signal object(s) currently selected in the Signals column of SPTool.

Using the Signal Browser you can:

- View and compare vector or array signals
- Zoom in on a range of signal data to examine it more closely
- Measure a variety of characteristics of signal data
- Play signal data on audio hardware
- Print signal data

Opening the Signal Browser

To open or activate the Signal Browser from SPTool:

1. Click on one or more signals in the Signals list of SPTool.
2. Press View in the Signals panel of SPTool.

The Signal Browser is activated and the selected signal(s) are loaded into the Signal Browser and displayed.
Basic Signal Browser Functions
The Signal Browser has the following components:

- A main axes (display) area for viewing signals graphically
- Display management controls: *Array Signals*... and the complex signal display pop-up menu
- Zoom controls for getting a closer look at signal features
- Rulers and line display controls for making signal measurements and comparisons
- A panner for seeing what part of the signal is currently being displayed and for quickly moving the view to other features of the signal
- Menu options for printing signal data
- A menu option for playing a selected signal through audio equipment
Menus

**File Menu.** Use **Page Setup...** from the **File** menu to open the standard MATLAB **Page Setup** dialog box, in which you can set the orientation, size and position, limits, and color of your printout.

![Page Setup dialog box](image)

See Using MATLAB Graphics for more information about the **Page Setup** dialog box.

Use **Print Preview...** from the **File** menu to open a MATLAB figure window with a preview of the information from the Signal Browser window that will appear in your printout. See “Signal Browser Settings” on page 5-23 and “Printing Signal Data” on page 5-54 for details about printing from the Signal Browser.

Use **Print...** from the **File** menu to open the standard print dialog box, from which you can print your signal data.

Use **Close** from the **File** menu to close the Signal Browser. All signal selection and ruler information will be lost. (You may want to save ruler information to the workspace using the **Save Rulers...** button, as described on page 5-37.) Settings you changed and saved using the **Preferences** dialog box in SPTool are saved and used the next time you open a Signal Browser.
Options Menu. Use Play from the Options menu to play the selected signal.

Play works only when you have sound capabilities on your computer. If your computer does not have sound capabilities, this menu choice does nothing.

The entire selected signal is played at either Fs (the sampling frequency of the signal) or at the default platform sampling frequency if Fs is less than 25 Hz. The real part and the imaginary part of a complex signal are played in separate channels.

Window Menu. Use the Window menu to select a currently open MATLAB window.

Zoom Controls
The available zoom controls in the Signal Browser are Mouse Zoom, Full View, Zoom In-Y, Zoom Out-Y, Zoom In-X, and Zoom Out-X. See “Zoom Controls” on page 5-31 for details on using the zoom controls in the Signal Browser.

Zoom persistence is off by default in the Signal Browser; use the Signal Browser settings panel in the Preferences dialog box in SPTool to toggle zoom persistence on and off. See “Signal Browser Settings” on page 5-23.

Ruler and Line Display Controls
Using the rulers and line display controls, you can measure a variety of characteristics of signals in the Signal Browser. See “Ruler Controls” on page 5-33 for details on using rulers and modifying line displays in the Signal Browser.

The rulers are displayed by default in the Signal Browser; you can turn off the ruler display in the Signal Browser settings panel in the Preferences dialog box. See “Signal Browser Settings” on page 5-23.

Help Button
To use context-sensitive help, press the Help button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in the Signal Browser, including menu items, to find out what it is and how to use it.
Display Management Controls

Array Signals… Button. Use this to enter a column index vector for a selected array signal. All array signals start out with only the first column displayed.

The Array Signals… button is enabled when at least one array signal is selected in SPTool:

1. Press Array Signals…

The Column Selection for Array Signals dialog box is displayed.

All array signals that are selected in SPTool are shown in the list.

2. Select a signal from the list.

3. Type a column index vector for the selected signal.

   Valid index vectors are of the form 1 or 1:3 or [1 3 5].

Complex Signal Display. Use to specify whether the Signal Browser plots the real part, the imaginary part, the magnitude, or the angle of a complex signal.

This menu is enabled when at least one of the signal variables selected in SPTool is complex. The Complex Display mode affects all of the variables in the current selection, even those that are strictly real.

Click and drag to select the plotting mode.

Main Axes Display Area

The Signals list in SPTool shows all signals in the current SPTool session. One or more signals may be selected. The signal data of all selected signals are displayed graphically in the main axes display area of the Signal Browser.
When there is only one signal displayed, its properties are reflected in the display management controls and its measurements are displayed in the ruler display panel. When more than one signal is displayed, select the line you want to focus on.

When a signal is selected, you can use the ruler controls on the selected line (see “Making Signal Measurements” on page 5-37), you can choose how to display the signal (see “Display Management Controls” on page 5-47), and you can play the signal (see “Options Menu” on page 5-46). The label of the selected signal (line) is displayed in the Selection pop-up menu.

Use one of three ways to select a signal (line) in the Signal Browser:

- Click on the Selection pop-up menu and drag to select the line to measure.
- Move the mouse pointer over any point in the line in the main axes display and click on it.
- Move the mouse pointer over any point in the line in the panner and click on it.

See “Selecting a Line to Measure” on page 5-34 for details.

**Axes Labels.** By default, the x-axis in the Signal Browser is labeled Time. You can change the x-axis label and add a y-axis label using the Signal Browser settings panel in the Preferences dialog box in SPTool. See “Signal Browser Settings” on page 5-23.

**Click-and-Drag Panning.** You can use the mouse to pan around the main axes display; click on a line in the main axes, hold down the mouse button, and drag the mouse.

Click-and-drag panning is not enabled in mouse zoom mode.

**Panner**

The panner gives a panoramic view of the signal(s) displayed in the main axes. The panner always displays the entire signal sample. When you zoom in on the main axes, a patch in the panner shows the section of the plot that is currently in view in the main axes. Click-and-drag the patch in the panner window to pan dynamically across the signal data in the main axes.

You can also select a line by clicking on it in the panner; the selected line is highlighted in both the panner and in the main axes display area.
See “Panner Display” on page 5-52 for more details.

The panner is displayed by default in the Signal Browser; you can turn off the panner in the Signal Browser settings panel in the Preferences dialog box in SPTool. See “Signal Browser Settings” on page 5-23.

Making Signal Measurements
Use the rulers to make a variety of measurements on the selected signal. See “Making Signal Measurements” on page 5-37 for details.

Viewing and Exploring Signals
You can open or activate the Signal Browser in SPTool by selecting one or more signals and pressing View in the Signal panel. The selected signals are loaded into the Signal Browser. See “Viewing a Signal” on page 5-17 for details.

Selecting and Displaying a Signal
When the Signal Browser is activated, all selected signals are displayed in the main axes display area and in the panner. The data of the selected signals are plotted against an equally spaced time vector in both the main axes display and the panner.
When the variable is a vector, one signal is displayed, as in the example above. It is automatically selected and its name, size, type, and sampling frequency are displayed above the main axes display; the name is also highlighted in the Selection pop-up menu.

When more than one signal is selected, each signal is displayed in a different color in both the main axes display and the panner.

The names of all signals are displayed above the main axes display. The first signal in the list is automatically selected in both the main axes display and the panner, its name is highlighted in the Selection pop-up menu, and its color is shown in the Selection display.
When the signal is an array, only the first column is initially displayed in both the main axes and the panner.

To display a different array column, or more than one column of the array, press the **Array Signals...** button and specify the columns to be displayed (see “Array Signals... Button” on page 5-47). All displayed columns of an array are shown in the same color; the selected column is emphasized with a heavier line.
in both the main axes and the panner, and its label is displayed in the Selection pop-up menu.

Panner Display
The panner displays the entire signal sample at all times.
Using the Signal Browser: Interactive Signal Analysis

When the signal in the main axes is zoomed, the part of the signal that is shown in the main axes is shown in the panner with a window around it.

![Signal Browser Image]

Each time you zoom, the panner is updated to frame the region of data displayed in the main axes.

Click-and-drag on the panner window to move it. As the window moves over the signal in the panner, the signal in the main axes area is panned.

**Manipulating Displays**

**Changing Signal Displays.** The signals are displayed in the default line colors and default line styles. You can change the defaults using the **Color Order** and **Line Style Order** fields in the **Colors** settings panel (see “Color Settings” on page 5-22).

**Changing the Sample Interval.** You can change the sample interval by selecting **Sampling Frequency...** from the **Edit** menu in SPTool. See “Editing Data Objects in SPTool” on page 5-15.

**Displaying Complex Signals.** You can change how complex numbers are displayed by selecting **Real**, **Imaginary**, **Magnitude**, or **Angle** from the pop-up menu. See “Array Signals... Button” on page 5-47.

**Changing Signal Browser Displays.** Using the **Signal Browser** settings panel in the **Preferences** dialog box in SPTool, you can set optional x-axis and y-axis labels,
enable and disable the display of the rulers and the panner, and toggle zoom persistence. See “Signal Browser Settings” on page 5-23.

Working with Signals
Once a signal is displayed, you can browse it in a variety of ways:

• You can zoom in on a specific area of the display (see “Zoom Controls” on page 5-31).
• You can mark off a segment of the display with the rulers (see “Ruler Controls” on page 5-33) and save ruler settings (see “Save Rulers... Button” on page 5-37).
• You can select a segment of the display with the panner (see “Panner Display” on page 5-52).
• You can make certain measurements on the displayed signals (see “Making Signal Measurements” on page 5-37).
• When there is more than one signal in the display, you can select which one you want to measure (see “Selecting a Line to Measure” on page 5-34).

You can use the other GUI tools to manipulate signals in a variety of ways:

• You can interactively design and analyze filters to be applied to signals (see “Using the Filter Designer: Interactive Filter Design” on page 5-59 and “Using the Filter Viewer: Interactive Filter Analysis” on page 5-84).
• You can create a spectrum for a signal and interactively analyze its spectral density with a variety of estimation methods (see “Using the Spectrum Viewer: Interactive PSD Analysis” on page 5-97).

You access the Filter Designer, Filter Viewer, and Spectrum Viewer tools from SPTool. You can access SPTool from the Signal Browser in one of two ways:

• Click on an active SPTool window
• Activate SPTool using the Window menu in the Signal Browser

Printing Signal Data
You can print all of the signal data that you can display and manipulate in the Signal Browser. Your printout will contain up to three components:
• The plots of one or more signals, as displayed in the main axes area
• The panner display of the signal(s) displayed in the main axes area
• The ruler measurements of the selected signal

You can preview your printout by choosing Print Preview... from the File menu in the Signal Browser window. This opens the Signal Browser Print Preview window, shown below with default settings for signal `mtlb`.

The Close button on the Signal Browser Print Preview window closes the print preview window and returns you to the Signal Browser.

The Print... button on the Signal Browser Print Preview window opens the standard print dialog box, from which you can print the contents of your print preview window.

If you choose to print from the print dialog, both that dialog and the Signal Browser Print Preview window will close, and the contents of the print preview will print.

If you choose not to print, the print dialog and the Signal Browser Print Preview window will close, and you will return to the Signal Browser.

Any changes you make, before opening the print preview window, that affect the display of data in the Signal Browser (see “Viewing and Exploring Signals”
on page 5-49) will affect the display of data in the **Signal Browser Print Preview** window in exactly the same way. For example, in the print preview shown below:

- The plot of the \texttt{mtlb} data has been mouse zoomed (see “Zoom Controls” on page 5-31).
- The panner display reflects the current zoom level by enclosing the zoomed portion of the whole signal in a window (see “Panner Display” on page 5-52).
- The rulers have been moved into the zoomed area and the ruler controls have been changed from Track to Slope (see “Ruler Controls” on page 5-33).
- The ruler measurements displayed reflect the changes to the rulers in the display area (see “Making Signal Measurements” on page 5-37).

Changes you make in the **Preferences** dialog box in SPTool, before opening the print preview window, will also affect the **Signal Browser Print Preview** window in the same way that they affect the Signal Browser itself. For example, in the print preview shown below, the panner display and the ruler measurements have been suppressed, and the axis labels have been changed,
by changes to the Signal Browser preferences (see “Signal Browser Settings” on page 5-23).

**NOTE** The `HandleVisibility` property of the **Print Preview** figure window is set to 'on'. This allows you to modify the preview using Handle Graphics commands from the MATLAB command line or the Plot Editor tools (see Using MATLAB Graphics for more information). Changes that you make in the preview window print exactly as they appear on the screen; however, they are not saved when the **Print Preview** figure window is closed.

### Saving Signal Data

After creating a signal in SPTool (by applying a filter to an imported signal, for example), you can export the signal information to the workspace or to disk using **Export...** from the **File** menu in SPTool. The signal information is stored in a structure that you can access to retrieve the signal data and sample frequency. The signal structure also contains a number of fields that are used internally by SPTool.

To see the fields of the signal structure, try exporting a signal to the workspace:
1 Import a signal into SPTool if there are none currently loaded (see “Importing Signals, Filters, and Spectra” on page 5-7). Label the imported signal sig1.

2 Export the signal. Select **Export...** from the **File** menu.

3 In the **Export** dialog box, select **sig1** and press the **Export to Workspace** button.

4 Type `who` at the MATLAB command line to look at the variables in the workspace. The variable called **sig1** is the signal structure you exported from SPTool.

5 Type `sig1` at the command line to list the fields of the signal structure.

The **data** and **Fs** fields of the signal structure contain the information that defines the signal. The other fields are used internally by SPTool and are subject to change in future releases:

- The **data** field is a vector or array containing the signal’s data.
- The **Fs** field contains the sample frequency of the signal in Hertz.
Using the Filter Designer: Interactive Filter Design

The Filter Designer provides an interactive graphical environment for the design of digital IIR and FIR filters based on specifications that you enter on a magnitude or pole-zero plot. Using the Filter Designer, you can design IIR and FIR filters of various lengths and types, with standard frequency band configurations (highpass, lowpass, bandpass, bandstop, and multiband).

Using the Filter Designer you can:

- Design IIR filters with standard band configurations, using the Butterworth, Chebyshev type I, Chebyshev type II, and elliptic design options
- Design FIR filters with standard band configurations, using the equiripple, least squares, and Kaiser window design options
- Design FIR and IIR filters with arbitrary band configurations, using the Pole/Zero Editor
- Redesign a filter by manually adjusting indicators in the magnitude plot, or by graphically repositioning the transfer function’s poles and zeros
- Overlay a spectrum on the filter’s magnitude response plot

When you have designed a filter to your specifications, you can apply the filter to a selected signal using the Apply button in SPTool (see “Applying a Filter” on page 5-18).

NOTE For information on using filter design functions from the command line or from M-files, see Chapter 2.

Opening the Filter Designer

You can open or activate the Filter Designer from SPTool by pressing the New Design or Edit Design buttons:

- To create a filter, press New Design in SPTool. A default filter is generated and displayed in the Filter Designer. You can view the filter in a variety of ways in the Filter Viewer and modify it in the Filter Designer.
- To edit a filter, select it in SPTool and press Edit Design. You can modify the filter in a variety of ways in the Filter Designer.
See “Designing a Filter” on page 5-17 for complete details.

**Basic Filter Designer Functions**
The Filter Designer has the following components:

- A pop-up menu for selecting different filter design methods
- A magnitude plot (display) area for graphically adjusting filter responses
- A pole-zero plot (display) area for graphically adjusting filter transfer functions
- A Specifications panel for viewing and modifying the design parameters or pole-zero locations of the current filter
- A Measurements panel for viewing the response characteristics and stability of the current filter
- Specification lines for graphically adjusting the magnitude response parameters of a filter
- Measurement lines for measuring the magnitude response parameters of a filter
- Zoom controls for getting a closer look at the magnitude response or pole-zero features

The following sections describe the different components of the Filter Designer’s interface and how you can use them together to create and edit filters.

**Menus**

**File Menu.** Use **Close** from the **File** menu to close the Filter Designer. Settings you changed and saved using the **Preferences** dialog box in SPTool are saved and used the next time you open the Filter Designer.

**Window Menu.** Use the **Window** menu to select a MATLAB figure window.

**Filter Pop-Up Menu**
The **Filter** pop-up menu displays all of the filters currently selected in SPTool. Select a filter in the menu to make it the current filter in the Filter Designer.
Using the Filter Designer: Interactive Filter Design

Zoom Controls
The available zoom controls in the Filter Designer are **Zoom In-Y**, **Zoom Out-Y**, **Zoom In-X**, **Zoom Out-X**, **Pass Band**, **Full View**, and **Mouse Zoom**.

See “Zoom Controls” on page 5-31 for details on using the zoom controls.

Zoom persistence is off by default in the Filter Designer; use the Filter Designer settings panel in the **Preferences** dialog box to toggle zoom persistence on and off. See “Filter Designer Settings” on page 5-27.

Help Button
To use context-sensitive help, press the **Help** button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on any...
Interactive Tools

object in the Filter Designer, including menu items, to find out what it is and how to use it.

General Controls

Beneath the zoom controls are several general controls for filter design and display. Some of these controls are available only when an optimal filter design method (e.g., Equiripple FIR, Butterworth IIR) is selected from the Algorithm menu, and are not available when the Pole/Zero Editor is selected.

Algorithm. Use the Algorithm pop-up menu to select a design method for the current filter. The first seven options in the Algorithm menu – the optimal design methods – all design filters based on a high-level specification of filter magnitude response characteristics (such as passband frequencies or stopband attenuation). The filter characteristics are specified numerically in the Specifications panel or graphically on the magnitude response plot.

The eighth option, Pole/Zero Editor, is not an optimal design method but allows low-level specification of the filter’s transfer function via numerical or graphical pole-zero placement.

When you select one of the optimal design methods from the menu, the magnitude response plot, Specifications panel, and Measurements panel all update to reflect the design parameters available for that method. When you select Pole/Zero Editor, the magnitude response plot is replaced by the z-plane for pole-zero placement.

Auto Design (optimal design methods only). When the Auto Design check box is checked, the filter’s magnitude response is redrawn whenever you change a filter specification, either by entering a value in the Specifications panel or by dragging a specification line on the plot. When the box is not checked, the new response is computed and redrawn only when you press the Apply button or release the specification line. Auto Design is initially off by default; use the
Filter Designer settings panel in the Preferences dialog box to change this default setting. See “Filter Designer Settings” on page 5-27.

Sampling Frequency. The Sampling Frequency field allows you to specify the filter’s sampling frequency in Hertz. To change the sampling frequency, type a value in the box and press Enter. This is the same as changing the sampling frequency by selecting Sampling Frequency... from the SPTool Edit menu (see “Editing Data Objects in SPTool” on page 5-15). The frequency axis of the magnitude response plot is updated to reflect the new sampling frequency. Changing the Sampling Frequency in the Pole/Zero Editor does not alter the pole-zero plot, but does scale the corresponding magnitude plot in the Filter Viewer (if the Viewer is open).

Overlay Spectrum (design methods only). The Filter Designer allows you to overlay a signal spectrum on the filter’s magnitude response plot. Press the Overlay Spectrum... button to display a list of the current spectra in SPTool. Select a spectrum from the list and press OK to overlay it on the current magnitude response plot. Note that the spectrum is plotted on the existing frequency axis, which is scaled to the filter’s sampling frequency.

Filter Specifications Panel—Design Methods
When you design a new filter with the design methods, the Filter Designer initially contains the specifications and magnitude response plot for an order 22, lowpass, equiripple filter.

Use the Type pop-up menu in the Specifications panel to select a band configuration. Use the edit boxes below it in the panel to change the band edge frequencies and the amount of ripple in the passband and attenuation in the stopband. Check the Minimum Order box to let the Filter Designer automatically determine the lowest filter order that achieves the current specifications.

The design parameters available in the Specifications panel depend on the filter design selected in the Algorithm pop-up menu, the band configuration selected in the Type pop-up menu, and the state of the Minimum Order check box.

Specifications Parameters – Automatic Order Selection. When the Minimum Order box is checked, all of the filter designs except Least Squares FIR display the same set of parameters in the Specifications panel. (The order for the Least Squares FIR design cannot be automatically computed). For lowpass and
highpass band configurations, these parameters include the passband edge frequency \( F_p \), the stopband edge frequency \( F_s \), the passband ripple \( R_p \), and the stopband attenuation \( R_s \). For bandpass and bandstop configurations, the parameters include the lower and upper passband edge frequencies, \( F_{p1} \) and \( F_{p2} \), the lower and upper stopband edge frequencies, \( F_{s1} \) and \( F_{s2} \), the passband ripple \( R_p \), and the stopband attenuation \( R_s \). Frequency values are in Hertz, and ripple and attenuation values are in dB.

**Specifications Parameters—Manual Order Selection.** When the **Minimum Order** box is not checked, an **Order** parameter for setting the filter order appears below it, and each filter design displays a unique set of parameters in the **Specifications** panel. These are shown in the table below, where \( W_p \) and \( W_s \) are the weights for the passband and stopband, \( \beta \) is the Kaiser window \( \beta \) parameter, \( F_c \) is the cutoff frequency, and \( F_{3db} \) is the -3 dB frequency.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Lowpass</th>
<th>Highpass</th>
<th>Bandpass</th>
<th>Bandstop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equiripple FIR</td>
<td>( F_p, F_s, W_p, W_s )</td>
<td>( F_p, F_s, W_p, W_s )</td>
<td>( F_{p1}, F_{p2}, F_{s1}, F_{s2}, W_p, W_s )</td>
<td>( F_{p1}, F_{p2}, F_{s1}, F_{s2}, W_p, W_s )</td>
</tr>
<tr>
<td>Least Squares FIR</td>
<td>( F_p, F_s, W_p, W_s )</td>
<td>( F_p, F_s, W_p, W_s )</td>
<td>( F_{p1}, F_{p2}, F_{s1}, F_{s2}, W_p, W_s )</td>
<td>( F_{p1}, F_{p2}, F_{s1}, F_{s2}, W_p, W_s )</td>
</tr>
<tr>
<td>Kaiser Window FIR</td>
<td>( F_c, \beta )</td>
<td>( F_c, \beta )</td>
<td>( F_{c1}, F_{c2}, \beta )</td>
<td>( F_{c1}, F_{c2}, \beta )</td>
</tr>
<tr>
<td>Butterworth IIR</td>
<td>( F_{3db} )</td>
<td>( F_{3db} )</td>
<td>( F_{3db1}, F_{3db2} )</td>
<td>( F_{3db1}, F_{3db2} )</td>
</tr>
<tr>
<td>Chebyshev Type I IIR</td>
<td>( F_p, R_p )</td>
<td>( F_p, R_p )</td>
<td>( F_{p1}, F_{p2}, R_p )</td>
<td>( F_{p1}, F_{p2}, R_p )</td>
</tr>
<tr>
<td>Chebyshev Type II IIR</td>
<td>( F_s, R_s )</td>
<td>( F_s, R_s )</td>
<td>( F_{s1}, F_{s2}, R_s )</td>
<td>( F_{s1}, F_{s2}, R_s )</td>
</tr>
<tr>
<td>Elliptic IIR</td>
<td>( F_p, R_p, R_s )</td>
<td>( F_p, R_p, R_s )</td>
<td>( F_{p1}, F_{p2}, R_p, R_s )</td>
<td>( F_{p1}, F_{p2}, R_p, R_s )</td>
</tr>
</tbody>
</table>

**Designing a New Filter With the Design Methods.** In general, follow these steps to design a new filter using the **Specifications** panel parameters:

1. Click-and-drag to select an appropriate optimal filter design method from the **Algorithm** pop-up menu.
2. Click-and-drag to select the band configuration from the **Type** pop-up menu.
3. Type in the **Specifications** panel’s editable fields to change the values for band edge frequencies, ripple, attenuation, and other filter characteristics.
4 Press Enter after typing in an editable text box to enter the value. If Auto Design is not checked, press the Apply button below the Specifications panel to update the magnitude plot.

5 To specify a filter order, press Minimize Order to deselect the check box and disable automatic filter order selection. Then type a value for the Order parameter. If Auto Design is not checked, press Apply.

Edit a new filter or an existing filter in the same way. Note that when Auto Design is disabled and you change a filter’s parameter values, the magnitude response plot changes from solid lines to dashed lines. This indicates that the plot no longer reflects the current specifications. When you press the Apply button, the new response is computed and the lines revert to the solid style. When Auto Design is enabled, the plot updates whenever you change a filter specification.

Filter Measurements Panel—Design Methods
When you design a filter, the Measurements panel (shown at left) displays the values of filter parameters that do not appear in the Specifications panel. For example, when the Filter Designer provides an Fs parameter in the Specifications panel, it displays the Actual Fs value in the Measurements panel. Similarly, when the Minimum Order option is selected in the Specifications panel, the computed filter order is displayed in the Order field of the Measurements panel. The values in the Measurements panel are updated whenever the magnitude plot is redrawn.

Measurement Parameters – Automatic Order Selection. The parameter combinations that appear in the Measurements panel are shown in the following two tables. The first table lists the parameters that appear when Minimum Order is checked (automatic order selection). The stopband edge frequency parameters listed (Fs, Fs1, Fs2) are the actual edge frequencies for the design, rather than the desired frequencies entered in the Specifications panel.
Measurement Parameters – Manual Order Selection. The next table shows the parameter sets that appear in the Measurements panel when Minimum Order is not selected. The measurements that can be interactively changed by dragging the red measurement lines on the response plot are shown shaded.

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Lowpass</th>
<th>Highpass</th>
<th>Bandpass</th>
<th>Bandstop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equiripple FIR</td>
<td>Order, Rp, Rs, Wp, Ws</td>
<td>Order, Rp, Rs, Wp, Ws</td>
<td>Order, Rp, Rs, Wp, Ws</td>
<td>Order, Rp, Rs, Wp, Ws</td>
</tr>
<tr>
<td>Least Squares FIR</td>
<td>Order cannot be automatically computed.</td>
<td>Order cannot be automatically computed.</td>
<td>Order cannot be automatically computed.</td>
<td>Order cannot be automatically computed.</td>
</tr>
<tr>
<td>Kaiser Window FIR</td>
<td>Order, Fc, Beta, Rp, Rs</td>
<td>Order, Fc, Beta, Rp, Rs</td>
<td>Order, Fc1, Fc2, Beta, Rp, Rs</td>
<td>Order, Fc1, Fc2, Beta, Rp, Rs</td>
</tr>
<tr>
<td>Butterworth IIR</td>
<td>Order, Rp, F3db</td>
<td>Order, Rp, F3db</td>
<td>Order, Rp, F3db 1, F3db 2</td>
<td>Order, Rp, F3db 1, F3db 2</td>
</tr>
<tr>
<td>Chebyshev Type I IIR</td>
<td>Order, Fs</td>
<td>Order, Fs</td>
<td>Order, Fs1, Fs2</td>
<td>Order, Fs1, Fs2</td>
</tr>
<tr>
<td>Chebyshev Type II IIR</td>
<td>Order, Fs</td>
<td>Order, Fs</td>
<td>Order, Fs1, Fs2</td>
<td>Order, Fs1, Fs2</td>
</tr>
<tr>
<td>Elliptic IIR</td>
<td>Order, Fs</td>
<td>Order, Fs</td>
<td>Order, Fs1, Fs2</td>
<td>Order, Fs1, Fs2</td>
</tr>
</tbody>
</table>

Filter Specifications Panel—Pole/Zero Editor

When you design a new filter using the Pole/Zero Editor, the Filter Designer initially displays a pole-zero plot for the default order-22 filter.

The general pole-zero (factored) form of the transfer function of a digital filter is:

$$H(z) = \frac{Z(z)}{P(z)} = k \frac{(z - z_1)(z - z_2)\cdots(z - z_m)}{(z - p_1)(z - p_2)\cdots(z - p_n)}$$

where $z_1, z_2, \ldots, z_m$ are the filter zeros, $p_1, p_2, \ldots, p_n$ are the filter poles, and $k$ is the overall transfer function gain. The number of zeros and poles do not need to be the same.

The controls in the Specifications panel allow you to set the overall gain and to numerically adjust the position of each pole and zero using polar or rectangular coordinates.
The value you enter in the Gain edit box specifies the gain, $k$. The Coordinates pop-up menu lets you position the poles and zeros using either polar or rectangular measurements. When Polar is selected, you can specify the coordinates of the currently selected pole or zero by entering values in the Mag and Angle edit boxes. The resulting location of the selected pole or zero is

$$z = Me^{j\theta}$$

where $M$ is the magnitude specified by Mag, and $\theta$ is the angle in radians specified by Angle.

When Rectangular is selected, you can specify the coordinates of the currently selected pole or zero by entering values in the X and Y edit boxes. The resulting location of the selected pole or zero is

$$z = X + jY$$

The Conjugate pair checkbox controls whether the current selection is a complex conjugate pair of poles or zeros, or a lone pole or zero. When a lone pole or zero is selected on the pole-zero plot, the Conjugate pair checkbox is inactive. If you now enable the Conjugate pair checkbox by clicking it, a new conjugate pole or zero (as appropriate) is created to complete the pair.

When a conjugate pair of poles or zeros is selected on the plot (by clicking one of the pair), the Conjugate pair checkbox is active. If you now disable the Conjugate pair checkbox by clicking it, the conjugate pair is broken into two lone poles or zeros at the same positions, and only one of these remains selected.

When the Conjugate pair checkbox is selected, the coordinates you specify (in either angular or rectangular form) control the location of the currently selected complex conjugate pair of poles or zeros, with locations

$$z = X \pm jY$$

or

$$z = Me^{\pm j\theta}$$

**Designing a New Filter with the Pole/Zero Editor.** In general, follow these steps to design a new filter using the Specifications panel parameters:
1. Click-and-drag to select the **Pole/Zero Editor** option from the **Algorithm** pop-up menu.

2. Enter the desired filter gain in the **Gain** edit box.

3. Select a pole or zero (or conjugate pair) by clicking on one of the **X** or **O** symbols on the plot.

4. Choose the coordinates to work in by specifying **Polar** or **Rectangular** from the **Coordinates** pop-up menu.

5. Specify the new location(s) of the selected pole, zero, or conjugate pair by typing values into the **Mag** and **Angle** (for angular coordinates) or **X** and **Y** (for rectangular coordinates) edit boxes.

6. Use the **Conjugate pair** checkbox to create a conjugate pair from a lone pole or zero or to break a conjugate pair into two individual poles or zeros.

Edit a new filter or an existing filter in the same way.

---

**TIP** Keep the Filter Viewer open while designing a filter with the Pole/Zero Editor. Any changes that you make to the filter transfer function in the Pole/Zero Editor are then simultaneously reflected in the response plots of the Filter Viewer.

---

**Filter Measurements Panel—Pole/Zero Editor**

When you design a filter, the **Measurements** panel (shown at left) displays useful information about the filter being designed. The panel always shows the number of zeros and poles (the larger of which is the order of the filter), and indicates whether the filter is stable or unstable (poles outside the unit circle).

The panel also shows the phase characteristics of the filter. Where m is the total number of zeros, the four possible phase conditions are:

- **Minimum Phase**, which indicates that between 1 and m-1 zeros are on the unit circle, while the rest of the zeros are inside the unit circle
  \[ z_i \leq 1 \quad i = 1, ..., m \]
- **Strictly Minimum Phase**, which indicates that all of the zeros are inside the unit circle
  \[ z_i < 1 \quad i = 1, \ldots, m \]

- **Maximum Phase**, which indicates that between 1 and \( m \) zeros are on the unit circle, while the rest of the zeros are outside the unit circle
  \[ z_i \geq 1 \quad i = 1, \ldots, m \]

- **Strictly Maximum Phase**, which indicates that all of the zeros are outside the unit circle
  \[ z_i > 1 \quad i = 1, \ldots, m \]

When the **Measurements** panel does not indicate any of the above conditions, zeros are both inside and outside the unit circle.

When a new pole or zero is being placed on the plot (using the **Add Poles** or **Add Zeros** tools, described on page 5-72) the **Measurements** panel shows additional positioning information. After pressing either the **Add Poles** or **Add Zeros** button, and before clicking on the plot to place the new pole(s) or zero(s), the **Measurements** panel displays the precise position of the pointer over the plot area.

![Coordinates](image)

All values in the **Measurements** panel are updated continuously as a pole or zero is repositioned.

**Magnitude Plot (Display) Area—Design Methods**

The response of the filter selected in the **Filter** pop-up menu is displayed graphically in the magnitude response plot area of the Filter Designer, and its characteristics are shown in the **Specifications** and **Measurements** panels.
You can zoom in on the displayed filter in the magnitude plot (see “Zoom Controls” on page 5-31) and you can drag the specification lines to redesign the displayed filter visually (see the next section).

**Specification Lines.** You can redesign a filter by manipulating the green specification lines in the magnitude plot. Using the specification lines, you can change the passband ripple, stopband attenuation, and edge frequencies of a filter:

- Set passband ripple or stopband attenuation by clicking on a green line and dragging it up or down. The Rp and Rs values displayed in the Specifications panel change as you drag. If Auto Design is checked, the response plot also updates as you drag the lines. If Auto Design is disabled, the response plot updates when you release the mouse.

**NOTE** With IIR filters you can only drag the lower passband bar and the stopband bar. The upper passband bar is fixed at 0.

- Set edge frequencies either by clicking on the edge of a horizontal green line (the mouse pointer changes to ![ ] and dragging the edge to a new frequency, or by clicking anywhere on a vertical green line (if the Filter Designer provides one) and dragging it horizontally to a new frequency. The Fp and Fs values displayed in the Specifications panel change as you drag. If Auto Design is checked, the response plot also updates as you drag the lines. If Auto Design is disabled, the response plot updates when you release the mouse.

See “Redesigning a Filter Using the Magnitude Plot” on page 5-78 for details.

**Measurement Lines.** A number of the filter designs provide rulers on the response plot that allow you to measure response magnitude levels. These measurement lines, which appear in red on the plot, are available for the Kaiser window, Butterworth, Chebyshev type I, and Chebyshev type II filters when the Minimum Order check box is not selected. As you drag a measurement line, the corresponding values in the Measurements panel change to reflect the measurement line’s current position.
Magnitude Plot (Display) Area—Pole/Zero Editor

The plot area of the Pole/Zero Editor displays the filter transfer function's pole and zeros on the complex $z$-plane. The unit circle is shown in dashed lines, along with the vertical and horizontal grid lines, to provide a visual reference for pole-zero placement.

You can zoom in on the $z$-plane (see “Zoom Controls” on page 5-31) and you can drag the pole and zero symbols to redesign the filter transfer function visually (see the next section).

Positioning Poles and Zeros. You can redesign the transfer function by simply clicking and dragging on one of the pole or zero symbols. Poles or zeros appear green (rather than the usual blue) when selected. In a conjugate pair of poles or zeros, both conjugates are selected when either is clicked; conjugate pairs move together when dragged, maintaining equal distance from the real axis at all times. To ungroup conjugates, select the desired pair and uncheck Conjugate pair in the Specifications panel.

When two or more pole symbols (or two or more zero symbols) are placed directly on top of each other, a number is displayed next to the symbols (on the left for poles, and on the right for zeros) indicating the number of poles or zeros at that location (e.g., $\circ3$). This number makes it easy to keep track of all the poles and zeros in the plot area, even when several are superimposed on each other and are not visually differentiable. Note, however, that this number does not indicate the multiplicity of the poles or zeros to which it is attached.
To help detect whether a set of poles or zeros are truly multiples, use the zoom tools to magnify the region around the poles or zeros in question. Because numerical limitations usually prevent any set of poles or zeros from sharing exactly the same value, at a high enough zoom level even truly multiple poles or zeros appear distinct from each other. A common way to assess whether a particular group of poles or zeros contains multiples is by comparing the mutual proximity of the group members against a selected threshold value. As an example, the residuez function defines a pole or zero as being a multiple of another pole or zero if the absolute distance separating them is less than 0.1% of the larger pole or zero's magnitude.

**Tools.** The Pole/Zero Editor provides several tools to assist with pole-zero placement. The tools are located at the top left of the pole-zero plot area.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Drag Poles/Zeros Tool" /></td>
<td>Drag Poles/Zeros Tool</td>
<td>Use this tool to select and drag poles and zeros on the plot.</td>
</tr>
<tr>
<td><img src="image" alt="Add Poles Tool" /></td>
<td>Add Poles Tool</td>
<td>Use this tool to add poles to the plot. Simply click on the plot where you want to place the new pole (or conjugate pair, when the Conjugate pair checkbox is selected).</td>
</tr>
<tr>
<td><img src="image" alt="Add Zeros Tool" /></td>
<td>Add Zeros Tool</td>
<td>Use this tool to add zeros to the plot. Simply click on the plot where you want to place the new zero (or conjugate pair, when the Conjugate pair checkbox is selected).</td>
</tr>
<tr>
<td><img src="image" alt="Delete Poles/Zeros Tool" /></td>
<td>Delete Poles/Zeros Tool</td>
<td>Use this tool to delete poles and zeros from the plot. Simply click on any pole or zero to delete it. When you click on a set of poles or zeros occupying the same location, only the top-most pole or zero is removed. You can also use the Delete or Backspace key on the keyboard to delete the currently selected pole or zero.</td>
</tr>
</tbody>
</table>

At the bottom of the plot area, there are two additional buttons, **Send To Back** and **Delete All**. Use **Send To Back** when you are unable to select a pole or zero because another pole or zero is in the way. Select the interfering pole or zero on the plot, and press **Send To Back** to move that pole or zero behind all other symbols on the plot (i.e., last in the stacking order). You can then select the desired pole or zero without the interference of the other symbol.
The **Delete All** button deletes every pole and zero in the plot area.

---

**NOTE** The **Delete All** operation cannot be undone. Once all the poles and zeros are removed, they cannot be restored.

---

**Designing Finite Impulse Response (FIR) Filters**

The Filter Designer provides three options for basic FIR filter design. These options allow you to create FIR filters with standard band configurations (lowpass, highpass, bandpass, or bandstop configurations only). The three options for FIR filter design in the **Algorithm** pop-up menu are:

- **Equiripple FIR**, which accesses the toolbox function `remez` to create an equiripple FIR filter.
- **Least Squares FIR**, which accesses the toolbox function `firls` to create an FIR filter using the least square design method.
- **Kaiser Window FIR**, which accesses the `fir1` function to create an FIR filter using a Kaiser window.

**Example: FIR Filter Design, Standard Band Configuration**

In the following example, use the Kaiser window filter design option:

1. Select **Kaiser Window FIR** as the filter design from the **Algorithm** pop-up menu.
2. Select **bandpass** from the **Type** pop-up menu as the configuration.
3. Set the filter’s sampling frequency to 2000 Hz by entering this value in the **Sampling Frequency** text box.
4. Press **Apply** to redraw the response with these settings.

**NOTE** You must press **Apply** before you change the following parameters.

5. Check the **Minimum Order** check box to enable automatic order selection.
6 Set $F_{p1}$ to 290 and $F_{p2}$ to 525.

These fields respectively define the lower and upper passband edge frequencies in Hertz.

7 Set $F_{s1}$ to 200 and $F_{s2}$ to 625.

These fields respectively define the lower and upper stopband edge frequencies in Hertz.

8 Set $R_p$ (passband ripple) to 4 and $R_s$ (stopband attenuation) to 30.

$R_p$ and $R_s$ are specified in dB.

9 Press the **Apply** button.

The Filter Designer calls `fir1` to create the filter using a Kaiser window. The Filter Designer updates the magnitude plot to show the new filter’s magnitude response.
Filter Design Options
When the **Minimum Order** option is disabled, you can specify parameters that define characteristics unique to certain filter types:

- For equiripple and least squares filters: the weights for error minimization
- For Kaiser window filters: the cutoff frequency and $\beta$ parameter of the Kaiser window

Order Selection for FIR Filter Design
As described earlier, the FIR filter design options available through the Filter Designer call the toolbox functions `remez`, `firls`, and `fir1`. In calculating filter order, the Filter Designer uses the same guidelines as the toolbox functions:

- The **Equiripple FIR** design option calls the `remezord` order estimation function to determine a filter order that meets a set of specifications. In some cases, `remezord` underestimates the filter order $n$. If the filter does not appear to meet the given specifications using **Minimum Order** order selection, deselect **Minimum Order** and manually specify a slightly larger order ($n+1$ or $n+2$).

- The **Least Squares FIR** design option calls the toolbox function `firls`. Because the toolbox does not provide an order estimation function for use with `firls`, you cannot use the **Minimum Order** option with the **Least Squares FIR** method.

- The **Kaiser Window FIR** design option calls `kaiserord`, the order estimation function, which sometimes underestimates the filter order $n$. If the filter does not appear to meet the given specifications using **Minimum Order** order selection, deselect **Minimum Order** and manually specify a slightly larger order ($n+1$ or $n+2$).

All of the FIR filter design options in the Filter Designer require an even filter order for the highpass and bandstop configurations. For more information on order selection with the FIR filter design options, see the reference descriptions of `remez`, `remezord`, `kaiserord`, `firls`, and `fir1` in Chapter 6.
Designing Infinite Impulse Response (IIR) Filters
The Filter Designer lets you design a number of classical IIR filters, including Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filters.

Example: Classical IIR Filter Design
In the following example, design a simple Chebyshev type I filter:

1. Select **Chebyshev Type I IIR** as the filter design from the **Algorithm** pop-up menu.
2. Select **highpass** from the **Type** menu as the configuration.
3. Set the filter's sample frequency to 2000 Hz by entering this value in the **Sampling Frequency** text box.
4. Press **Apply** to redraw the response with these settings.

**NOTE** You must press **Apply** before you change the following parameters.

5. Check the **Minimum Order** check box.
6. Set **Fp** (passband edge frequency) to 800 and **Fs** (stopband edge frequency) to 700.
   - **Fp** and **Fs** are specified in Hertz.
7. Set **Rp** (passband ripple) to 2.5 and **Rs** (stopband attenuation) to 35.
   - **Rp** and **Rs** are specified in dB.
8. Press **Apply** to draw the magnitude response.
9. Deselect **Minimum Order** and specify a filter order of 7. Press **Apply** to redraw the magnitude response.
The Filter Designer calls the appropriate filter design function to create the filter.

To zoom in on plot details, use the zoom control buttons as described in “Zoom Controls” on page 5-31.

**Filter Design Options**

When the **Minimum Order** option is disabled, you can specify parameters that define characteristics unique to certain filter types:

- For Butterworth filters: the -3 dB frequencies
- For Chebyshev type I filters: the passband edge frequencies
- For Chebyshev type II filters: the stopband edge frequencies
- For elliptic filters: the passband edge frequencies

In the following example, redesign the Chebyshev type I filter from the previous example as a Butterworth filter, using a -3 dB frequency of 800 Hz:
1. Select the **Butterworth IIR** filter design from the **Algorithm** menu. The magnitude response plot changes to reflect the new design.

2. Type a value of 800 in the **F3db** text field.

3. Press **Apply** to update the response plot.

### Order Selection for IIR Filter Design

The IIR filter design options available through the Filter Designer call the toolbox filter design functions. In calculating the order for a given filter, the Filter Designer uses the corresponding order estimation function if the **Minimum Order** check box is selected.

For details on order selection with the IIR filter design options, see the reference descriptions of `buttord`, `cheb1ord`, `cheb2ord`, and `ellipord` in Chapter 6.

### Redesigning a Filter Using the Magnitude Plot

After designing a filter in the Filter Designer, you can redesign it by dragging the specification lines in the magnitude plot. Use the specification lines to change passband ripple, stopband attenuation, and edge frequencies (see “Specification Lines” on page 5-70 for details). In the following example, create a Chebyshev filter and modify it by dragging the specification lines:

1. Create a Chebyshev type I highpass filter with a sample frequency of 2000 Hz. Set the following parameters:
   
   $\textbf{Fp} = 800$
   
   $\textbf{Fs} = 700$
   
   $\textbf{Rp} = 2.5$
   
   $\textbf{Rs} = 35$

2. Check **Minimum Order** so the Filter Designer can calculate the lowest filter order that produces the desired characteristics.

3. Press **Apply** to update the response plot.

4. Position the cursor over the green line specifying the stopband.
   
   The cursor changes to the up/down drag indicator.
5 Drag the line until the Rs (stopband attenuation) field reads 100.

Note that the Order value in the Measurements panel changes because a higher filter order is needed to meet the new specifications.

**Saving Filter Data**

After designing a filter in the Filter Designer, you can export the filter information to the workspace or to disk using Export... from the File menu in SPTool. The filter information is stored in a structure that you can access to retrieve the coefficients and design parameters of the filter you created. The filter structure also contains a number of fields that are used internally by SPTool.

To see the fields of the filter structure, first export a filter to the workspace:

1 Create a new filter by pressing New Design in SPTool. The new filter is called filt1.

2 Select Export... from the File menu.

3 In the Export from SPTool dialog box, select filt1 and press the Export to Workspace button.

4 Type who at the MATLAB command line to look at the variables in the workspace. The variable called filt1 is the filter structure you exported from SPTool.

5 At the command line type filt1 to list the fields of the filter structure.

The tf, Fs, and specs fields of the filter structure contain the information that describes the filter. These fields are discussed below. The other fields in the structure are used internally by SPTool, and are subject to change in future releases.

**tf.** The tf field is a structure containing the transfer function representation of the filter:

- tf.num contains the numerator coefficients
- tf.den contains the denominator coefficients
both in descending powers of $z$

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(nb + 1)z^{-m}}{a(1) + a(2)z^{-1} + \ldots + a(na + 1)z^{-n}}$$

where $b$ is a vector containing the coefficients from the `tf.num` field, $a$ is a vector containing the coefficients from the `tf.den` field, $m$ is the numerator order, and $n$ is the denominator order. You can change the filter representation from the default transfer function to another form by using the `tf2ss` or `tf2zp` functions.

`Fs`. The `Fs` field contains the sampling frequency of the filter in Hertz.

`specs`. The `specs` field is a structure containing information about the filter design. The first field, `specs.currentModule`, contains a string representing the design selected for the filter in the Filter Designer’s `Algorithm` pop-up menu. The possible contents of the `currentModule` field, and the corresponding designs, are shown below.

<table>
<thead>
<tr>
<th><code>currentModule</code></th>
<th><code>Algorithm</code></th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fdbutter</code></td>
<td>Butterworth IIR</td>
</tr>
<tr>
<td><code>fdcheby1</code></td>
<td>Chebyshev Type I IIR</td>
</tr>
<tr>
<td><code>fdcheby2</code></td>
<td>Chebyshev Type II IIR</td>
</tr>
<tr>
<td><code>fdellip</code></td>
<td>Elliptic IIR</td>
</tr>
<tr>
<td><code>fdfirls</code></td>
<td>Least Squares FIR</td>
</tr>
<tr>
<td><code>fdkaiser</code></td>
<td>Kaiser Window FIR</td>
</tr>
<tr>
<td><code>fdremez</code></td>
<td>Equiripple FIR</td>
</tr>
</tbody>
</table>

Following the `specs.currentModule` field, there may be up to seven additional fields, with labels such as `specs.fdremez`, `specs.fdfirls`, etc. The design specifications for the most recently exported filter are contained in the field
whose label matches the currentModule string. For example, if the specs structure is

```plaintext
currentModule: 'fdkaiser'
fdrmeme: [1x1 struct]
fdfirls: [1x1 struct]
fdkaiser: [1x1 struct]
```

the filter specifications are contained in the fdkaiser field, which is itself a data structure.

The specifications include the parameter values from the Specifications panel of the Filter Designer, such as band edges and filter order. For example, the Kaiser window filter above has the following specifications stored in specs.fdkaiser.

```plaintext
setOrderFlag: 0
type: 1
f: [0 0.1000 0.1500 1]
Rp: 3
Rs: 20
Wn: 0.1250
order: 34
Beta: 0
wind: [35x1 double]
```

Because certain filter parameters are unique to a particular design, this structure has a different set of fields for each filter design. For example, the Beta field above only appears in the specs structure if the design is a Kaiser window FIR filter.

The table below lists the possible specifications fields that can appear in the export structure, and describes their contents.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>Kaiser window β parameter.</td>
</tr>
<tr>
<td>f</td>
<td>Contains a vector of band-edge frequencies, normalized to 1 (i.e., 1 = Nyquist).</td>
</tr>
</tbody>
</table>
Interactive Tools

Viewing Frequency Response Plots

It is often useful to view a filter’s frequency response, impulse response, and step response during the filter design process. You can use the Filter Viewer to view frequency-domain information about filters in the Filter Designer:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fpass</td>
<td>Passband cutoff frequencies. Scalar for lowpass and highpass designs, two-element vector for bandpass and bandstop designs.</td>
</tr>
<tr>
<td>Fstop</td>
<td>Stopband cutoff frequencies. Scalar for lowpass and highpass designs, two-element vector for bandpass and bandstop designs.</td>
</tr>
<tr>
<td>m</td>
<td>The response magnitudes corresponding to the band-edge frequencies in f.</td>
</tr>
<tr>
<td>order</td>
<td>Filter order.</td>
</tr>
<tr>
<td>Rp</td>
<td>Passband ripple (dB)</td>
</tr>
<tr>
<td>Rs</td>
<td>Stopband attenuation (dB)</td>
</tr>
<tr>
<td>setOrderFlag</td>
<td>Contains 1 if the filter order was specified manually (i.e., the Minimum Order box in the Specifications panel was not checked). Contains 0 if the filter order was computed automatically.</td>
</tr>
<tr>
<td>type</td>
<td>Contains 1 for lowpass, 2 for highpass, 3 for bandpass, or 4 for bandstop.</td>
</tr>
<tr>
<td>w3db</td>
<td>-3 dB frequency for Butterworth IIR designs.</td>
</tr>
<tr>
<td>wind</td>
<td>Vector of Kaiser window coefficients.</td>
</tr>
<tr>
<td>Wn</td>
<td>Cutoff frequency for the Kaiser window FIR filter when setOrderFlag = 1.</td>
</tr>
<tr>
<td>wt</td>
<td>Vector of weights, one weight per frequency band.</td>
</tr>
</tbody>
</table>
1 Activate SPTool from the Window menu.

2 Make sure the filters you want to analyze are selected in the Filters list.

3 Press View in the Filter panel.

   The Filter Viewer is activated with the selected filters displayed.

4 To edit one of the filters you're viewing, you can reactivate the Filter Designer from the Window menu in the Filter Viewer.

5 When you want to review a filter's characteristics after you've edited it, reactivate the Filter Viewer from the Window menu in the Filter Designer.

When the Filter Viewer is open at the same time that the Filter Designer is open, they both display the same filter. You can move back and forth between the Filter Designer and the Filter Viewer until the filter design is finished.

You can apply the filter to a signal by activating SPTool, selecting the filter in the Filters list, and the signal to apply it to from the Signals list, and pressing Apply. See “Applying a Filter” on page 5-18 for details.

See “Using the Filter Viewer: Interactive Filter Analysis” on page 5-84 for more information on the Filter Viewer.
Using the Filter Viewer: Interactive Filter Analysis

An important aspect of filter design is filter analysis, which encompasses both frequency and time-domain analysis of a filter. The Filter Viewer is a GUI-based frequency analysis tool that provides an interactive environment for the graphical display of digital filter characteristics.

The Filter Viewer can display six different characteristics subplots of a selected filter. Any combination of the six subplots may be displayed.

Using the Filter Viewer you can:

- View magnitude-response plots for one or more filters
- View phase-response plots for one or more filters
- View group-delay plots for one or more filters
- View zero-pole plots for one or more filters
- View impulse-response plots for one or more filters
- View step-response plots for one or more filters
- Zoom in to explore filter response details
- Modify selected plot parameters and display characteristics
- Measure a variety of characteristics of the filter response

For information on frequency analysis using toolbox functions from the command line or from M-files, see “Frequency Response” in Chapter 1 of this manual.

Opening the Filter Viewer

Open or activate the Filter Viewer from SPTool:

1. Select one or more filters from the Filters list in SPTool.
2. Press View in the Filters panel in SPTool.

   The Filter Viewer is activated and the selected filters are loaded into the Filter Viewer and displayed.

Basic Filter Viewer Functions

The Filter Viewer has the following components:
• A **Plots** panel for selecting which subplots display in the main plots window
• A **Rulers** panel and line display controls for making signal measurements and comparisons
• A **Frequency Axis** panel for specifying x-axis scaling in the main plots window
• A filter identification panel that displays information about the currently selected filter(s)
• A main plots (display) area for viewing one or more frequency-domain plots for the selected filter(s)
• Zoom controls for getting a closer look at filter response characteristics

When you first open or activate the Filter Viewer, it displays the default plot configuration for the selected filter(s):

The filters’ magnitude and phase plots are displayed. The frequency axis of the plots is set to **linear**, and the frequency axis range is set to **[0,Fs/2]**.
You can choose to display one or any combination of the six available subplots by using the check boxes in the Plots panel, and you can modify many of the plot display characteristics using the pop-up menus in the Plots panel and the Frequency Axis panel.

**Menus**

**File Menu.** Use Close from the File menu to close the Filter Viewer. Settings you changed and saved using the Preferences dialog box in SPTool are saved and used the next time you open a Filter Viewer.

**Window Menu.** Use the Window menu to select a currently open MATLAB figure window.

**Filter Identification Panel**

This panel displays the variable names and the highest sampling frequency of the currently selected filters. To change names or sampling frequencies, use Name... or Sampling Frequency... from the Edit menu in SPTool.

**Plots Panel**

The check boxes in this panel select the subplots to display in the main plots area. Any combination of subplots may be displayed.

To display a subplot, check the box at the left of the plot description.

There are six available subplots:

- **Magnitude:** displays the magnitude of the frequency response of the currently selected filter(s).
- **Phase:** displays the phase of the frequency response.
- **Group Delay:** displays the negative of the derivative of the phase response.
- **Zeros and Poles:** displays the poles and zeros of the filter transfer function(s) and their proximity to the unit circle.
- **Impulse Response:** displays the response of the currently selected filter(s) to a discrete-time unit-height impulse at $t=0$.
- **Step Response:** displays the response of the currently selected filter(s) to a discrete-time unit-height step function.
You can customize the display characteristics of the magnitude and phase subplots using the Magnitude and Phase pop-up menus. The options include:

- **Magnitude**: Scaling for the magnitude plot may be linear, log, or decibels.
- **Phase**: Phase units may be degrees or radians.

You can also change the magnitude and phase display characteristics for the Filter Viewer using the Filter settings panel of the Preferences dialog box in SPTool.

**Frequency Axis Settings**

You can change frequency axis scaling and range parameters for plots in the Filter Viewer.

Click on the option in the Frequency Axis panel you want to edit and drag to select a value. The options include:

- **Scale**: Scaling for the frequency axis may be linear or log.
- **Range**: The range for the frequency axis may be [0,Fs/2], [0,Fs], or [-Fs/2,Fs/2], where Fs represents the filter’s sampling frequency.
  
  The frequency range cannot be negative if Scale is set to log.

You can also change the frequency axis display characteristics for the Filter Viewer using the Filter Viewer settings panel of the Preferences dialog box in SPTool.

**Zoom Controls**

The available zoom controls in the Filter Viewer are Mouse Zoom and Full View. You can zoom independently in each displayed subplot.

By default, persistent zooming is disabled in the Filter Viewer. You can turn persistent zooming on from the Filter Viewer settings panel of the Preferences dialog box in SPTool.

See “Zoom Controls” on page 5-31 for details on using the zoom controls in the Filter Viewer.

**Help Button**

To use context-sensitive help, press the Help button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything
in the Filter Viewer, including menu items, to find out what it is and how to use it.

**Main Plots Area**
One or more of the six filter response subplots may be displayed graphically in the main plots area of the Filter Viewer. You can specify how the subplots are arranged by selecting **Filter Viewer Tiling** from the **Preferences** dialog box in SPTool. The options are **2-by-3 Grid**, **3-by-2 Grid**, **Vertical (6-by-1 Grid)**, and **Horizontal (1-by-6 Grid)**.

The following figure shows the Filter Viewer when four subplots are turned on and the 2-by-3 grid option is selected.

You can experiment to find the tiling option that works best for each specific combination and number of subplots.

You can zoom in on a subplot by pressing **Mouse Zoom** and then clicking on or dragging over a selected area of the subplot. By default, mouse zooming in the Filter Viewer is not persistent; after you click once, the zoom mode is turned off. You can make zooming persistent by checking **Stay in Zoom-mode after**
Zoom in the SPTool Preferences dialog box. This allows you to click repeatedly in a subplot to continue to zoom in on a particular feature of the display.

After you zoom in on a subplot, you can click and drag to pan around the subplot:

1. Press Mouse Zoom to turn on mouse zoom mode.
2. Click on a feature of a subplot to zoom in on it.
3. If persistent zooming is enabled, press Mouse Zoom again to turn off mouse zoom mode.
4. Click again in the same subplot, hold down the mouse button until the hand cursor is displayed, and drag the mouse to pan around the subplot.

**Viewing Filter Plots**

This section has a brief description and picture of each of the six filter response plots available in the Filter Viewer. A sequence of connected examples shows you how to display each plot on its own; you can also display any combination of plots, as needed.

Each plot in the example sequence displays the response of an order 22 equiripple lowpass filter with a sampling frequency of 1 Hz.

Regardless of how many or what combination of plots is displayed, you can zoom in on and pan each subplot independently.

**Viewing Magnitude Response**

A magnitude response plot is generally the simplest way to obtain a high-level view of a filter’s shape and fit to specifications. In the following example, use the Filter Designer to create a standard default filter and then view its magnitude response plot in the Filter Viewer:
1 From SPTool, press Create. The Filter Designer is activated and a standard default filter is created and displayed.

This is an order 22 equiripple lowpass filter with a sampling frequency of 1 Hz.

2 Use the Window menu in the Filter Designer to activate SPTool.

3 Press View from the Filters panel in SPTool to activate the Filter Viewer. The Filter Viewer is displayed with a magnitude response plot and a phase response plot.
4 Click the check box next to the **Phase** option to turn off the phase plot.

The magnitude plot for the default filter is displayed.

By default, this plot uses the default scaling (**linear**) for both axes and the default range for the frequency axis.

You can change the following display characteristics of the magnitude plot:

- Use the **Magnitude** pop-up menu to choose between **linear**, **log**, or **decibels** scaling of the y-axis.
- Use the **Scale** pop-up menu to choose between **linear** and **log** scaling of the x-axis.
- Use the **Range** pop-up menu to choose between the following ranges for the x-axis: $[0, F_s/2]$, $[0, F_s]$, or $[-F_s/2, F_s/2]$, where $F_s$ represents the filter’s sampling frequency.

**Viewing Phase Response**

In addition to displaying magnitude response, the Filter Viewer can calculate and plot the filter’s phase response. Phase response is the angular component of a filter’s frequency response. To display only a phase response plot for the current filter:
1. Click the check box next to the **Magnitude** option to turn off the magnitude plot.

2. Click the check box next to the **Phase** option to turn on the phase plot and update the display.

By default, this plot uses the default phase (degrees) and the default scaling and range for the frequency axis.

You can change the following display characteristics of the phase plot:

- Use the **Phase** pop-up menu to choose between displaying phase in degrees or radians.
- Use the **Scale** pop-up menu to choose between linear and log scaling of the x-axis.
- Use the **Range** pop-up menu to choose between the following ranges for the x-axis: 
  - \([0,F_s/2]\),
  - \([0,F_s]\), or
  - \([-F_s/2,F_s/2]\),
  where \(F_s\) represents the filter’s sampling frequency.
Viewing Group Delay

Group delay is a measure of the average delay of a filter as a function of frequency. To display only a group delay plot for the currently selected filter(s):

1. Click the check box next to the Phase option to turn off the phase plot.

2. Click the check box next to the Group Delay option to turn on the group delay plot and update the display.

By default, this plot uses the default scaling and range for the frequency axis.

You can change the following display characteristics of the group delay plot:

- Use the Scale pop-up menu to choose between linear and log scaling of the x-axis.
- Use the Range pop-up menu to choose between the following ranges for the x-axis: [0,Fs/2], [0,Fs], or [-Fs/2,Fs/2], where Fs represents the highest sampling frequency of the currently selected filters.
Viewing a Zero-Pole Plot
The zero-pole plot displays the poles and zeros of the transfer function and their proximity to the unit circle. An X represents a pole of the transfer function; a O represents a zero of the transfer function. To display only a zero-pole plot for the currently selected filter(s):

1. Click the check box next to the Group Delay option to turn off the group delay plot.

2. Click the check box next to the Zeros and Poles option to turn on the zero-pole plot and update the display.

Viewing Impulse Response
The impulse response plot displays the response of the current filter(s) to a discrete-time unit-height impulse at t=0.

To display only an impulse response plot for the currently selected filter(s):
1. Click the check box next to the **Zeros and Poles** option to turn off the zero-pole plot.

2. Click the check box next to the **Impulse Response** option to turn on the impulse response plot and update the display.

You can change the following display characteristics of the impulse response plot:

Edit the **Time Response Length** field in the **Filter Viewer Preferences** panel to set the number of samples used to display the impulse response.

**Viewing Step Response**

The step response plot displays the response of the current filter(s) to a discrete-time unit-height step function. To display only a step response plot for the currently selected filter(s):
1 Click the check box next to the **Impulse Response** option to turn off the impulse response plot.

2 Click the check box next to the **Step Response** option to turn on the step response plot and update the display.

You can change the following display characteristics of the step response plot:

Edit the **Time Response Length** field in the **Filter Viewer Preferences** panel to set the number of samples used to display the step response.
Using the Spectrum Viewer: Interactive PSD Analysis

The Spectrum Viewer provides an interactive environment for the estimation of power spectral density for one data channel. It allows you to view and modify spectra created in SPTool.

Using the Spectrum Viewer you can:

- View and compare spectral density plots
- Use different estimation methods, including Burg, FFT, MTM, MUSIC, Welch, and Yule-Walker AR
- Modify spectrum parameters such as FFT length, window type, and sample frequency
- Print spectrum data

For information on spectral analysis using toolbox functions from the command line or from M-files, see Chapter 3 of this manual.

Opening the Spectrum Viewer

You can open or activate the Spectrum Viewer from SPTool by pressing one of the following buttons: Create, View, and Update. See “Creating a Spectrum” on page 5-19, “Viewing a Spectrum” on page 5-19, and “Updating a Spectrum” on page 5-19 for complete details.

Here is a brief summary of each method of activating the Spectrum Viewer:

- To create a spectrum, select a signal in SPTool and press Create. Press Apply in the Spectrum Viewer.
  A default spectrum of the selected signal is generated and displayed. You can view it in a variety of ways, measure it, and modify it in the Spectrum Viewer.
- To view a spectrum, select one or more spectra in SPTool and press View in the Spectra panel.
- To update a spectrum, select exactly one signal and one spectrum in SPTool and press Update. Press Apply in the Spectrum Viewer.

The spectrum is updated to reflect the data in the currently selected signal.
**Basic Spectrum Viewer Functions**

The Spectrum Viewer window has the following components:

- A **Parameters** panel for viewing and modifying the parameters or method of the current spectrum
- A signal identification panel that displays information about the signal linked to the current spectrum
- A main axes (display) area for viewing spectra graphically
- Zoom controls for getting a closer look at spectral features
- Rulers and line-display controls for making spectral measurements and comparisons
- Spectrum management controls: **Inherit from...**, **Revert**, and **Apply**
- Menu options for modifying plot display characteristics
- Menu options for printing spectrum data
Menus

File Menu. Use Page Setup... from the File menu to open the standard MATLAB Page Setup dialog box (see Using MATLAB Graphics), in which you can set the orientation, size and position, limits, and color of your printout.

Use Print Preview... from the File menu to open a MATLAB figure window with a preview of the information from the Spectrum Viewer window that will appear in your printout. See “Spectrum Viewer Settings” on page 5-24 and “Printing Spectrum Data” on page 5-107 for details of printing from the Spectrum Viewer.

Use Print... from the File menu to open the standard operating system print dialog box, from which you can print your spectrum data.

Use Close from the File menu to close the Spectrum Viewer. All spectrum selection and ruler information will be lost. Settings you changed and saved using the Preferences dialog box in SPTool are saved and used the next time you open the Spectrum Viewer.

Options Menu. Use these options to change scaling and range parameters for plots in the Spectrum Viewer.

Click on the option you want to edit and drag to select a value.
The options include:

- **Magnitude Scale**: Scaling for the magnitude plot may be **dB** or **linear**.

- **Frequency Range**: The range for the frequency axis may be **[0, Fs/2]**, **[0, Fs]**, or **[-Fs/2, Fs/2]**, where **Fs** is the sampling frequency. If multiple spectra are displayed, the value of **Fs** is the maximum of all the sampling frequencies. **Fs** is not defined for the case of a spectrum whose signal is <None>, that is, a spectrum whose associated signal has been deleted from SPTool. In this case, a value twice the highest frequency in the spectrum’s frequency vector is chosen.

  The frequency range cannot be negative if **Frequency Scale** is set to **log**.

- **Frequency Scale**: Scaling for the frequency axis may be **linear** or **log**.

**Window Menu.** Use the **Window** menu to select a currently open MATLAB figure window.

**Signal ID Panel**
This panel displays information about the signal linked to the currently selected spectrum. The information includes the signal’s name, size, data type (**real** or **complex**), and sampling frequency. To change any of these signal properties, use SPTool.

To associate a completely new signal with a displayed spectrum, select the signal in SPTool and press **Update** in the **Spectra** panel.

**Spectrum Management Buttons**

**Inherit from...** Choose a spectrum from this menu to let the active spectrum inherit its parameters (not including the associated signal).

Press **Inherit from...** and drag to select the spectrum from which you want to inherit parameters.

**Revert.** Restores the properties of the current spectrum to what they were the last time **Apply** was pressed.

**Apply.** Compute and display the active spectrum using the parameters set in the **Parameters** panel.
Zoom Controls
The available zoom controls in the Spectrum Viewer are Mouse Zoom, Full View, Zoom In-Y, Zoom Out-Y, Zoom In-X, and Zoom Out-X. See “Zoom Controls” on page 5-31 for details on using the zoom controls in the Spectrum Viewer.

Ruler and Line Display Controls
Using the rulers and line-display controls, you can measure a variety of characteristics of spectra in the Spectrum Viewer. See “Ruler Controls” on page 5-33 for details on using rulers and modifying line displays in the Spectrum Viewer.

Help Button
To use context-sensitive help, press the Help button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in the Spectrum Viewer, including menu items, to find out what it is and how to use it.

Main Axes Display Area
The Spectra list in SPTool shows all spectra in the current SPTool session. One or more spectra may be selected. The spectral data of all selected spectra are displayed graphically in the main axes display area of the Spectrum Viewer.

NOTE If a spectrum is not displayed, or if it is displayed with the wrong signal information, press Apply to recompute the spectral data.

When there is only one spectrum displayed, its properties are displayed in the Parameters panel and its measurements are displayed in the Rulers panel. When more than one spectrum is displayed, select the line you want to focus on.

When a spectrum is selected, you can use the ruler controls on the selected line (see “Making Signal Measurements” on page 5-37) and you can modify its parameters (see below). The label of the selected spectrum (line) is displayed in the Selection pop-up menu.
Select a spectrum (line) in one of two ways:

- Click on the Selection pop-up menu and drag to select the line to measure
- Move the mouse pointer over any point in the line you want to select and click on it

See “Selecting a Line to Measure” on page 5-34 for details.

Click-and-Drag Panning. You can use the mouse to pan around the main axes display:

Click on a line in the main axes, hold down the mouse button, and drag the mouse.

Click-and-drag panning is not enabled in mouse zoom mode.

Making Spectrum Measurements
Use the rulers to make a variety of measurements on the selected spectrum. See “Making Signal Measurements” on page 5-37 for details.

Viewing Spectral Density Plots
Spectral density estimation is a technique that finds the approximate frequency content of a signal. The Spectrum Viewer calculates single-channel power spectral density (PSD). When you first generate a spectrum, the Spectrum Viewer shows a default power spectral density function of the input data. By default, the Spectrum Viewer uses the Welch method of PSD estimation with a length-256 Hanning window and an FFT length of 1024.

You can change plot properties and computation parameters for a displayed spectrum, and you can set confidence intervals.

Controlling and Manipulating Plots

Changing Plot Properties
You can control the axes units and scaling properties that affect the Spectrum Viewer’s plots.
Use the **Options** menu to select:

- Linear or decibel scaling for the magnitude axis
- Linear or logarithmic scaling for the frequency axis
- The frequency range to view

See “Options Menu” on page 5-99 for details.

You can also zoom in on any of the Spectrum Viewer’s plots. See “Zoom Controls” on page 5-31 for details.

You can set other scaling properties in the **Parameters** panel, depending on the PSD method computation parameters you choose.

**Choosing Computation Parameters**

The Spectrum Viewer lets you control the PSD estimation parameters of the selected spectrum. Different parameters are available, depending on which method of PSD computation you choose. Set these parameters from the **Parameters** panel, as illustrated in the following steps:

1. Click on the **Method** pop-up menu and drag to select one of the following methods:
   - Burg
   - Covariance
   - FFT
   - Modified Covariance
   - MTM
   - MUSIC
   - Welch
   - Yule AR

   Appropriate parameter selections are displayed for each method you choose.
2 Modify the appropriate parameters.
   - When a parameter is in a pop-up menu, click on the parameter label and drag to select a value from the menu.
   - When a parameter is in an edit box, type the value or variable into the box.

You can also modify the parameters by using **Inherit from** to copy the parameters of another spectrum in SPTool. See “Inherit from...” on page 5-100 for details.

3 If you change your mind, you can discard changes you make by pressing **Revert**.

4 To apply the modified parameters, press **Apply**.

The new parameters are applied to the selected spectrum; the Spectrum Viewer recalculates the spectral density function and displays the modified spectrum.

**Computation Methods and Parameters**
You can choose from seven PSD computation methods. Each method has its own set of parameters.

This section shows the **Parameters** panel for each of the PSD computation methods. For detailed definitions and values for each parameter, use context-sensitive help (see “Help Button” on page 5-101).

**Burg.** For the Burg method, you can specify the following parameters:

- **Order**
- **Nfft**
**Covariance.** For the Covariance method, you can specify the following parameters:

- Order
- Nfft

**FFT.** For the FFT method, you can specify the following parameter:

- Nfft

**Mod. Covar.** For the Modified Covariance method, you can specify the following parameters:

- Order
- Nfft

**MTM.** For the MTM method, you can specify the following parameters:

- NW
- Nfft
• **Weights**
  Select one of the following from the pop-up menu:
  - adapt
  - unity
  - eigen

• **Conf. Int.**
  Check to compute a confidence interval and type in a value (see “Setting Confidence Intervals” on page 5-107).

**MUSIC.** For the MUSIC method, you can specify the following parameters:

• **Signal Dim.**
• **Threshold**
• **Nfft**
• **Nwind**
• **Window**
• **Overlap**
• **Corr. Matrix**
  Check if selected signal is a correlation matrix.
• **Eigenvector Weights**
  Check to select eigenvector weights.

**Welch.** For Welch’s method, you can specify the following parameters:

• **NFFT**
• **Nwind**
• **Window**
• **Overlap**
• **Scaling**
  Select one of the following from the pop-up menu:
  - Unbiased
  - Peaks
• **Conf. Int.**
  Check to compute a confidence interval and type in a value (see “Setting Confidence Intervals” on page 5-107).

**Yule AR.** For the Yule AR method, you can specify the following parameters:

• **Order**
• **Nfft**
• **Corr. Matrix**
  Check if selected signal is a correlation matrix.

**Setting Confidence Intervals**

By default, the Spectrum Viewer does not compute confidence intervals for spectral density. You can enable the computation of confidence intervals for the Welch and MTM methods by following these steps:

1. Click the **Conf. Int.** check box so that it is selected.

2. Type a value for the confidence level in the **Conf. Int.** edit box.
   
   This value must be a scalar between 0 and 1.

3. Press **Apply**.

**NOTE**  Confidence intervals are reliable only for nonoverlapping sections.

**Printing Spectrum Data**

You can print all of the spectrum data that you can display and manipulate in the Spectrum Viewer. Your printout will always contain:

• The plots of one or more spectra, as displayed in the main axes area
• A legend displayed in the main axes area to label each spectrum by name, line color/style, method, and FFT length

The ruler measurements of the currently selected spectrum will be displayed at the bottom of the figure, unless you suppress them with the **Rulers** check
box in the Spectrum Viewer preferences panel of the Preferences dialog box in SPTool (see “Spectrum Viewer Settings” on page 5-24).

You can preview your printout by choosing Print Preview... from the File menu in the Spectrum Viewer window. This opens the Spectrum Viewer Print Preview window, shown below with default settings for the spectrum mtlbse.

![Spectrum Viewer Print Preview](image)

The Close button on the Spectrum Viewer Print Preview window closes the print preview window and returns you to the Spectrum Viewer.

The Print... button on the Spectrum Viewer Print Preview window opens the standard print dialog box, from which you can print the contents of your print preview window.

If you choose to print from the print dialog, both that dialog and the Spectrum Viewer Print Preview window will close, and the contents of the print preview will print.

If you choose not to print, the print dialog and the Spectrum Viewer Print Preview window will close, and you will return to the Spectrum Viewer.

Any changes you make, prior to opening the print preview window, that affect the display of data in the Spectrum Viewer (see “Controlling and Manipulating Plots” on page 5-102) will affect the display of data in the Spectrum Viewer.
Print Preview window in exactly the same way. For example, in the print preview shown below:

- Data from three different sample spectra have been selected for display in the same main axes area (see “Opening the Filter Viewer” on page 5-84).
- Using the tools in the Edit Line pop-up menu (see “Ruler and Line Display Controls” on page 5-101), the spectrum lines have all been changed to black and their line styles have been differentiated, as is reflected in the legend.
- The plots of all three spectra have been zoomed to an interval between their rulers (see “Zoom Controls” on page 5-101). Note that the ruler measurements displayed are for the train spectrum, which is the currently selected spectrum.

Changes you make in the Preferences dialog box in SPTool will also affect the Spectrum Viewer Print Preview window in the same way that they affect the Spectrum Viewer itself. For example, in the print preview shown below, the
ruler measurements have been suppressed by changes in the Spectrum Viewer preferences (see “Spectrum Viewer Settings” on page 5-24).

![Spectrum Viewer Print Preview](image)

**NOTE**  The `HandleVisibility` property of the `Print Preview` figure window is set to 'on'. This allows you to modify the preview using Handle Graphics commands from the MATLAB command line or the Plot Editor tools (see Using MATLAB Graphics for more information). For example, you can drag the legend to a different position within the figure, as has been done in the figure shown above.

Changes that you make in the preview window print exactly as they appear on the screen; however, they are not saved when the `Print Preview` figure window is closed.

**Saving Spectrum Data**

After creating a spectrum in SPTool, you can export spectrum information to the workspace or to disk using `Export...` from the `File` menu in SPTool. The spectrum information is stored in a structure that you can access to retrieve the spectral power and frequency data. The spectrum structure also contains a number of fields that are used internally by SPTool.
To see the fields of the spectrum structure, try exporting a spectrum to the workspace:

1. Create a new spectrum if none are currently loaded. Label the spectrum `spect1`.
2. In SPTool, select `Export...` from the `File` menu.
3. In the `Export` dialog box, select `spect1` and press the `Export to Workspace` button.
4. Type `who` at the MATLAB command line to look at the variables in the workspace. The variable called `spect1` is the spectrum structure you exported from SPTool.
5. Type `spect1` to list the fields of the spectrum structure.

The following structure fields describe the spectrum.

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>The spectral power vector.</td>
</tr>
<tr>
<td>f</td>
<td>The spectral frequency vector.</td>
</tr>
<tr>
<td>confid</td>
<td>A structure containing the confidence intervals data:</td>
</tr>
<tr>
<td></td>
<td>• The <code>confid.level</code> field contains the chosen confidence level.</td>
</tr>
<tr>
<td></td>
<td>• The <code>confid.Pc</code> field contains the spectral power data for the confidence intervals.</td>
</tr>
<tr>
<td></td>
<td>• The <code>confid.enable</code> field contains a 1 if confidence levels are enabled for the spectrum.</td>
</tr>
<tr>
<td>signalLabel</td>
<td>The name of the signal from which the spectrum was generated.</td>
</tr>
<tr>
<td>Fs</td>
<td>The associated signal’s sample rate.</td>
</tr>
</tbody>
</table>
The other fields are used internally by SPTool, and are subject to change in future releases.
Example: Generation of Bandlimited Noise

This section provides a complete example of using the GUI-based interactive tools to design and implement an FIR digital filter, apply it to a signal, and display signals and spectra. The steps include:

- Importing and naming a signal using SPTool
- Designing a filter using the Filter Designer
- In SPTool, applying the filter to the signal to create another signal
- Viewing the time domain information of the original and filtered signals using the Signal Browser
- Comparing the spectra of both signals using the Spectrum Viewer

Create, Import, and Name a Signal

You can import an existing signal into SPTool, or you can create a new signal and edit and name it in SPTool. In this step, you’ll create a new signal at the command line and then import it into SPTool.

1. At the command line, create a random signal by typing:
   \[ x = \text{randn}(5000,1); \]

2. Activate SPTool by typing:
   sptool

   The SPTool window is displayed.
3 Select **Import**... from the **File** menu:

The **Import to SPTool** window is displayed.

Notice that the variable `x` is displayed in the **Workspace Contents** list. (If it is not, click the **From Workspace** radio button to display the contents of the workspace.)

4 Name the signal and import it into SPTool:
   a. Make sure that **Signal** is selected in the **Import As** pop-up menu.
   b. Click in the **Data** field and type `x`.
      
      You can also move the variable `x` into the **Data** field by clicking on `x` in the **Workspace Contents** list and then clicking on the arrow to the left of the **Data** field.
   c. Click in the **Sampling Frequency** field and type `5000`.
   d. Name the signal by clicking in the **Name** field and typing `noise`.
   e. Press **OK**.

The SPTool window is reactivated, and the signal `noise` is selected in the **Signals** list.
Design a Filter

You can import an existing filter into SPTool, or you can design and edit a new filter using the Filter Designer. In this step, you’ll create a default filter and customize it in the Filter Designer.

1 Press **New Design** in SPTool to activate the Filter Designer and generate a default filter.

   The Filter Designer window is displayed with the default filter `filt1`.

2 Change the filter sampling frequency to 5000 by entering this value in the **Sampling Frequency** text box in the Filter Designer.

3 Specify the filter parameters shown at left:
   - Make sure **Equiripple FIR** is selected in the **Algorithm** pop-up menu.
   - Select **bandpass** from the **Type** pop-up menu.
   - Set the passband edge frequencies by entering 750 for **Fp1** and 1250 for **Fp2**.
   - Set the stopband edge frequencies by entering 500 for **Fs1** and 1500 for **Fs2**.
   - Type .01 into the **Rp** field and 75 into the **Rs** field. **Rp** sets the maximum passband ripple and **Rs** sets the stopband attenuation for the filter.
   - Press the **Apply** button to compute the new filter.
When the new filter is computed, the magnitude response of the filter is displayed with a solid line in the main axes display area.

The resulting filter is an order 78 bandpass equiripple filter.

**Apply the Filter to a Signal**

In this step, you apply the filter to the signal in SPTool. The new, filtered signal is automatically created in SPTool.
Example: Generation of Bandlimited Noise

1. Activate SPTool from the **Window** menu in the Filter Designer.

2. Click to select the signal `noise(vector)` from the **Signals** list and click to select the filter (named `filt1[design]`) from the **Filters** list, as shown below.

3. Press **Apply** to apply the filter `filt1` to the signal `noise`.

   The **Apply Filter** dialog box is displayed.

4. Name the new signal by clicking in the **Output Signal** field and typing `blnoise`.

5. Press **OK**.

   The filter is applied to the selected signal. The new, filtered signal `blnoise(vector)` is displayed in the **Signals** list.

**View, Play, and Print the Signals**

You can view and print the time domain information of the signals using the Signal Browser. You can also play the signals, if your computer has audio output capabilities. In this step, you'll display both signals in the Signal Browser, select and play each signal, and print both signals.
1 **Shift**-click on the `noise` and `blnoise` signals in the **Signals** list of SPTool to select both signals.

2 Press **View** in the **Signals** panel.

The Signal Browser is activated and both signals are displayed in the main axes display area. Initially, the `noise` signal covers up the bandlimited `blnoise` signal, but you can see that both signals are displayed because the names of both signals are shown above the main axes display area.

3 **Click-and-drag** in the **Selection** pop-up menu to select the `blnoise` signal.

The main axes display area is redisplayed. Now you can see the `blnoise` signal superimposed on top of the `noise` signal. The signals are displayed in different colors in both the main axes display area and the panner. Notice
that the color of the line in the Selection display changes to correspond to the color of the signal that you've selected.

The signal that's displayed in the Selection pop-up menu and in the Selection display is the active signal. When you select Play, or use the rulers, the active signal is the one that is played or measured.

4 To hear the active signal, select Play from the Options menu.

5 To hear the other signal, select it as in step 3 above (you can also select the signal by clicking on it in the main axes display area). Select Play from the Options menu again.

6 Before printing the two signals together, use the line controls to maximize the visual contrast between the signals by setting the line color for noise to
gray and the line color for \texttt{bnoise} to white. (See “Ruler Controls” on page 5-33 for details on using these controls.)

Use the Signal Browser panel in the \texttt{Preferences} dialog box in SPTool to suppress printing of both the panner and the ruler settings. (See “Signal Browser Settings” on page 5-23 for details on these preferences.)

To print both signals, as seen in the picture below, select \texttt{Print...} from the \texttt{File} menu in the Signal Browser.

![Signal Browser](image)

\section*{Compare Spectra of Both Signals}

You can get an idea of the frequency content of the two signals by displaying their power spectra using the Spectrum Viewer, as described below.

1. Reactivate SPTool by selecting it from the \texttt{Window} menu of the Signal Browser.

2. Click on the \texttt{noise} signal in the \texttt{Signals} list of SPTool to select it.
3 Press Create in the Spectra panel.

The Spectrum Viewer is activated, and a spectrum object (spect1) corresponding to the noise signal is created in the Spectra list. The spectrum is not computed or displayed yet.

4 Press Apply in the Spectrum Viewer to compute and display spect1. The spectrum of the noise signal is displayed in the main axes display area.

Notice that the spectrum's signal identification information – including its name, its type, and its sampling frequency – is displayed above the Parameters panel, and the spectrum's name is displayed both above the main axes display area and in the Selection pop-up menu.

The spectrum estimate is within 2 or 3 dB of 0, so the noise has a fairly “flat” spectrum.

5 Reactivate SPTool by selecting it from the Window menu in the Spectrum Viewer.

6 Click on the blnoise signal in the Signals list of SPTool to select it.
7 Press **Create** in the **Spectra** panel.

The Spectrum Viewer is again activated, and a spectrum object (spect2) corresponding to the blnoise signal is created in the **Spectra** list. The spectrum is not computed or displayed yet.

8 Press **Apply** in the Spectrum Viewer to display spect2.

The spectrum of the blnoise signal is displayed in the main axes display area.

The new spectrum’s signal identification information – including its name, its type, and its sampling frequency – is displayed above the **Parameters** panel, and the spectrum’s name is displayed both above the main axes display area and in the **Selection** pop-up menu.

This spectrum is flat between 750 and 1250 Hz and has 75 dB less power in the stopband regions of filt1.

9 Reactivate SPTool again, as in step 5 above.
10 Shift-click on spect1 and spect2 in the Spectra list to select them both.

11 Press View in the Spectra panel to reactivate the Spectrum Viewer and display both spectra together.

12 To select one of the spectra for measuring or editing, use the Selection pop-up menu, or click on the spectrum in the main axes display area.

The color of the line in the Selection display changes to correspond to the color of the spectrum that you’ve selected.

The spectrum that’s displayed in the Selection pop-up menu and in the Selection display is the active spectrum. When you use the rulers or change parameters, the active spectrum is the one that is measured or modified.

13 Before printing the two spectra together, use the selection and ruler controls to differentiate the two plots by line style, rather than by color. Drag the rulers to demark the stopband edge frequencies and change the rulers from
**Track to Vertical.** (See “Ruler Controls” on page 5-33 for details on using these controls.)

Select **Print Preview...** from the **File** menu in the Spectrum Viewer. From the **Spectrum Viewer Print Preview** window, drag the legend out of the axes display area so that it doesn’t obscure part of the plot.

To print both signals, as seen in the picture below, select the **Print...** button on the **Spectrum Viewer Print Preview** window.
Reference
This chapter contains detailed descriptions of all Signal Processing Toolbox functions. It begins with a list of functions grouped by subject area and continues with the reference entries in alphabetical order. For more information, see the online MATLAB Function Reference.

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<td>diric</td>
<td>Dirichlet or periodic sinc function.</td>
</tr>
<tr>
<td>gauspuls</td>
<td>Gaussian-modulated sinusoidal pulse generator.</td>
</tr>
<tr>
<td>pulstran</td>
<td>Pulse train generator.</td>
</tr>
<tr>
<td>rectpuls</td>
<td>Sampled aperiodic rectangle generator.</td>
</tr>
<tr>
<td>sawtooth</td>
<td>Sawtooth or triangle wave generator.</td>
</tr>
<tr>
<td>sinc</td>
<td>Sinc function.</td>
</tr>
<tr>
<td>square</td>
<td>Square wave generator.</td>
</tr>
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<td>strips</td>
<td>Strip plot.</td>
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<td>Phase angle.</td>
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<tr>
<td>conv</td>
<td>Convolution and polynomial multiplication.</td>
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<tr>
<td>conv2</td>
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<td>fftfilt</td>
<td>FFT-based FIR filtering using the overlap-add method.</td>
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<tr>
<td>filter</td>
<td>Filter data with a recursive (IIR) or nonrecursive (FIR) filter.</td>
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## Filter Analysis and Implementation

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<td>filtfilt</td>
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<tr>
<td>filtic</td>
<td>Find initial conditions for a transposed direct form II filter implementation.</td>
</tr>
<tr>
<td>freqs</td>
<td>Frequency response of analog filters.</td>
</tr>
<tr>
<td>freqspace</td>
<td>Frequency spacing for frequency response.</td>
</tr>
<tr>
<td>freqz</td>
<td>Frequency response of digital filters.</td>
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<td>Average filter delay (group delay).</td>
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<td>latcfilt</td>
<td>Lattice and lattice-ladder filter implementation.</td>
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<tr>
<td>sgolayfilt</td>
<td>Savitzky-Golay filtering.</td>
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<th>Function</th>
<th>Description</th>
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<td>Conversion of autocorrelation sequence to prediction polynomial.</td>
</tr>
<tr>
<td>ac2rc</td>
<td>Conversion of autocorrelation sequence to reflection coefficients.</td>
</tr>
<tr>
<td>convmtx</td>
<td>Convolution matrix.</td>
</tr>
<tr>
<td>latc2tf</td>
<td>Lattice filter to transfer function conversion.</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>poly2ac</td>
<td>Conversion of prediction polynomial to autocorrelation sequence.</td>
</tr>
<tr>
<td>poly2rc</td>
<td>Conversion of prediction polynomial to reflection coefficients.</td>
</tr>
<tr>
<td>rc2ac</td>
<td>Conversion of reflection coefficients to autocorrelation sequence.</td>
</tr>
<tr>
<td>rc2poly</td>
<td>Conversion of reflection coefficients to prediction polynomial.</td>
</tr>
<tr>
<td>residuez</td>
<td>z-transform partial-fraction expansion.</td>
</tr>
<tr>
<td>sos2ss</td>
<td>Conversion of second-order sections to state-space.</td>
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<tr>
<td>sos2tf</td>
<td>Conversion of second-order sections to transfer function.</td>
</tr>
<tr>
<td>sos2zp</td>
<td>Conversion of second-order sections to zero-pole-gain.</td>
</tr>
<tr>
<td>ss2sos</td>
<td>Conversion of state-space to second-order sections.</td>
</tr>
<tr>
<td>ss2tf</td>
<td>Conversion of state-space to transfer function.</td>
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<tr>
<td>ss2zp</td>
<td>Conversion of state-space to zero-pole-gain.</td>
</tr>
<tr>
<td>tf2latc</td>
<td>Conversion of transfer function to lattice filter.</td>
</tr>
<tr>
<td>tf2sos</td>
<td>Conversion of transfer function to second-order sections.</td>
</tr>
<tr>
<td>tf2ss</td>
<td>Conversion of transfer function to state-space.</td>
</tr>
<tr>
<td>tf2zp</td>
<td>Conversion of transfer function to zero-pole-gain.</td>
</tr>
<tr>
<td>zp2sos</td>
<td>Conversion of zero-pole-gain to second-order sections.</td>
</tr>
<tr>
<td>zp2ss</td>
<td>Conversion of zero-pole-gain to state-space.</td>
</tr>
<tr>
<td>zp2tf</td>
<td>Conversion of zero-pole-gain to transfer function.</td>
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<td>Bessel analog filter design.</td>
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<td>butter</td>
<td>Butterworth analog and digital filter design.</td>
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<tr>
<td>cheby1</td>
<td>Chebyshev type I filter design (passband ripple).</td>
</tr>
<tr>
<td>cheby2</td>
<td>Chebyshev type II filter design (stopband ripple).</td>
</tr>
<tr>
<td>ellip</td>
<td>Elliptic (Cauer) filter design.</td>
</tr>
<tr>
<td>maxflat</td>
<td>Generalized digital Butterworth filter design.</td>
</tr>
<tr>
<td>yulewalk</td>
<td>Recursive digital filter design.</td>
</tr>
</tbody>
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### IIR Filter Order Selection

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<td>Butterworth filter order selection.</td>
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<tr>
<td>cheb1ord</td>
<td>Chebyshev type I filter order selection.</td>
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<tr>
<td>cheb2ord</td>
<td>Chebyshev type II filter order selection.</td>
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<tr>
<td>ellipord</td>
<td>Elliptic filter order selection.</td>
</tr>
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</table>

### FIR Filter Design

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<th>Function</th>
<th>Description</th>
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<td>Complex and nonlinear-phase equiripple FIR filter design.</td>
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<tr>
<td>fir1</td>
<td>Window-based finite impulse response filter design – standard response.</td>
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<td>fir2</td>
<td>Window-based finite impulse response filter design – arbitrary response.</td>
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<tr>
<td>fircls</td>
<td>Constrained least square FIR filter design for multiband filters.</td>
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</tbody>
</table>
### FIR Filter Design

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<tr>
<th>Function</th>
<th>Description</th>
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<td>Constrained least square filter design for lowpass and highpass linear phase FIR filters.</td>
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<td>Least square linear-phase FIR filter design.</td>
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<td>firrcos</td>
<td>Raised cosine FIR filter design.</td>
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<td>Interpolation FIR filter design.</td>
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<tr>
<td>kaiserord</td>
<td>Estimate parameters for an FIR filter design with Kaiser window.</td>
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<td>remez</td>
<td>Parks-McClellan optimal FIR filter design.</td>
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<td>sgolay</td>
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### Transforms

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<td>Chirp z-transform.</td>
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<td>dct</td>
<td>Discrete cosine transform (DCT).</td>
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<tr>
<td>dftmtx</td>
<td>Discrete Fourier transform matrix.</td>
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<tr>
<td>fft</td>
<td>One-dimensional fast Fourier transform.</td>
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<tr>
<td>fft2</td>
<td>Two-dimensional fast Fourier transform.</td>
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<tr>
<td>fftshift</td>
<td>Rearrange the outputs of the FFT functions.</td>
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<td>hilbert</td>
<td>Hilbert transform.</td>
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<td>idct</td>
<td>Inverse discrete cosine transform.</td>
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<td>ifft</td>
<td>One-dimensional inverse fast Fourier transform.</td>
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<td>Two-dimensional inverse fast Fourier transform.</td>
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<tr>
<td>Function</td>
<td>Description</td>
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<td>----------</td>
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<tr>
<td>cohere</td>
<td>Estimate magnitude squared coherence function between two signals.</td>
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<tr>
<td>corrcoef</td>
<td>Correlation coefficient matrix.</td>
</tr>
<tr>
<td>cov</td>
<td>Covariance matrix.</td>
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<tr>
<td>csd</td>
<td>Estimate the cross spectral density (CSD) of two signals.</td>
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<tr>
<td>pburg</td>
<td>Power spectrum estimate using the Burg method.</td>
</tr>
<tr>
<td>pcov</td>
<td>Power spectrum estimate using the covariance method.</td>
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<tr>
<td>pmcov</td>
<td>Power spectrum estimate using the modified covariance method.</td>
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<td>pmtm</td>
<td>Power spectrum estimate using the multitaper method (MTM).</td>
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<td>pmusic</td>
<td>Power spectrum estimate using MUSIC eigenvector method.</td>
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<td>pwelch</td>
<td>Estimate the power spectral density (PSD) of a signal using Welch’s method.</td>
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<td>tfe</td>
<td>Transfer function estimate from input and output.</td>
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<tr>
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<td>Rectangular window.</td>
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<td>hanning</td>
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<td>kaiser</td>
<td>Kaiser window.</td>
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<td>triang</td>
<td>Triangular window.</td>
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### Parametric Modeling

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<tr>
<td>arcov</td>
<td>Compute an estimate of AR model parameters using the covariance method.</td>
</tr>
<tr>
<td>armcov</td>
<td>Compute an estimate of AR model parameters using the modified covariance method.</td>
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<td>Compute an estimate of AR model parameters using the Yule-Walker method.</td>
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<td>invfreqs</td>
<td>Continuous-time (analog) filter identification from frequency data.</td>
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<td>Levinson-Durbin recursion.</td>
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<td>Prony's method for time domain IIR filter design.</td>
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### Parametric Modeling

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<td>rlevinson</td>
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<td>Buffer a signal vector into a matrix of data frames.</td>
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<td>cceps</td>
<td>Complex cepstral analysis.</td>
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<td>Decrease the sampling rate for a sequence (decimation).</td>
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<td>dpss</td>
<td>Discrete prolate spheroidal sequences (Slepian sequences).</td>
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<td>dpssclear</td>
<td>Remove discrete prolate spheroidal sequences from database.</td>
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<td>Discrete prolate spheroidal sequences database directory.</td>
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<td>Load discrete prolate spheroidal sequences from database.</td>
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<td>Increase sampling rate by an integer factor (interpolation).</td>
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<td>medfilt1</td>
<td>One-dimensional median filtering.</td>
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<td>polystab</td>
<td>Stabilize polynomial.</td>
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<td>rceps</td>
<td>Real cepstrum and minimum phase reconstruction.</td>
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### Specialized Operations

<table>
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<tr>
<th>Function</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>resample</td>
<td>Change sampling rate by any rational factor.</td>
</tr>
<tr>
<td>specgram</td>
<td>Time-dependent frequency analysis (spectrogram).</td>
</tr>
<tr>
<td>upfirdn</td>
<td>Upsample, apply an FIR filter, and downsample.</td>
</tr>
<tr>
<td>vco</td>
<td>Voltage controlled oscillator.</td>
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### Analog Prototype Design

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<th>Function</th>
<th>Description</th>
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<td>besselap</td>
<td>Bessel analog lowpass filter prototype.</td>
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<tr>
<td>buttap</td>
<td>Butterworth analog lowpass filter prototype.</td>
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<tr>
<td>cheb1ap</td>
<td>Chebyshev type I analog lowpass filter prototype.</td>
</tr>
<tr>
<td>cheb2ap</td>
<td>Chebyshev type II analog lowpass filter prototype.</td>
</tr>
<tr>
<td>ellipap</td>
<td>Elliptic analog lowpass filter prototype.</td>
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### Frequency Translation

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<tr>
<th>Function</th>
<th>Description</th>
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<td>lp2bp</td>
<td>Lowpass to bandpass analog filter transformation.</td>
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<td>lp2bs</td>
<td>Lowpass to bandstop analog filter transformation.</td>
</tr>
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<td>lp2hp</td>
<td>Lowpass to highpass analog filter transformation.</td>
</tr>
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<td>Lowpass to lowpass analog filter transformation.</td>
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<tr>
<td>Filter Discretization</td>
<td></td>
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<tr>
<td>-----------------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>bilinear</td>
<td>Bilinear transformation method of analog-to-digital filter conversion.</td>
</tr>
<tr>
<td>impinvar</td>
<td>Impulse invariance method of analog-to-digital filter conversion.</td>
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<th>Interactive Tools</th>
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<tr>
<td>sptool</td>
<td>Interactive digital signal processing tool (SPTool).</td>
</tr>
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</table>
abs

**Purpose**
Absolute value (magnitude).

**Syntax**
y = abs(x)

**Description**
y = abs(x) returns the absolute value of the elements of x. If x is complex, abs returns the complex modulus (magnitude):

\[ \text{abs}(x) = \sqrt{\text{real}(x)^2 + \text{imag}(x)^2} \]

If x is a MATLAB string, abs returns the numeric values of the ASCII characters in the string. The display format of the string changes; the internal representation does not.

The abs function is part of the standard MATLAB language.

**Example**
Calculate the magnitude of the FFT of a sequence:

```matlab
t = (0:99)/100; % time vector
x = sin(2*pi*15*t) + sin(2*pi*40*t); % signal
y = fft(x); % compute DFT of x
m = abs(y); % magnitude

f = (0:length(y)-1)'/length(y)*100; % frequency vector
plot(f,m)
```

**See Also**
angle Phase angle.
### Purpose
Conversion of autocorrelation sequence to prediction polynomial.

### Syntax
```
a = ac2poly(r)
[a,efinal] = ac2poly(r)
```

### Description
`a = ac2poly(r)` finds the prediction polynomial, `a`, corresponding to the autocorrelation sequence `r`. `a` is the same length as `r`, and `a(1) = 1`.

`[a,efinal] = ac2poly(r)` returns the final prediction error, `efinal`.

### Example
Consider the autocorrelation sequence
```
r = [5.0000  -1.5450  -3.9547  3.9331  1.4681  -4.7500];
```

The equivalent prediction polynomial is
```
a = ac2poly(r)
a =
1.0000   0.6147   0.9898   0.0004   0.0034  -0.0077
```

### See Also
- `ac2rc` Conversion of autocorrelation sequence to reflection coefficients.
- `poly2ac` Conversion of prediction polynomial to autocorrelation sequence.
- `rc2poly` Conversion of prediction polynomial to reflection coefficients.

### References
Purpose          Conversion of autocorrelation sequence to reflection coefficients.

Syntax          \[ [k, r0] = \text{ac2rc}(r) \]

Description     \([k, r0] = \text{ac2rc}(r)\) finds the reflection coefficients, \(k\), corresponding to the autocorrelation sequence \(r\). \(r0\) contains the zero-lag autocorrelation.

See Also         \(\text{ac2poly}\) Conversion of autocorrelation sequence to prediction polynomial.
                 \(\text{poly2rc}\) Conversion of prediction polynomial to reflection coefficients.
                 \(\text{rc2ac}\) Conversion of reflection coefficients to autocorrelation sequence.

Purpose

Phase angle.

Syntax

\[ p = \text{angle}(h) \]

Description

\[ p = \text{angle}(h) \] returns the phase angles, in radians, of the elements of complex vector or array \( h \). The phase angles lie between \(-\pi\) and \(\pi\).

For complex sequence \( h = x + iy = me^{ip} \), the magnitude and phase are given by

\[
\begin{align*}
  m &= \text{abs}(h) \\
  p &= \text{angle}(h)
\end{align*}
\]

To convert to the original \( h \) from its magnitude and phase:

\[
\begin{align*}
  i &= \sqrt{-1} \\
  h &= m \cdot \exp(i\cdot p)
\end{align*}
\]

The \text{angle} function is part of the standard MATLAB language.

Example

Calculate the phase of the FFT of a sequence:

```matlab
  t = (0:99)/100; % time vector
  x = sin(2*pi*15*t) + sin(2*pi*40*t); % signal
  y = fft(x); % compute DFT of x
  p = unwrap(angle(y)); % phase
  
  f = (0:length(y)-1)/length(y)*100; % frequency vector
  plot(f,p)
```

Algorithm

\[ \text{angle} \] can be expressed as:

\[
\text{angle}(x) = \text{imag}(\log(x)) = \text{atan2}(\text{imag}(x),\text{real}(x))
\]

See Also

\[ \text{abs} \]  
Absolute value (magnitude).
Purpose
Compute an estimate of AR model parameters using the Burg method.

Syntax
a = arburg(x,p)
[a,e] = arburg(x,p)
[a,e,k] = arburg(x,p)

Description
a = arburg(x,p) uses the Burg method to fit a p-th order autoregressive (AR) model to the input signal, x, by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise. Vector a contains the normalized estimate of the AR system parameters, A(z), in descending powers of z.

\[
H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_2z^{-1} + \ldots + a_{(p+1)}z^{-p}}
\]

Since the method characterizes the input data using an all-pole model, the correct choice of the model order p is important.

[a,e] = arburg(x,p) returns the variance estimate, e, of the white noise input to the AR model.

[a,e,k] = arburg(x,p) returns a vector, k, of reflection coefficients.

See Also
arcov
Compute an estimate of AR model parameters using the covariance method.
armcov
Compute an estimate of AR model parameters using the modified covariance method.
aryule
Compute an estimate of AR model parameters using the Yule-Walker method.
lpc
Linear prediction coefficients.
pburg
Power spectrum estimate using the Burg method.
prony
Prony's method for time domain IIR filter design.
Purpose
Compute an estimate of AR model parameters using the covariance method.

Syntax

\[ a = \text{arcov}(x,p) \]
\[ [a,e] = \text{arcov}(x,p) \]

Description
\( a = \text{arcov}(x,p) \) uses the covariance method to fit a \( p \)-th order autoregressive (AR) model to the input signal, \( x \), which is assumed to be the output of an AR system driven by white noise. This method minimizes the forward prediction error in the least-squares sense. Vector \( a \) contains the normalized estimate of the AR system parameters, \( A(z) \), in descending powers of \( z \).

\[
H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_2z^{-1} + \ldots + a_{(p+1)}z^{-p}}
\]

Because the method characterizes the input data using an all-pole model, the correct choice of the model order \( p \) is important.

\[ [a,e] = \text{arcov}(x,p) \] returns the variance estimate, \( e \), of the white noise input to the AR model.

See Also
- arburg: Compute an estimate of AR model parameters using the Burg method.
- armcov: Compute an estimate of AR model parameters using the modified covariance method.
- aryule: Compute an estimate of AR model parameters using the Yule-Walker method.
- lpc: Linear prediction coefficients.
- pcov: Power spectrum estimate using the covariance method.
- prony: Prony’s method for time domain IIR filter design.
Purpose
Compute an estimate of AR model parameters using the modified covariance method.

Syntax
\[
a = \text{armcov}(x,p)\\n[a,e] = \text{armcov}(x,p)
\]

Description
\(a = \text{armcov}(x,p)\) uses the modified covariance method to fit a \(p\)-th order autoregressive (AR) model to the input signal, \(x\), which is assumed to be the output of an AR system driven by white noise. This method minimizes the forward and backward prediction errors in the least-squares sense. Vector \(a\) contains the normalized estimate of the AR system parameters, \(A(z)\), in descending powers of \(z\).

\[
H(z) = \frac{\sqrt{\hat{e}}}{A(z)} = \frac{\sqrt{\hat{e}}}{1 + a_2z^{-1} + \ldots + a_{(p+1)}z^{-p}}
\]

Because the method characterizes the input data using an all-pole model, the correct choice of the model order \(p\) is important.

\([a,e] = \text{armcov}(x,p)\) returns the variance estimate, \(e\), of the white noise input to the AR model.

See Also
- arburg
  - Compute an estimate of AR model parameters using the Burg method.
- arcov
  - Compute an estimate of AR model parameters using the covariance method.
- aryule
  - Compute an estimate of AR model parameters using the Yule-Walker method.
- lpc
  - Linear prediction coefficients.
- pmcov
  - Power spectrum estimate using the modified covariance method.
- prony
  - Prony’s method for time domain IIR filter design.
Purpose

Compute an estimate of AR model parameters using the Yule-Walker method.

Syntax

\[
a = \text{aryule}(x,p) \\
[a,e] = \text{aryule}(x,p) \\
[a,e,k] = \text{aryule}(x,p)
\]

Description

\(a = \text{aryule}(x,p)\) uses the Yule-Walker method, also called the autocorrelation method, to fit a \(p\)-th order autoregressive (AR) model to the windowed input signal, \(x\), by minimizing the forward prediction error in the least-squares sense. This formulation leads to the Yule-Walker equations, which are solved by the Levinson-Durbin recursion. \(x\) is assumed to be the output of an AR system driven by white noise. Vector \(a\) contains the normalized estimate of the AR system parameters, \(A(z)\), in descending powers of \(z\).

\[
H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_2z^{-1} + \ldots + a_{p+1}z^{-p}}
\]

Because the method characterizes the input data using an all-pole model, the correct choice of the model order \(p\) is important.

\([a,e] = \text{aryule}(x,p)\) returns the variance estimate, \(e\), of the white noise input to the AR model.

\([a,e,k] = \text{aryule}(x,p)\) returns a vector, \(k\), of reflection coefficients.

See Also

arburg | Compute an estimate of AR model parameters using the Burg method.
arcov | Compute an estimate of AR model parameters using the covariance method.
armcov | Compute an estimate of AR model parameters using the modified covariance method.
lpc | Linear prediction coefficients.
prony | Prony's method for time domain IIR filter design.
pyulear | Power spectrum estimate using Yule-Walker AR method.
**bartlett**

**Purpose**
Bartlett window.

**Syntax**

\[ w = \text{bartlett}(n) \]

**Description**

\[ w = \text{bartlett}(n) \]
returns an \( n \)-point Bartlett window in the column vector \( w \).

The coefficients of a Bartlett window are:

- For \( n \) odd

\[ w[k] = \begin{cases} 
\frac{2(k-1)}{n-1}, & 1 \leq k \leq \frac{n+1}{2} \\
2 - \frac{2(k-1)}{n-1}, & \frac{n+1}{2} \leq k \leq n 
\end{cases} \]

- For \( n \) even

\[ w[k] = \begin{cases} 
\frac{2(k-1)}{n-1}, & 1 \leq k \leq \frac{n}{2} \\
\frac{2(n-k)}{n-1}, & \frac{n}{2} + 1 \leq k \leq n 
\end{cases} \]

The Bartlett window is very similar to a triangular window as returned by the \text{triang} function. The Bartlett window always ends with zeros at samples 1 and \( n \), however, while the triangular window is nonzero at those points. For \( n \) odd, the center \( n-2 \) points of \text{bartlett}(n) are equivalent to \text{triang}(n-2).

**See Also**

- \text{blackman} Blackman window.
- \text{boxcar} Rectangular window.
- \text{chebwin} Chebyshev window.
- \text{hamming} Hamming window.
- \text{hanning} Hanning window.
- \text{kaiser} Kaiser window.
- \text{triang} Triangular window.

**References**

**Purpose**

Bessel analog lowpass filter prototype.

**Syntax**

`[z,p,k] = besselap(n)`

**Description**

`[z,p,k] = besselap(n)` returns the zeros, poles, and gain of an order `n` Bessel analog lowpass filter prototype. It returns the poles in the length `n` column vector `p` and the gain in scalar `k`. `z` is an empty matrix, because there are no zeros. `n` must be less than or equal to 25. The transfer function is

\[
H(s) = \frac{k}{(s-p(1))(s-p(2))\cdots(s-p(n))}
\]

`besselap` normalizes the poles and gain so that at low frequency and high frequency the Bessel prototype is asymptotically equivalent to the Butterworth prototype of the same order [1]. The magnitude of the filter is less than \(\sqrt{1/2}\) at the unity cutoff frequency \(\Omega_c = 1\).

Analog Bessel filters are characterized by a group delay that is maximally flat at zero frequency and almost constant throughout the passband. The group delay at zero frequency is

\[
\left(\frac{2n!}{2^{n}n!}\right)^{1/n}
\]

**Algorithm**

`besselap` finds the filter roots from a look-up table constructed using the Symbolic Math Toolbox.

**See Also**

- `besself`: Bessel analog filter design.
- `buttap`: Butterworth analog lowpass filter prototype.
- `cheb1ap`: Chebyshev type I analog lowpass filter prototype.
- `cheb2ap`: Chebyshev type II analog lowpass filter prototype.
- `ellipap`: Elliptic analog lowpass filter prototype.

Also see the Symbolic Math Toolbox User's Guide.

**References**

**besself**

**Purpose**  
Bessel analog filter design.

**Syntax**  

(b,a) = besself(n,Wn)
(b,a) = besself(n,Wn,'ftype')
[z,p,k] = besself(...)
[A,B,C,D] = besself(...)

**Description**  
besself designs lowpass, bandpass, highpass, and bandstop analog Bessel filters. Analog Bessel filters are characterized by almost constant group delay across the entire passband, thus preserving the wave shape of filtered signals in the passband. Digital Bessel filters do not retain this quality, and besself therefore does not support the design of digital Bessel filters.

(b,a) = besself(n,Wn) designs an order n lowpass analog filter with cutoff frequency Wn. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of s:

\[
H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \cdots + b(n+1)}{s^n + a(2)s^{n-1} + \cdots + a(n+1)}
\]

Cutoff frequency is the frequency at which the magnitude response of the filter begins to decrease significantly. For besself, the cutoff frequency Wn must be greater than 0. The magnitude response of a Bessel filter designed by besself is always less than \(\sqrt{1/2}\) at the cutoff frequency, and it decreases as the order n increases.

If Wn is a two-element vector, Wn = [w1 w2] with w1 < w2, besself(n,Wn) returns an order 2*n bandpass analog filter with passband w1 < \(\omega\) < w2.

(b,a) = besself(n,Wn,'ftype') designs a highpass or bandstop filter, where ftype is

- high for a highpass analog filter with cutoff frequency Wn
- stop for an order 2*n bandstop analog filter if Wn is a two-element vector,
  
  Wn = [w1 w2]

  The stopband is w1 < \(\omega\) < w2.

With different numbers of output arguments, besself directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:
[z,p,k] = besself(n,Wn) or
[z,p,k] = besself(n,Wn,'ftype')
besself returns the zeros and poles in length n or 2*n column vectors z and p
and the gain in the scalar k.

To obtain state-space form, use four output arguments:

[A,B,C,D] = besself(n,Wn) or
[A,B,C,D] = besself(n,Wn,'ftype') where A, B, C, and D are

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

and u is the input, x is the state vector, and y is the output.

**Example**

Design a fifth-order analog lowpass Bessel filter that suppresses frequencies
greater than 10,000 rad/sec and plot the frequency response of the filter using
freqs:

\[
[b,a] = \text{besself}(5,10000); \\
freqs(b,a) \quad \% \text{plot frequency response}
\]
**besself**

**Limitations**

Lowpass Bessel filters have a monotonically decreasing magnitude response, as do lowpass Butterworth filters. Compared to the Butterworth, Chebyshev, and elliptic filters, the Bessel filter has the slowest rolloff and requires the highest order to meet an attenuation specification.

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

**Algorithm**

besself performs a four-step algorithm:

1. It finds lowpass analog prototype poles, zeros, and gain using the besselap function.
2. It converts the poles, zeros, and gain into state-space form.
3. It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
4. It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

**See Also**

besselap Bessel analog lowpass filter prototype.
butter Butterworth analog and digital filter design.
cheby1 Chebyshev type I filter design (passband ripple).
cheby2 Chebyshev type II filter design (stopband ripple).
ellip Elliptic (Cauer) filter design.
Purpose

Bilinear transformation method of analog-to-digital filter conversion.

Syntax

\[
[zd, pd, kd] = \text{bilinear}(z, p, k, Fs)
\]
\[
[zd, pd, kd] = \text{bilinear}(z, p, k, Fs, Fp)
\]
\[
[numd, dend] = \text{bilinear}(num, den, Fs)
\]
\[
[numd, dend] = \text{bilinear}(num, den, Fs, Fp)
\]
\[
[Ad, Bd, Cd, Dd] = \text{bilinear}(A, B, C, D, Fs)
\]
\[
[Ad, Bd, Cd, Dd] = \text{bilinear}(A, B, C, D, Fs, Fp)
\]

Description

The bilinear transformation is a mathematical mapping of variables. In digital filtering, it is a standard method of mapping the \(s\) or analog plane into the \(z\) or digital plane. It transforms analog filters, designed using classical filter design techniques, into their discrete equivalents.

The bilinear transformation maps the \(s\)-plane into the \(z\)-plane by

\[
H(z) = H(s) \bigg|_{s = \frac{2}{T_s} \frac{z - 1}{z + 1}}
\]

This transformation maps the \(j\Omega\) axis (from \(\Omega = -\infty\) to \(+\infty\)) repeatedly around the unit circle (\(\exp(j\omega)\), from \(\omega = -\pi\) to \(\pi\)) by

\[
\omega = 2\tan^{-1}\left(\frac{\Omega}{2f_p}\right)
\]

bilinear can accept an optional parameter \(F_p\) that specifies prewarping. \(F_p\), in Hertz, indicates a "match" frequency, that is, a frequency for which the frequency responses before and after mapping match exactly. In prewarped mode, the bilinear transformation maps the \(s\)-plane into the \(z\)-plane with

\[
H(z) = H(s) \bigg|_{s = \frac{2\pi f_p}{\tan(\pi f_p/2z + 1)}(z - 1)}
\]
With the prewarping option, \texttt{bilinear} maps the $j\Omega$ axis (from $\Omega = -\infty$ to $+\infty$) repeatedly around the unit circle ($\exp(j\omega)$, from $\omega = -\pi$ to $\pi$) by

$$
\omega = 2\tan^{-1}\left(\frac{\Omega \tan\left(\frac{f_p}{2f_s}\right)}{2\pi f_p}\right)
$$

In prewarped mode, \texttt{bilinear} matches the frequency $2\pi f_p$ (in radians per second) in the $s$-plane to the normalized frequency $2\pi f_p/f_s$ (in radians per second) in the $z$-plane.

The \texttt{bilinear} function works with three different linear system representations: zero-pole-gain, transfer function, and state-space form.

**Zero-Pole-Gain**

\[
[zd, pd, kd] = \texttt{bilinear}(z, p, k, Fs)
\]

and

\[
[zd, pd, kd] = \texttt{bilinear}(z, p, k, Fs, Fp)
\]

convert the $s$-domain transfer function specified by $z$, $p$, and $k$ to a discrete equivalent. Inputs $z$ and $p$ are column vectors containing the zeros and poles, and $k$ is a scalar gain. $Fs$ is the sampling frequency in Hertz. \texttt{bilinear} returns the discrete equivalent in column vectors $zd$ and $pd$ and scalar $kd$. $Fp$ is the optional match frequency, in Hertz, for prewarping.

**Transfer Function**

\[
[numd, dend] = \texttt{bilinear}(num, den, Fs)
\]

and

\[
[numd, dend] = \texttt{bilinear}(num, den, Fs, Fp)
\]

convert an $s$-domain transfer function given by $num$ and $den$ to a discrete equivalent. Row vectors $num$ and $den$ specify the coefficients of the numerator and denominator, respectively, in descending powers of $s$

\[
\frac{\text{num}(s)}{\text{den}(s)} = \frac{\text{num}(1)s^{\text{nn}} + \cdots + \text{num}(\text{nn})s + \text{num}(\text{nn+1})}{\text{den}(1)s^{\text{nd}} + \cdots + \text{den}(\text{nd})s + \text{den}(\text{nd+1})}
\]
Fs is the sampling frequency in Hertz. `bilinear` returns the discrete equivalent in row vectors `numd` and `dend` in descending powers of $z$ (ascending powers of $z^{-1}$). $F_p$ is the optional match frequency, in Hertz, for prewarping.

**State-Space**

$[A_d, B_d, C_d, D_d] = \text{bilinear}(A, B, C, D, F_s)$ and

$[A_d, B_d, C_d, D_d] = \text{bilinear}(A, B, C, D, F_s, F_p)$ convert the continuous-time state-space system in matrices $A$, $B$, $C$, $D$,

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

to the discrete-time system

$$x[n+1] = A_d x[n] + B_d u[n]$$

$$y[n] = C_d x[n] + D_d u[n]$$

Fs is the sampling frequency in Hertz. `bilinear` returns the discrete equivalent in matrices $A_d$, $B_d$, $C_d$, $D_d$. $F_p$ is the optional match frequency, in Hertz, for prewarping.

**Algorithm**

`bilinear` uses one of two algorithms, depending on the format of the input linear system you supply. One algorithm works on the zero-pole-gain format and the other on the state-space format. For transfer function representations, `bilinear` converts to state-space form, performs the transformation, and converts the resulting state-space system back to transfer function form.

**Zero-Pole-Gain Algorithm**

For a system in zero-pole-gain form, `bilinear` performs four steps:

1. If $F_p$ is present, $k = 2\pi F_p / \tan(\pi F_p / F_s)$; otherwise $k = 2 F_s$.
2. It strips any zeros at plus or minus infinity using
   $$z = z(\text{find}(\text{finite}(z)))$$;
3 It transforms the zeros, poles, and gain using

\[
pd = (1+p/k)/(1–p/k);
zd = (1+z/k)/(1–z/k);
kd = \text{real}(k*\text{prod}(fs–z)/\text{prod}(fs–p));
\]

4 It adds extra zeros at -1 so the resulting system has equivalent numerator and denominator order.

**State-Space Algorithm**

For a system in state-space form, `bilinear` performs two steps:

1 If \( Fp \) is present, \( k = 2*\pi*Fp/\tan(\pi*Fp/Fs) \); else \( k = 2*Fs \).
2 It computes \( A_d, B_d, C_d, \) and \( D_d \) in terms of \( A, B, C, \) and \( D \) using

\[
A_d = \left(1 + \frac{1}{k}\right)A\left(1 - \frac{1}{k}\right)A^{-1}
\]

\[
B_d = \frac{2k}{k}\left(1 - \frac{1}{k}\right)A^{-1}B
\]

\[
C_d = rC\left(1 - \frac{1}{k}\right)A^{-1}
\]

\[
D_d = \frac{1}{k}C\left(1 - \frac{1}{k}\right)A^{-1}B + D
\]

`bilinear` implements these relations using conventional MATLAB statements. The scalar \( r \) is arbitrary; `bilinear` uses \( \sqrt{2/k} \) to ensure good quantization noise properties in the resulting system.

**Diagnostics**

`bilinear` requires that the numerator order be no greater than the denominator order. If this is not the case, `bilinear` displays:

```
Numerator cannot be higher order than denominator.
```

For `bilinear` to distinguish between the zero-pole-gain and transfer function linear system formats, the first two input parameters must be vectors with the same orientation in these cases. If this is not the case, `bilinear` displays:

```
First two arguments must have the same orientation.
```
See Also

- **impinvar**: Impulse invariance method of analog-to-digital filter conversion.
- **lp2bp**: Lowpass to bandpass analog filter transformation.
- **lp2bs**: Lowpass to bandstop analog filter transformation.
- **lp2hp**: Lowpass to highpass analog filter transformation.
- **lp2lp**: Lowpass to lowpass analog filter transformation.

References


Purpose  Blackman window.

Syntax  

\[
\begin{align*}
w & = \text{blackman}(n) \\
w & = \text{blackman}(n, \text{sflag})
\end{align*}
\]

Description  \(w = \text{blackman}(n)\) returns the \(n\)-point symmetrically sampled Blackman window in the column vector \(w\). \(n\) should be a nonnegative integer. The equation for a Blackman window is

\[
w[k] = 0.42 - 0.5 \cos\left(\frac{2\pi k}{n \cdot 1}\right) + 0.08 \cos\left(\frac{4\pi k}{n \cdot 1}\right) \quad k = 1, \ldots, n
\]

Blackman windows have slightly wider central lobes and less sideband leakage than equivalent length Hamming and Hanning windows.

\(w = \text{blackman}(n, \text{sflag})\) returns an \(n\)-point Blackman window using the window sampling specified by \(\text{sflag}\), which can be either 'periodic' or 'symmetric' (the default). When 'periodic' is specified, \text{blackman} computes a length \(n+1\) window and returns the first \(n\) points.

Algorithm  

\[
w = (0.42 - 0.5 \cos(2\pi(0:N-1)/(N-1)) + \ldots \\
0.08 \cos(4\pi(0:N-1)/(N-1)))';
\]

Diagnostics  An error message is displayed when incorrect arguments are used:

Order cannot be less than zero.
Sampling must be either 'symmetric' or 'periodic'.

A warning message is displayed for noninteger \(n\):

Warning: Rounding order to nearest integer.
See Also

<table>
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<th>Description</th>
</tr>
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<tr>
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</tr>
<tr>
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<td>Triangular window.</td>
</tr>
</tbody>
</table>

References

boxcar

Purpose
Rectangular window.

Syntax
w = boxcar(n)

Description
w = boxcar(n) returns a rectangular window of length n in the column vector w. This function is provided for completeness; a rectangular window is equivalent to no window at all.

Algorithm
w = ones(n,1);

See Also
bartlett Bartlett window.
blackman Blackman window.
chebwin Chebyshev window.
hamming Hamming window.
hanning Hanning window.
kaiser Kaiser window.
triang Triangular window.

References
buffer

**Purpose**
Buffer a signal vector into a matrix of data frames.

**Syntax**

```plaintext
y = buffer(x,n)
y = buffer(x,n,p)
y = buffer(x,n,p,opt)
[y,z] = buffer(...)  
[y,z,opt] = buffer(...)  
```

**Description**

`y = buffer(x,n)` partitions a length-\(L\) signal vector \(x\) into nonoverlapping data segments (frames) of length \(n\). Each data frame occupies one column of matrix output \(y\), which has \(n\) rows and \(\text{ceil}(L/n)\) columns. If \(L\) is not evenly divisible by \(n\), the last column is zero-padded to length \(n\).

`y = buffer(x,n,p)` overlaps or underlaps successive frames in the output matrix by \(p\) samples:

- For \(0 < p < n\) (overlap), `buffer` repeats the final \(p\) samples of each frame at the beginning of the following frame. For example, if \(x=1:30\) and \(n=7\), an overlap of \(p=3\) looks like this:

\[
\begin{array}{cccccccc}
0 & 2 & 6 & 10 & 14 & 18 & 22 & 26 \\
0 & 3 & 7 & 11 & 15 & 19 & 23 & 27 \\
0 & 4 & 8 & 12 & 16 & 20 & 24 & 28 \\
1 & 5 & 9 & 13 & 17 & 21 & 25 & 29 \\
2 & 6 & 10 & 14 & 18 & 22 & 26 & 30 \\
3 & 7 & 11 & 15 & 19 & 23 & 27 & 0 \\
4 & 8 & 12 & 16 & 20 & 24 & 28 & 0
\end{array}
\]

The first frame starts with \(p\) zeros (the default initial condition), and the number of columns in \(y\) is \(\text{ceil}(L/(n-p))\).
buffer

- For \( p < 0 \) (underlap), buffer skips \( p \) samples between consecutive frames. For example, if \( x = 1:30 \) and \( n = 7 \), a buffer with underlap of \( p = -3 \) looks like this:

\[
\begin{array}{ccc}
1 & 11 & 21 \\
2 & 12 & 22 \\
3 & 13 & 23 \\
4 & 14 & 24 \\
5 & 15 & 25 \\
6 & 16 & 26 \\
7 & 17 & 27 \\
\end{array}
\]

The number of columns in \( y \) is \( \lceil L/(n-p) \rceil \).

\( y = \text{buffer}(x, n, p, \text{opt}) \) specifies a vector of samples to precede \( x(1) \) in an overlapping buffer, or the number of initial samples to skip in an underlapping buffer:

- For \( 0 < p < n \) (overlap), \( \text{opt} \) specifies a length-\( p \) vector to insert before \( x(1) \) in the buffer. This vector can be considered an initial condition, which is needed when the current buffering operation is one in a sequence of consecutive buffering operations. To maintain the desired frame overlap from one buffer to the next, \( \text{opt} \) should contain the final \( p \) samples of the previous buffer in the sequence. See “Continuous Buffering” below.

- By default, \( \text{opt} \) is \( \text{zeros}(p,1) \) for an overlapping buffer. Set \( \text{opt} \) to ‘noDelay’ to skip the initial condition and begin filling the buffer immediately with \( x(1) \). In this case, \( L \) must be \( \text{length}(p) \) or longer. For example, if \( x = 1:30 \) and \( n = 7 \), a buffer with overlap of \( p = 3 \) looks like this:

\[
\begin{array}{ccccccc}
1 & 4 & 7 & 10 & 13 & 17 & 21 & 25 \\
2 & 5 & 8 & 11 & 14 & 18 & 22 & 26 \\
3 & 6 & 9 & 12 & 15 & 19 & 23 & 27 \\
4 & 7 & 10 & 13 & 16 & 19 & 22 & 26 \\
5 & 8 & 11 & 14 & 17 & 20 & 23 & 27 \\
6 & 9 & 12 & 15 & 18 & 21 & 24 & 28 \\
7 & 10 & 13 & 16 & 19 & 22 & 25 & 29 \\
\end{array}
\]

- For \( p < 0 \) (underlap), \( \text{opt} \) is an integer value in the range \( [0,-p] \) specifying the number of initial input samples, \( x(1:\text{opt}) \), to skip before adding samples to
the buffer. The first value in the buffer is therefore \( x(\text{opt}+1) \). By default, \( \text{opt} \) is zero for an underlapping buffer.

This option is especially useful when the current buffering operation is one in a sequence of consecutive buffering operations. To maintain the desired frame underlap from one buffer to the next, \( \text{opt} \) should equal the difference between the total number of points to skip between frames (\( \text{p} \)) and the number of points that were available to be skipped in the previous input to buffer. If the previous input had fewer than \( \text{p} \) points that could be skipped after filling the final frame of that buffer, the remaining \( \text{opt} \) points need to be removed from the first frame of the current buffer. See “Continuous Buffering” below for an example of how this works in practice.

\[
[y, z] = \text{buffer}(...) \quad \text{partitions the length-}L \text{ signal vector } x \text{ into frames of length } n, \text{ and outputs only the full frames in } y. \text{ If } y \text{ is an overlapping buffer, it has } n \text{ rows and } m \text{ columns, where}
\]

\[
m = \text{floor}(L/(n-p)) \quad \% \text{ when length(opt) = } p
\]

or

\[
m = \text{floor}((L-n)/(n-p)) + 1 \quad \% \text{ when opt} = \text{'nodelay'}
\]

If \( y \) is an underlapping buffer, it has \( n \) rows and \( m \) columns, where

\[
m = \text{floor}((L-\text{opt})/(n-p)) + (\text{rem}((L-\text{opt}),(n-p)) \geq n)
\]

If the number of samples in the input vector (after the appropriate overlapping or underlapping operations) exceeds the number of places available in the \( n \)-by-\( m \) buffer, the remaining samples in \( x \) are output in vector \( z \), which for an overlapping buffer has length

\[
\text{length}(z) = L - m^*(n-p) \quad \% \text{ when length(opt) = } p
\]

or

\[
\text{length}(z) = L - ((m-1)^*(n-p)+n) \quad \% \text{ when opt} = \text{'nodelay'}
\]

and for an underlapping buffer has length

\[
\text{length}(z) = (L-\text{opt}) - m^*(n-p)
\]
Output \( z \) shares the same orientation (row or column) as \( x \). If there are no remaining samples in the input after the buffer with the specified overlap or underlap is filled, \( z \) is an empty vector.

\[
[y, z, \text{opt}] = \text{buffer}(...) \quad \text{returns the last } p \text{ samples of a overlapping buffer in output } \text{opt}. \text{In an underlapping buffer, } \text{opt is the difference between the total number of points to skip between frames (} -p \text{) and the number of points in } x \text{ that were available to be skipped after filling the last frame.}
\]

- For \( 0 < p < n \) (overlap), \( \text{opt} \) (as an output) contains the final \( p \) samples in the last frame of the buffer. This vector can be used as the initial condition for a subsequent buffering operation in a sequence of consecutive buffering operations. This allows the desired frame overlap to be maintained from one buffer to the next. See “Continuous Buffering” below.

- For \( p < 0 \) (underlap), \( \text{opt} \) (as an output) is the difference between the total number of points to skip between frames \( (-p) \) and the number of points in \( x \) that were available to be skipped after filling the last frame.

\[
\text{opt} = m*(n–p) + \text{opt} – L \quad \% \text{ for } z = \text{empty vector}
\]

where \( \text{opt} \) on the right-hand side is the input argument to \text{buffer}, and \( \text{opt} \) on the left-hand side is the output argument. Here \( m \) is the number of columns in the buffer, which is

\[
m = \text{floor}((L–\text{opt})/(n–p)) + (\text{rem}((L–\text{opt}),(n–p)) >= n)
\]

Note that for an underlapping buffer output \( \text{opt} \) is always zero when output \( z \) contains data.

The \( \text{opt} \) output for an underlapping buffer is especially useful when the current buffering operation is one in a sequence of consecutive buffering operations. The \( \text{opt} \) output from each buffering operation specifies the number of samples that need to be skipped at the start of the next buffering operation to maintain the desired frame underlap from one buffer to the next. If fewer than \( p \) points were available to be skipped after filling the final frame of the current buffer, the remaining \( \text{opt} \) points need to be removed from the first frame of the next buffer.

In a sequence of buffering operations, the \( \text{opt} \) output from each operation should be used as the \( \text{opt} \) input to the subsequent buffering operation. This ensures that the desired frame overlap or underlap is maintained from buffer

---

**buffer**
to buffer, as well as from frame to frame within the same buffer. See “Continuous Buffering” below for an example of how this works in practice.

**Continuous Buffering**

In a continuous buffering operation, the vector input to the `buffer` function represents one frame in a sequence of frames that make up a discrete signal. These signal frames can originate in a frame-based data acquisition process, or within a frame-based algorithm like the FFT.

As an example, you might acquire data from an A/D card in frames of 64 samples. In the simplest case, you could rebuffer the data into frames of 16 samples; `buffer` with \( n = 16 \) creates a buffer of four frames from each 64-element input frame. The result is that the signal of frame size 64 has been converted to a signal of frame size 16; no samples were added or removed.

In the general case where the original signal frame size, \( L \), is not equally divisible by the new frame size, \( n \), the overflow from the last frame needs to be captured and recycled into the following buffer. You can do this by iteratively calling `buffer` on input \( x \) with the two-output-argument syntax:

\[
[y, z] = \text{buffer}([z;x], n) \quad \text{for column vector } x
\]
\[
[y, z] = \text{buffer}([z,x], n) \quad \text{for row vector } x
\]

This simply captures any buffer overflow in \( z \), and prepends the data to the subsequent input in the next call to `buffer`. Again, the input signal, \( x \), of frame size \( L \), has been converted to a signal of frame size \( n \) without any insertion or deletion of samples.

Note that continuous buffering cannot be done with the single-output syntax \( y = \text{buffer}(...) \), because the last frame of \( y \) in this case is zero padded, which adds new samples to the signal.

Continuous buffering in the presence of overlap and underlap is handled with the `opt` parameter, which is used as both an input and output to `buffer`. The following two examples demonstrate how the `opt` parameter should be used.

**Examples**

**Example 1: Continuous Overlapping Buffers**

First create a buffer containing 100 frames, each with 11 samples.

```plaintext
data = buffer(1:1100,11); % 11 samples per frame
```
Imagine that the frames (columns) in the matrix called `data` are the sequential outputs of a data acquisition board sampling a physical signal: `data(:,1)` is the first D/A output, containing the first 11 signal samples; `data(:,2)` is the second output, containing the next 11 signal samples, and so on.

You want to rebuffer this signal from the acquired frame size of 11 to a frame size of 4 with an overlap of 1. To do this, you will repeatedly call `buffer` to operate on each successive input frame, using the `opt` parameter to maintain consistency in the overlap from one buffer to the next.

Set the buffer parameters.

```matlab
n = 4;    % new frame size
p = 1;    % overlap
opt = -5; % value of y(1)
z = [];   % initialize the carry-over vector
```

Now repeatedly call `buffer`, each time passing in a new signal frame from `data`. Note that overflow samples (returned in `z`) are carried over and prepended to the input in the subsequent call to `buffer`.

```matlab
for i=1:size(data,2),    % Loop over each source frame (column)
x = data(:,i);        % A single frame of the D/A output
[y,z,opt] = buffer([z;x],n,p,opt);
disp(y); % Do something with the buffer of data
pause
end
```
Here’s what happens during the first four iterations.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Input frame ([z;x])'</th>
<th>opt (input)</th>
<th>opt (output)</th>
<th>Output buffer ((y))</th>
<th>Overflow ((z))</th>
</tr>
</thead>
</table>
| i=1       | [1:11]                   | -5          | 9            | \([-5 3 6
1 4 7
2 5 8
3 6 9]\) | [10 11]         |
| i=2       | [10 11 12:22]            | 9           | 21           | \([-5 12 15 18
10 13 16 19
11 14 17 20
12 15 18 21]\) | [22]            |
| i=3       | [22 23:33]               | 21          | 33           |                         | [ ]             |
| i=4       | [34:44]                  | 33          | 42           | \([-5 3 6 9
12 15 18 21
10 13 16 19
11 14 17 20]\) | [43 44]         |

Note that the size of the output matrix, \(y\), can vary by a single column from one iteration to the next. This is typical for buffering operations with overlap or underlap.

**Example 2: Continuous Underlapping Buffers**

Again create a buffer containing 100 frames, each with 11 samples.

```matlab
    data = buffer(1:1100,11); % 11 samples per frame
```

Again, imagine that \(data(:,1)\) is the first D/A output, containing the first 11 signal samples; \(data(:,2)\) is the second output, containing the next 11 signal samples, and so on.

You want to rebuffer this signal from the acquired frame size of 11 to a frame size of 4 with an underlap of 2. To do this, you will repeatedly call `buffer` to operate on each successive input frame, using the `opt` parameter to maintain consistency in the underlap from one buffer to the next.
Set the buffer parameters:

\[
\begin{align*}
n &= 4; & \text{\% new frame size} \\
p &= -2; & \text{\% underlap} \\
\text{opt} &= 1; & \text{\% skip the first input element, } x(1) \\
z &= []; & \text{\% initialize the carry-over vector}
\end{align*}
\]

Now repeatedly call `buffer`, each time passing in a new signal frame from `data`. Note that overflow samples (returned in `z`) are carried over and prepended to the input in the subsequent call to `buffer`.

```matlab
for i=1:size(data,2),
    x = data(:,i); % A single frame of the D/A output
    [y,z,opt] = buffer([z;x],n,p,opt);
    disp(y); % Do something with the buffer of data
    pause
end
```
Here's what happens during the first three iterations.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Input frame ([z; x]')</th>
<th>opt (input)</th>
<th>opt (output)</th>
<th>Output buffer ((y))</th>
<th>Overflow ((z))</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=1</td>
<td>([1:11])</td>
<td>1</td>
<td>2</td>
<td>([1 - \ldots \text{skip}])</td>
<td>([])</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>([2 8 3 9 4 10 5 11 6 - 7 - \text{skip}])</td>
<td></td>
</tr>
<tr>
<td>i=2</td>
<td>([12:22])</td>
<td>2</td>
<td>0</td>
<td>([\ldots \text{skip} 12 13 14 15 16 17 18 19 \text{skip}])</td>
<td>([20 21 22])</td>
</tr>
<tr>
<td>i=3</td>
<td>([20 21 22 23:33])</td>
<td>0</td>
<td>0</td>
<td>([\ldots \text{skip} 20 26 21 27 22 28 23 29 24 30 25 31 \text{skip}])</td>
<td>([32 33])</td>
</tr>
</tbody>
</table>

**Diagnostics**

Error messages are displayed when \(p \leq n\) or \(\text{length}(\text{opt}) \neq \text{length}(p)\) in an overlapping buffer case:

Frame overlap \(P\) must be less than the buffer size \(N\). Initial conditions must be specified as a length-\(P\) vector.

**See Also**

reshape Reshape array.
Purpose
Butterworth analog lowpass filter prototype.

Syntax
[z,p,k] = buttap(n)

Description
[z,p,k] = buttap(n) returns the zeros, poles, and gain of an order n
Butterworth analog lowpass filter prototype. It returns the poles in the length
n column vector p and the gain in scalar k. z is an empty matrix, because there
are no zeros. The transfer function is

\[ H(s) = \frac{Z(s)}{P(s)} = \frac{k}{(s-p(1))(s-p(2))\cdots(s-p(n))} \]

Butterworth filters are characterized by a magnitude response that is
maximally flat in the passband and monotonic overall. In the lowpass case, the
first 2n-1 derivatives of the squared magnitude response are zero at \( \omega = 0 \). The
squared magnitude response function is

\[ |H(\omega)|^2 = \frac{1}{1 + (\omega/\omega_0)^{2n}} \]

corresponding to a transfer function with poles equally spaced around a circle
in the left half plane. The magnitude response at the cutoff frequency \( \omega_0 \) is
always 1/sqrt(2), regardless of the filter order. buttap sets \( \omega_0 \) to 1 for a
normalized result.

Algorithm
\[
z = []; \\
p = \exp(\sqrt{-1}*(pi*(1:2:2*n-1)/(2*n)+pi/2)).'; \\
k = \text{real(prod(-p))};
\]

See Also
besselap Bessel analog lowpass filter prototype.
butter Butterworth analog and digital filter design.
cheb1ap Chebyshev type I analog lowpass filter prototype.
cheb2ap Chebyshev type II analog lowpass filter prototype.
ellipap Elliptic analog lowpass filter prototype.

References
Purpose
Butterworth analog and digital filter design.

Syntax
[b,a] = butter(n,Wn)
[b,a] = butter(n,Wn,'ftype')
[b,a] = butter(n,Wn,'s')
[b,a] = butter(n,Wn,'ftype','s')
[z,p,k] = butter(...)
[A,B,C,D] = butter(...)

Description
butter designs lowpass, bandpass, highpass, and bandstop digital and analog Butterworth filters. Butterworth filters are characterized by a magnitude response that is maximally flat in the passband and monotonic overall.

Butterworth filters sacrifice rolloff steepness for monotonicity in the pass- and stopbands. Unless the smoothness of the Butterworth filter is needed, an elliptic or Chebyshev filter can generally provide steeper rolloff characteristics with a lower filter order.

Digital Domain
[b,a] = butter(n,Wn) designs an order n lowpass digital Butterworth filter with cutoff frequency Wn. It returns the filter coefficients in length n + 1 row vectors b and a, with coefficients in descending powers of z:

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \ldots + a(n+1)z^{-n}}
\]

Cutoff frequency is that frequency where the magnitude response of the filter is \(\sqrt{1/2}\). For butter, the cutoff frequency Wn must be a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

If Wn is a two-element vector, Wn = [w1 w2], butter returns an order 2*n digital bandpass filter with passband \(w1 < \omega < w2\).

[b,a] = butter(n,Wn,'ftype') designs a highpass or bandstop filter, where ftype is
• high for a highpass digital filter with cutoff frequency \( W_n \)
• stop for an order \( 2n \) bandstop digital filter if \( W_n \) is a two-element vector,
  \( W_n = [w_1 \ w_2] \)

  The stopband is \( w_1 < \omega < w_2 \).

With different numbers of output arguments, \texttt{butter} directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

\[
[z,p,k] = \texttt{butter}(n,Wn) \quad \text{or} \quad [z,p,k] = \texttt{butter}(n,Wn,'ftype')
\]

\texttt{butter} returns the zeros and poles in length \( n \) column vectors \( z \) and \( p \), and the gain in the scalar \( k \).

To obtain state-space form, use four output arguments:

\[
[A,B,C,D] = \texttt{butter}(n,Wn) \quad \text{or} \quad [A,B,C,D] = \texttt{butter}(n,Wn,'ftype')
\]

where \( A, B, C, \) and \( D \) are

\[
x[n+1] = Ax[n] + Bu[n] \\
y[n] = Cx[n] + Du[n]
\]

and \( u \) is the input, \( x \) is the state vector, and \( y \) is the output.

**Analog Domain**

\[
[b,a] = \texttt{butter}(n,Wn,'s')
\]

designs an order \( n \) lowpass analog Butterworth filter with cutoff frequency \( W_n \). It returns the filter coefficients in the length \( n + 1 \) row vectors \( b \) and \( a \), in descending powers of \( s \):

\[
H(s) = \frac{b(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \ldots + b(n+1)}{s^n + a(2)s^{n-1} + \ldots + a(n+1)}
\]

\texttt{butter}’s cutoff frequency \( W_n \) must be greater than 0.

If \( W_n \) is a two-element vector with \( w_1 < w_2 \), \texttt{butter}(n,Wn,'s') returns an order \( 2n \) bandpass analog filter with passband \( w_1 < \omega < w_2 \).
[b,a] = butter(n,Wn, 'ftype', 's') designs a highpass or bandstop filter, where ftype is:

- **high** for a highpass analog filter with cutoff frequency Wn
- **stop** for an order 2*n bandstop analog filter if Wn is a two-element vector, Wn = [w1 w2]

The stopband is w1 < ω < w2.

With different numbers of output arguments, butter directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

[z,p,k] = butter(n,Wn, 's') or

[z,p,k] = butter(n,Wn, 'ftype', 's') returns the zeros and poles in length n or 2*n column vectors z and p and the gain in the scalar k.

To obtain state-space form, use four output arguments:

[A,B,C,D] = butter(n,Wn, 's') or

[A,B,C,D] = butter(n,Wn, 'ftype', 's') where A, B, C, and D are

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]

and u is the input, x is the state vector, and y is the output.

**Examples**

For data sampled at 1000 Hz, design a 9th-order highpass Butterworth filter with cutoff frequency of 300 Hz:

[b,a] = butter(9,300/500, 'high');
The filter’s frequency response is

\[ \text{freqz}(b,a,128,1000) \]

Design a 10th-order bandpass Butterworth filter with a passband from 100 to 200 Hz and plot its impulse response, or unit sample response:

\[
\begin{align*}
\text{n} & = 5; \ Wn = [100 \ 200]/500; \\
[b,a] & = \text{butter}(n,Wn); \\
[y,t] & = \text{impz}(b,a,101); \\
\text{stem}(t,y)
\end{align*}
\]
Limitations
For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm
butter uses a five-step algorithm:

1. It finds the lowpass analog prototype poles, zeros, and gain using the buttap function.
2. It converts the poles, zeros, and gain into state-space form.
3. It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
4. For digital filter design, butter uses bilinear to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at \( W_0 \) or \( w_1 \) and \( w_2 \).
5. It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also
- besself: Bessel analog filter design.
- buttap: Butterworth analog lowpass filter prototype.
- buttord: Butterworth filter order selection.
- cheby1: Chebyshev type I filter design (passband ripple).
- cheby2: Chebyshev type II filter design (stopband ripple).
- ellip: Elliptic (Cauer) filter design.
- maxflat: Generalized digital Butterworth filter design.
buttord

Purpose
Butterworth filter order selection.

Syntax
[n,Wn] = buttord(Wp,Ws,Rp,Rs)
[n,Wn] = buttord(Wp,Ws,Rp,Rs,'s')

Description
buttord selects the minimum order digital or analog Butterworth filter required to meet a set of filter design specifications:

- **Wp**: Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- **Ws**: Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- **Rp**: Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is 0 < w < Wp.
- **Rs**: Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is Ws < w < 1.

Digital Domain

[n,Wn] = buttord(Wp,Ws,Rp,Rs) returns the order n of the lowest order digital Butterworth filter that loses no more than Rp dB in the passband and has at least Rs dB of attenuation in the stopband. The passband runs from 0 to Wp and the stopband runs from Ws to 1, the Nyquist frequency. buttord also returns Wn, the Butterworth cutoff frequency that allows butter to achieve the given specifications (the “3 dB” frequency).

Use buttord for highpass, bandpass, and bandstop filters. For highpass filters, Wp is greater than Ws. For bandpass and bandstop filters, Wp and Ws are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, buttord returns Wn as a two-element row vector for input to butter.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.
Analog Domain

\[ [n, Wn] = \text{buttord}(Wp, Ws, Rp, Rs, 's') \]

finds the minimum order \( n \) and cutoff frequencies \( Wn \) for an analog filter. In this case the frequencies in \( Wp \) and \( Ws \) are in radians per second and may be greater than 1.

Use \text{buttord} \ for highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

**Examples**

For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz, and attenuation at least 15 dB from 150 Hz to the Nyquist frequency. Plot the filter’s frequency response:

\[
\begin{align*}
Wp &= 100/500; \quad Ws = 150/500; \\
[n, Wn] &= \text{buttord}(Wp, Ws, 3, 15)
\end{align*}
\]

\[
\begin{align*}
n &= 4 \\
Wn &= 0.2042
\end{align*}
\]

\[
[b, a] = \text{butter}(n, Wn);
\]

\[
\text{freqz}(b, a, 512, 1000); \quad \text{title('n=4 Butterworth Lowpass Filter')}
\]
Next design a bandpass filter with passband of 100 Hz to 200 Hz, less than 3 dB of attenuation at the passband corners, and attenuation down 30 dB by 50 Hz out on both sides of the passband:

\[
W_p = \frac{[100 \ 200]}{500}; \ W_s = \frac{[50 \ 250]}{500};
\]
\[
R_p = 3; \ R_s = 30;
\]
\[
[n,W_n] = \text{buttord}(W_p,W_s,R_p,R_s); \ [b,a] = \text{butter}(n,W_n);
\]
\[
\text{freqz}(b,a,128,1000)
\]

**Algorithm**

`buttord`'s order prediction formula is described in [1]. It operates in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s-domain before estimating the order and natural frequency, then converts back to the z-domain.

`buttord` initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.
### See Also

- **butter**: Butterworth analog and digital filter design.
- **cheb1ord**: Chebyshev type I filter order selection.
- **cheb2ord**: Chebyshev type II filter order selection.
- **ellipord**: Elliptic filter order selection.
- **kaiserord**: Estimate parameters for an FIR filter design with Kaiser window.

### References

Purpose

Complex cepstral analysis.

Syntax

\[
xhat = \text{cceps}(x)
\]

\[
[xhat, nd] = \text{cceps}(x)
\]

\[
[xhat, nd, xhat1] = \text{cceps}(x)
\]

\[
[\ldots] = \text{cceps}(x, n)
\]

Description

Cepstral analysis is a nonlinear signal processing technique that is applied most commonly in speech processing and homomorphic filtering [1].

\(xhat = \text{cceps}(x)\) returns the complex cepstrum of the (assumed real) sequence \(x\). The input is altered, by the application of a linear phase term, to have no phase discontinuity at \(\pm \pi\) radians. That is, it is circularly shifted (after zero padding) by some samples, if necessary, to have zero phase at \(\pi\) radians.

\([xhat, nd] = \text{cceps}(x)\) returns the number of samples \(nd\) of (circular) delay added to \(x\) prior to finding the complex cepstrum.

\([xhat, nd, xhat1] = \text{cceps}(x)\) returns a second complex cepstrum, computed using an alternate rooting algorithm, in \(xhat1\). The alternate method ([1] p.795) is useful for short sequences that can be rooted and do not have zeros on the unit circle. For these signals, \(xhat1\) can provide a verification of \(xhat\).

\([\ldots] = \text{cceps}(x, n)\) zero pads \(x\) to length \(n\) and returns the length \(n\) complex cepstrum of \(x\).

Algorithm

\text{cceps}, in its basic form, is an M-file implementation of algorithm 7.1 in [2]. A lengthy Fortran program reduces to three lines of MATLAB code:

\[
\begin{align*}
    h &= \text{fft}(x); \\
    \log h &= \log(\text{abs}(h)) + \text{sqrt}(-1) \times \text{rcunwrap}(\text{angle}(h)); \\
    y &= \text{real}(\text{ifft}(\log h));
\end{align*}
\]

\text{rcunwrap} is a special version of \text{unwrap} that subtracts a straight line from the phase.
See Also

icceps  Inverse complex cepstrum.
hilbert  Hilbert transform.
rceps  Real cepstrum and minimum phase reconstruction.
unwrap  Unwrap phase angles.

References


Purpose
Chebyshev type I analog lowpass filter prototype.

Syntax
\[ [z,p,k] = \text{cheb1ap}(n,Rp) \]

Description
\[ [z,p,k] = \text{cheb1ap}(n,Rp) \]
returns the zeros, poles, and gain of an order \( n \) Chebyshev type I analog lowpass filter prototype with \( Rp \) dB of ripple in the passband. It returns the poles in the length \( n \) column vector \( p \) and the gain in scalar \( k \). \( z \) is an empty matrix, because there are no zeros. The transfer function is

\[
H(s) = \frac{z(s)}{p(s)} = \frac{k}{(s - p(1))(s - p(2)) \cdots (s - p(n))}
\]

Chebyshev type I filters are equiripple in the passband and monotonic in the stopband. The poles are evenly spaced about an ellipse in the left half plane. The Chebyshev type I cutoff frequency \( \omega_0 \) is set to 1.0 for a normalized result. This is the frequency at which the passband ends and the filter has magnitude response of \( 10^{Rp/20} \).

See Also
- \text{besselap}
- \text{buttap}
- \text{cheb2ap}
- \text{cheby1}
- \text{ellipap}

References
Purpose

Chebyshev type I filter order selection.

Syntax

\[ [n,Wn] = \text{cheb1ord}(Wp,Ws,Rp,Rs) \]
\[ [n,Wn] = \text{cheb1ord}(Wp,Ws,Rp,Rs,'s') \]

Description

\text{cheb1ord} selects the minimum order digital or analog Chebyshev type I filter required to meet a set of filter design specifications:

- passband corner frequency. \( Wp \), the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- stopband corner frequency. \( Ws \) is in the same units as \( Wp \); it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is \( 0 < w < Wp \).
- stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is \( Ws < w < 1 \).

Digital Domain

\[ [n,Wn] = \text{cheb1ord}(Wp,Ws,Rp,Rs) \] returns the order \( n \) of the lowest order Chebyshev filter that loses no more than \( Rp \) dB in the passband and has at least \( Rs \) dB of attenuation in the stopband. The passband runs from 0 to \( Wp \) and the stopband runs from \( Ws \) to 1, the Nyquist frequency. \text{cheb1ord} also returns \( Wn \), the Chebyshev type I cutoff frequency that allows \text{cheby1} to achieve the given specifications.

Use \text{cheb1ord} for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, \( Wp > Ws \). For bandpass and bandstop filters, \( Wp \) and \( Ws \) are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, \text{cheb1ord} returns \( Wn \) as a two-element row vector for input to \text{cheby1}.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.
cheb1ord

Analog Domain

\[ [n, W_n] = \text{cheb1ord}(W_p, W_s, R_p, R_s, 's') \]
finds the minimum order \( n \) and cutoff frequencies \( W_n \) for an analog filter. In this case the frequencies in \( W_p \) and \( W_s \) are in radians per second and may be greater than 1.

Use `cheb1ord` for lowpass, highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

Examples

For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz and attenuation at least 15 dB from 150 Hz to the Nyquist frequency:

\[
W_p = 100/500; \quad W_s = 150/500;
\]
\[
R_p = 3; \quad R_s = 15;
\]
\[ [n, W_n] = \text{cheb1ord}(W_p, W_s, R_p, R_s) \]

\[
n = 3
\]
\[
W_n = 0.2000
\]

\[ [b, a] = \text{cheby1}(n, R_p, W_n); \]
\[ \text{freqz}(b, a, 512, 1000); \quad \text{title('n=3 Chebyshev Type I Lowpass Filter')} \]
Next design a bandpass filter with a passband of 100 Hz to 200 Hz, less than 3 dB of attenuation throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

\[
W_p = \begin{bmatrix} 100 & 200 \end{bmatrix}/500; \quad W_s = \begin{bmatrix} 50 & 250 \end{bmatrix}/500;
\]

\[R_p = 3; \quad R_s = 30;\]

\[[n, W_n] = \text{cheb1ord}(W_p, W_s, R_p, R_s)\]

\[
n = 4
\]

\[
W_n = \begin{bmatrix} 0.2000 & 0.4000 \end{bmatrix}
\]

\[[b, a] = \text{cheby1}(n, R_p, W_n);\]

\text{freqz}(b, a, 512, 1000);

\text{title('n=4 Chebyshev Type I Bandpass Filter')}

**Algorithm**

\text{cheb1ord} uses the Chebyshev lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s-domain before the order and natural frequency estimation process, then converts them back to the z-domain.
cheb1ord initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- or highpass filters) or to -1 and 1 rad/sec (for bandpass or bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.

See Also

- `buttord`: Butterworth filter order selection.
- `cheby1`: Chebyshev type I filter design (passband ripple).
- `cheb2ord`: Chebyshev type II filter order selection.
- `ellipord`: Elliptic filter order selection.
- `kaiserord`: Estimate parameters for an FIR filter design with Kaiser window.

References

Purpose

Chebyshev type II analog lowpass filter prototype.

Syntax

\[ [z, p, k] = \text{cheb2ap}(n, Rs) \]

Description

\[ [z, p, k] = \text{cheb2ap}(n, Rs) \] finds the zeros, poles, and gain of an order \( n \)
Chebyshev type II analog lowpass filter prototype with stopband ripple \( Rs \) dB down from the passband peak value. \text{cheb2ap} returns the zeros and poles in length \( n \) column vectors \( z \) and \( p \) and the gain in scalar \( k \). If \( n \) is odd, \( z \) is length \( n-1 \). The transfer function is

\[
H(s) = \frac{z(s)}{p(s)} = k\frac{(s - z(1))(s - z(2))\cdots(s - z(n))}{(s - p(1))(s - p(2))\cdots(s - p(n))}
\]

Chebyshev type II filters are monotonic in the passband and equiripple in the stopband. The pole locations are the inverse of the pole locations of \text{cheb1ap}, whose poles are evenly spaced about an ellipse in the left half plane. The Chebyshev type II cutoff frequency \( \omega_0 \) is set to 1 for a normalized result. This is the frequency at which the stopband begins and the filter has magnitude response of \( 10^{-Rs/20} \).

Algorithm

Chebyshev type II filters are sometimes called inverse Chebyshev filters because of their relationship to Chebyshev type I filters. The \text{cheb2ap} function is a modification of the Chebyshev type I prototype algorithm:

1. \text{cheb2ap} replaces the frequency variable \( \omega \) with \( 1/\omega \), turning the lowpass filter into a highpass filter while preserving the performance at \( \omega = 1 \).
2. \text{cheb2ap} subtracts the filter transfer function from unity.

See Also

- \text{besselap} Bessel analog lowpass filter prototype.
- \text{buttap} Butterworth analog lowpass filter prototype.
- \text{cheb1ap} Chebyshev type I analog lowpass filter prototype.
- \text{cheby2} Chebyshev type II filter design (stopband ripple).
- \text{ellipap} Elliptic analog lowpass filter prototype.

References

Purpose
Chebyshev type II filter order selection.

Syntax

\[ [n, Wn] = \text{cheb2ord}(Wp, Ws, Rp, Rs) \]

\[ [n, Wn] = \text{cheb2ord}(Wp, Ws, Rp, Rs, 's') \]

Description
cheb2ord selects the minimum order digital or analog Chebyshev type II filter required to meet a set of filter design specifications:

- **Wp** (Passband corner frequency). Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- **Ws** (Stopband corner frequency). Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
- **Rp** (Passband ripple, in decibels). This value is the maximum permissible passband loss in decibels. The passband is \(0 < w < Wp\).
- **Rs** (Stopband attenuation, in decibels). This value is the number of decibels the stopband is down from the passband. The stopband is \(Ws < w < 1\).

**Digital Domain**

\[ [n, Wn] = \text{cheb2ord}(Wp, Ws, Rp, Rs) \] returns the order \(n\) of the lowest order Chebyshev filter that loses no more than \(Rp\) dB in the passband and has at least \(Rs\) dB of attenuation in the stopband. The passband runs from 0 to \(Wp\) and the stopband runs from \(Ws\) to 1, the Nyquist frequency. cheb2ord also returns \(Wn\), the Chebyshev type II cutoff frequency that allows cheby2 to achieve the given specifications.

Use cheb2ord for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, \(Wp\) is greater than \(Ws\). For bandpass and bandstop filters, \(Wp\) and \(Ws\) are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, cheb2ord returns \(Wn\) as a two-element row vector for input to cheby2.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.
Analog Domain

\[ [n, Wn] = \text{cheb2ord}(Wp, Ws, Rp, Rs, 's') \]

finds the minimum order \( n \) and cutoff frequencies \( Wn \) for an analog filter. In this case the frequencies in \( Wp \) and \( Ws \) are in radians per second and may be greater than 1.

Use \text{cheb2ord} for lowpass, highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

Examples

For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz, and attenuation at least 15 dB from 150 Hz to the Nyquist frequency:

\[
\begin{align*}
Wp &= 100/500; \\
Ws &= 150/500; \\
Rp &= 3; \\
Rs &= 15;
\end{align*}
\]

\[ [n, Wn] = \text{cheb2ord}(Wp, Ws, Rp, Rs) \]

\[ n = 3 \]

\[ Wn = 0.2609 \]

\[ [b, a] = \text{cheby2}(n, Rs, Wn); \]
\[ \text{freqz}(b, a, 512, 1000); \]
\[ \text{title}('n=3 \text{ Chebyshev Type II Lowpass Filter}') \]
Next design a bandpass filter with a passband of 100 Hz to 200 Hz, less than 3 dB of attenuation throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

\[
W_p = \frac{[100 \ 200]}{500}; \quad W_s = \frac{[50 \ 250]}{500};
\]

\[
R_p = 3; \quad R_s = 30;
\]

\[
[n,W_n] = \text{cheb2ord}(W_p,W_s,R_p,R_s)
\]

\[
n = 4
\]

\[
W_n =
\]

\[
0.1633 \quad 0.4665
\]

\[
[b,a] = \text{cheby2}(n,R_s,W_n);
\]

\[
\text{freqz}(b,a,512,1000)
\]

title('n=4 Chebyshev Type II Bandpass Filter')

Algorithm

`cheb2ord` uses the Chebyshev lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s-domain before the order and natural frequency estimation process, then converts them back to the z-domain.
cheb2ord initially develops a lowpass filter prototype by transforming the stopband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the passband specification.

**See Also**

- **butterord**: Butterworth filter order selection.
- **cheb1ord**: Chebyshev type I filter order selection.
- **cheby2**: Chebyshev type II filter design (stopband ripple).
- **ellipord**: Elliptic filter order selection.
- **kaiserord**: Estimate parameters for an FIR filter design with Kaiser window.

**References**

Purpose
Chebyshev window.

Syntax
w = chebwin(n,r)

Description
w = chebwin(n,r) returns the column vector w, containing the length n Chebyshev window whose Fourier transform magnitude sidelobe ripple is r dB below the mainlobe magnitude.

See Also
bartlett Bartlett window.
blackman Blackman window.
boxcar Rectangular window.
hamming Hamming window.
hanning Hanning window.
kaiser Kaiser window.
triang Triangular window.

References
Purpose
Chebyshev type I filter design (passband ripple).

Syntax
[b,a] = cheby1(n,Rp,Wn)
[b,a] = cheby1(n,Rp,Wn,'ftype')
[b,a] = cheby1(n,Rp,Wn,'s')
[b,a] = cheby1(n,Rp,Wn,'ftype','s')
[z,p,k] = cheby1(...)
[A,B,C,D] = cheby1(...)

Description
cheby1 designs lowpass, bandpass, highpass, and bandstop digital and analog Chebyshev type I filters. Chebyshev type I filters are equiripple in the passband and monotonic in the stopband. Type I filters roll off faster than type II filters, but at the expense of greater deviation from unity in the passband.

Digital Domain
[b,a] = cheby1(n,Rp,Wn) designs an order n lowpass digital Chebyshev filter with cutoff frequency Wn and Rp dB of ripple in the passband. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of z:

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \ldots + a(n+1)z^{-n}}
\]

Cutoff frequency is the frequency at which the magnitude response of the filter is equal to –Rp dB. For cheby1, the cutoff frequency Wn is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). Smaller values of passband ripple Rp lead to wider transition widths (shallower rolloff characteristics).

If Wn is a two-element vector, Wn = [w1 w2], cheby1 returns an order 2*n bandpass filter with passband w1 < ω < w2.

[b,a] = cheby1(n,Rp,Wn,'ftype') designs a highpass or bandstop filter, where ftype is
cheby1

- high for a highpass digital filter with cutoff frequency \( W_n \)
- stop for an order \( 2n \) bandstop digital filter if \( W_n \) is a two-element vector, \( W_n = [w_1 \ w_2] \)

The stopband is \( w_1 < \omega < w_2 \).

With different numbers of output arguments, cheby1 directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

\[
[z, p, k] = \text{cheby1}(n, R_p, W_n) \quad \text{or} \\
[z, p, k] = \text{cheby1}(n, R_p, W_n, 'ftype')
\]

returns the zeros and poles in length \( n \) column vectors \( z \) and \( p \) and the gain in the scalar \( k \).

To obtain state-space form, use four output arguments:

\[
[A, B, C, D] = \text{cheby1}(n, R_p, W_n) \quad \text{or} \\
[A, B, C, D] = \text{cheby1}(n, R_p, W_n, 'ftype') 
\]

where \( A, B, C, \) and \( D \) are

\[
x[n + 1] = Ax[n] + Bu[n] \\
y[n] = Cx[n] + Du[n] 
\]

and \( u \) is the input, \( x \) is the state vector, and \( y \) is the output.

Analog Domain

\[
[b, a] = \text{cheby1}(n, R_p, W_n, 's') \quad \text{designs an order } n \text{ lowpass analog Chebyshev type I filter with cutoff frequency } W_n. \text{ It returns the filter coefficients in length } n + 1 \text{ row vectors } b \text{ and } a, \text{ in descending powers of } s:
\]

\[
H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \ldots + b(n + 1)}{s^n + a(2)s^{n-1} + \ldots + a(n + 1)}
\]

Cutoff frequency is the frequency at which the magnitude response of the filter is \( -R_p \) dB. For cheby1, the cutoff frequency \( W_n \) must be greater than 0.

If \( W_n \) is a two-element vector, \( W_n = [w_1 \ w_2] \), with \( w_1 < w_2 \), then cheby1 \((n, R_p, W_n, 's')\) returns an order \( 2n \) bandpass analog filter with passband \( w_1 < \omega < w_2 \).
[b,a] = cheby1(n,Rp,Wn, 'ftype', 's') designs a highpass or bandstop filter, where *ftype* is

- **high** for a highpass analog filter with cutoff frequency *Wn*
- **stop** for an order 2*n bandstop analog filter if *Wn* is a two-element vector,  
  \[ Wn = [w1 \ w2] \]

The stopband is \( \omega_1 < \omega < \omega_2 \).

You can supply different numbers of output arguments for *cheby1* to directly obtain other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

\[ [z,p,k] = \text{cheby1}(n,Rp,Wn, 's') \]

\[ [z,p,k] = \text{cheby1}(n,Rp,Wn, 'ftype', 's') \] returns the zeros and poles in length *n* or 2*n column vectors \( z \) and \( p \) and the gain in the scalar \( k \).

To obtain state-space form, use four output arguments:

\[ [A,B,C,D] = \text{cheby1}(n,Rp,Wn, 's') \] or

\[ [A,B,C,D] = \text{cheby1}(n,Rp,Wn, 'ftype', 's') \] where \( A, B, C, \) and \( D \) are defined as

\[
\dot{x} = Ax + Bu \\
y =Cx + Du
\]

and \( u \) is the input, \( x \) is the state vector, and \( y \) is the output.

**Examples**

For data sampled at 1000 Hz, design a 9th-order lowpass Chebyshev type I filter with 0.5 dB of ripple in the passband and a cutoff frequency of 300 Hz:

\[ [b,a] = \text{cheby1}(9,0.5,300/500); \]
The frequency response of the filter is

\[
\text{freqz}(b, a, 512, 1000)
\]

Design a 10th-order bandpass Chebyshev type I filter with a passband from 100 to 200 Hz and plot its impulse response:

\[
\begin{align*}
\text{n} &= 10; \quad \text{Rp} = 0.5; \\
\text{Wn} &= \left[\frac{100 \ 200}{500}\right]; \\
[b, a] &= \text{cheby1}(n, \text{Rp}, \text{Wn}); \\
[y, t] &= \text{impz}(b, a, 101); \quad \text{stem}(t, y)
\end{align*}
\]
Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

cheby1 uses a five-step algorithm:

1. It finds the lowpass analog prototype poles, zeros, and gain using the cheb1ap function.
2. It converts the poles, zeros, and gain into state-space form.
3. It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
4. For digital filter design, cheby1 uses bilinear to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at \( W_n \) or \( w_1 \) and \( w_2 \).
5. It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

- besself: Bessel analog filter design.
- butter: Butterworth analog and digital filter design.
- cheb1ap: Chebyshev type I analog lowpass filter prototype.
- cheb1ord: Chebyshev type I filter order selection.
- cheby2: Chebyshev type II filter design (stopband ripple).
- ellip: Elliptic (Cauer) filter design.
cheby2

Purpose
Chebyshev type II filter design (stopband ripple).

Syntax
[b,a] = cheby2(n,Rs,Wn)
[b,a] = cheby2(n,Rs,Wn,'ftype')
[b,a] = cheby2(n,Rs,Wn,'s')
[b,a] = cheby2(n,Rs,Wn,'ftype', 's')
[z,p,k] = cheby2(...)  
[A,B,C,D] = cheby2(...)

Description
cheby2 designs lowpass, highpass, bandpass, and bandstop digital and analog Chebyshev type II filters. Chebyshev type II filters are monotonic in the passband and equiripple in the stopband. Type II filters do not roll off as fast as type I filters, but are free of passband ripple.

Digital Domain
[b,a] = cheby2(n,Rs,Wn) designs an order n lowpass digital Chebyshev type II filter with cutoff frequency \( W_n \) and stopband ripple \( R_s \) dB down from the peak passband value. It returns the filter coefficients in the length \( n + 1 \) row vectors \( b \) and \( a \), with coefficients in descending powers of \( z \):

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \ldots + a(n+1)z^{-n}}
\]

Cutoff frequency is the beginning of the stopband, where the magnitude response of the filter is equal to \(-R_s\) dB. For cheby2, the cutoff frequency \( W_n \) is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). Larger values of stopband attenuation \( R_s \) lead to wider transition widths (shallower rolloff characteristics).

If \( W_n \) is a two-element vector, \( W_n = [w_1 \ w_2] \), cheby2 returns an order \( 2n \) bandpass filter with passband \( w_1 < \omega < w_2 \).

\[ [b,a] = \text{cheby2}(n,Rs,Wn,'ftype') \] designs a highpass or bandstop filter, where \( ftype \) is
• **high** for a highpass digital filter with cutoff frequency $W_n$
• **stop** for an order $2n$ bandstop digital filter if $W_n$ is a two-element vector, $W_n = [w1 \ w2]$.

  The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `cheby2` directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

$$[z, p, k] = \text{cheby2}(n, Rs, Wn) \text{ or } [z, p, k] = \text{cheby2}(n, Rs, Wn, 'ftype')$$

returns the zeros and poles in length $n$ column vectors $z$ and $p$ and the gain in the scalar $k$.

To obtain state-space form, use four output arguments:

$$[A, B, C, D] = \text{cheby2}(n, Rs, Wn) \text{ or } [A, B, C, D] = \text{cheby2}(n, Rs, Wn, 'ftype')$$

where $A$, $B$, $C$, and $D$ are

$$\begin{align*}
x[n+1] &= Ax[n] + Bu[n] \\
y[n] &= Cx[n] + Du[n]
\end{align*}$$

and $u$ is the input, $x$ is the state vector, and $y$ is the output.

**Analog Domain**

$$[b, a] = \text{cheby2}(n, Rs, Wn, 's')$$

designs an order $n$ lowpass analog Chebyshev type II filter with cutoff frequency $W_n$. It returns the filter coefficients in the length $n + 1$ row vectors $b$ and $a$, with coefficients in descending powers of $s$:

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \ldots + b(n + 1)}{s^n + a(2)s^{n-1} + \ldots + a(n + 1)}$$

Cutoff frequency is the frequency at which the magnitude response of the filter is equal to $-Rs$ dB. For `cheby2`, the cutoff frequency $W_n$ must be greater than 0.

If $W_n$ is a two-element vector, $W_n = [w1 \ w2]$, with $w1 < w2$, then

$$\text{cheby2}(n, Rs, Wn, 's')$$

returns an order $2n$ bandpass analog filter with passband $w1 < \omega < w2$. 

cheby2

[b,a] = cheby2(n,Rs,Wn,'ftype','s') designs a highpass or bandstop filter, where ftype is

• high for a highpass analog filter with cutoff frequency Wn
• stop for an order 2*n bandstop analog filter if Wn is a two-element vector, Wn = [w1 w2]

The stopband is w1 < ω < w2.

With different numbers of output arguments, cheby2 directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

[z,p,k] = cheby2(n,Rs,Wn,'s') or

[z,p,k] = cheby2(n,Rs,Wn,'ftype','s') returns the zeros and poles in length n or 2*n column vectors z and p and the gain in the scalar k.

To obtain state-space form, use four output arguments:

[A,B,C,D] = cheby2(n,Rs,Wn,'s') or

[A,B,C,D] = cheby2(n,Rs,Wn,'ftype','s') where A, B, C, and D are

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]

and u is the input, x is the state vector, and y is the output.

Examples

For data sampled at 1000 Hz, design a ninth-order lowpass Chebyshev type II filter with stopband attenuation 20 dB down from the passband and a cutoff frequency of 300 Hz:

[b,a] = cheby2(9,20,300/500);
The frequency response of the filter is

\[
\text{freqz}(b, a, 512, 1000)
\]

Design a fifth-order bandpass Chebyshev type II filter with passband from 100 to 200 Hz and plot the impulse response of the filter:

\[
n = 5; \quad r = 20;
\]

\[
Wn = \frac{[100 \ 200]}{500};
\]

\[
[b, a] = \text{cheby2}(n, r, Wn);
\]

\[
[y, t] = \text{impz}(b, a, 101); \quad \text{stem}(t, y)
\]
Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

cheby2 uses a five-step algorithm:

1. It finds the lowpass analog prototype poles, zeros, and gain using the cheb2ap function.
2. It converts poles, zeros, and gain into state-space form.
3. It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
4. For digital filter design, cheby2 uses bilinear to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at $w_n$ or $w_1$ and $w_2$.
5. It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

besself  Bessel analog filter design.
butter  Butterworth analog and digital filter design.
cheb2ap  Chebyshev type II analog lowpass filter prototype.
cheb2ord  Chebyshev type II filter order selection.
cheby1  Chebyshev type I filter design (passband ripple).
ellip  Elliptic (Cauer) filter design.
Purpose
Swept-frequency cosine generator.

Syntax
- \( y = \text{chirp}(t,f_0,t_1,f_1) \)
- \( y = \text{chirp}(t,f_0,t_1,f_1,'method') \)
- \( y = \text{chirp}(t,f_0,t_1,f_1,'method',\phi) \)

Description
\( y = \text{chirp}(t,f_0,t_1,f_1) \) generates samples of a linear swept-frequency cosine signal at the time instances defined in array \( t \), where \( f_0 \) is the instantaneous frequency at time 0, and \( f_1 \) is the instantaneous frequency at time \( t_1 \). \( f_0 \) and \( f_1 \) are both in Hertz. If unspecified, \( f_0 \) is 0, \( t_1 \) is 1, and \( f_1 \) is 100.

\( y = \text{chirp}(t,f_0,t_1,f_1,'method') \) specifies alternative sweep method options, where \( method \) can be

- **linear**, which specifies an instantaneous frequency sweep \( f_i(t) \) given by
  \[
  f_i(t) = f_0 + \beta t
  \]
  - where
  \[
  \beta = (f_1 - f_0)/t_1
  \]
  - \( \beta \) ensures that the desired frequency breakpoint \( f_1 \) at time \( t_1 \) is maintained.

- **quadratic**, which specifies an instantaneous frequency sweep \( f_i(t) \) given by
  \[
  f_i(t) = f_0 + \beta t^2
  \]
  - where
  \[
  \beta = (f_1 - f_0)/t_1
  \]

- **logarithmic** specifies an instantaneous frequency sweep \( f_i(t) \) given by
  \[
  f_i(t) = f_0 + 10^{\beta t}
  \]
  where
  \[
  \beta = [\log_{10}(f_1 - f_0)]/t_1
  \]
  - For a log-sweep, \( f_1 \) must be greater than \( f_0 \).
y = chirp(t,f0,t1,f1,'method',phi) allows an initial phase phi to be specified in degrees. If unspecified, phi is 0. Default values are substituted for empty or omitted trailing input arguments.

Examples

Compute the spectrogram of a chirp with linear instantaneous frequency deviation:

t = 0:0.001:2; % 2 secs @ 1kHz sample rate
y = chirp(t,0,1,150); % Start @ DC, cross 150Hz at t=1 sec
specgram(y,256,1e3,256,250) % Display the spectrogram
Compute the spectrogram of a chirp with quadratic instantaneous frequency deviation:

```matlab
t = -2:0.001:2;           % ±2 secs @ 1kHz sample rate
y = chirp(t,100,1,200,'quadratic'); % Start @ 100Hz, cross 200Hz at t=1 sec
specgram(y,128,1e3,128,120) % Display the spectrogram
```

![Spectrogram of a chirp with quadratic frequency deviation](image)
### See Also

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>cos</td>
<td>Cosine of vector/matrix elements (see the online MATLAB Function Reference).</td>
</tr>
<tr>
<td>diric</td>
<td>Dirichlet or periodic sinc function.</td>
</tr>
<tr>
<td>gauspuls</td>
<td>Gaussian-modulated sinusoidal pulse generator.</td>
</tr>
<tr>
<td>pulstran</td>
<td>Pulse train generator.</td>
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<tr>
<td>rectpuls</td>
<td>Sampled aperiodic rectangle generator.</td>
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<tr>
<td>sawtooth</td>
<td>Sawtooth or triangle wave generator.</td>
</tr>
<tr>
<td>sin</td>
<td>Sine of vector/matrix elements (see the online MATLAB Function Reference).</td>
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<tr>
<td>sinc</td>
<td>Sinc function.</td>
</tr>
<tr>
<td>square</td>
<td>Square wave generator.</td>
</tr>
<tr>
<td>tripuls</td>
<td>Sampled aperiodic triangle generator.</td>
</tr>
</tbody>
</table>
**Purpose**

Estimate magnitude squared coherence function between two signals.

**Syntax**

- `Cxy = cohere(x,y)`
- `Cxy = cohere(x,y,nfft)`
- `[Cxy,f] = cohere(x,y,nfft,Fs)`
- `Cxy = cohere(x,y,nfft,Fs,window)`
- `Cxy = cohere(x,y,nfft,Fs,window,overlap)`
- `Cxy = cohere(x,y,...,'dflag')`
- `cohere(x,y)`

**Description**

`Cxy = cohere(x,y)` finds the magnitude squared coherence between length `n` signal vectors `x` and `y`. The coherence is a function of the power spectra of `x` and `y` and the cross spectrum of `x` and `y`:

\[
C_{xy}(f) = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)}
\]

`x` and `y` must be the same length. `Cxy = cohere(x,y)` uses the following default values:

- `nfft = min(256,length(x))`
- `Fs = 2`
- `window = hanning(nfft)`
- `overlap = 0`

`nfft` specifies the FFT length that `cohere` uses. This value determines the frequencies at which the coherence is estimated. `Fs` is a scalar that specifies the sampling frequency. `window` specifies a windowing function and the number of samples `cohere` uses in its sectioning of the `x` and `y` vectors. `overlap` is the number of samples by which the sections overlap. Any arguments that you omit from the end of the parameter list use the default values shown above.

If `x` is real, `cohere` estimates the coherence function at positive frequencies only; in this case, the output `Cxy` is a column vector of length `nfft/2 + 1` for `nfft` even and `(nfft + 1)/2` for `n` odd. If `x` or `y` is complex, `cohere` estimates the coherence function at both positive and negative frequencies, and `Cxy` has length `nfft`. 
cohere

Cxy = cohere(x,y,nfft) uses the FFT length nfft in estimating the power spectrum for x. Specify nfft as a power of 2 for fastest execution.

[Cxy,f] = cohere(x,y,nfft,Fs) returns a vector f of frequencies at which the function evaluates the coherence. Fs is the sampling frequency. f is the same size as Cxy, so plot(f,Cxy) plots the coherence function versus properly scaled frequency. Fs has no effect on the output Cxy; it is a frequency scaling multiplier.

Cxy = cohere(x,y,nfft,Fs,window) specifies a windowing function and the number of samples per section of the vectors x and y. If you supply a scalar for window, cohere uses a Hanning window of that length. The length of the window must be less than or equal to nfft; cohere zero pads the sections if the window length exceeds nfft.

Cxy = cohere(x,y,nfft,Fs,window,noverlap) overlaps the sections of x by noverlap samples.

You can use the empty matrix [] to specify the default value for any input argument except x or y. For example,

Cxy = cohere(x,y,[],[],kaiser(128,5));

uses 256 as the value for nfft and 2 as the value for Fs.

Cxy = cohere(x,y,...,'dflag') specifies a detrend option, where dflag is

- linear, to remove the best straight-line fit from the prewindowed sections of x and y
- mean, to remove the mean from the prewindowed sections of x and y
- none, for no detrending (default)

The dflag parameter must appear last in the list of input arguments. cohere recognizes a dflag string no matter how many intermediate arguments are omitted.

cohere with no output arguments plots the coherence estimate versus frequency in the current figure window.
Example

Compute and plot the coherence estimate between two colored noise sequences $x$ and $y$:

\[
\begin{align*}
    h &= \text{fir1}(30,0.2,\text{boxcar}(31)); \\
    h1 &= \text{ones}(1,10)/\sqrt{10}; \\
    r &= \text{randn}(16384,1); \\
    x &= \text{filter}(h1,1,r); \\
    y &= \text{filter}(h,1,x); \\
    \text{cohere}(x,y,1024,[],[],512)
\end{align*}
\]

Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector (either row or column) input.
- Requires inputs $X$ and $Y$ to have the same length.

Algorithm

\text{cohere} estimates the magnitude squared coherence function [1] using Welch's method of power spectrum estimation (see references [2] and [3]), as follows:
cohere

1 It divides the signals x and y into overlapping sections, detrends each section, and multiplies each section by window.
2 It calculates the length nfft fast Fourier transform of each section.
3 It averages the squares of the spectra of the x sections to form Pxx, averages the squares of the spectra of the y sections to form Pyy, and averages the products of the spectra of the x and y sections to form Pxy. It calculates Cxy by

\[
C_{xy} = \frac{|P_{xy}|^2}{P_{xx} \cdot P_{yy}}
\]

See Also
csd Estimate the cross spectral density (CSD) of two signals.
pwelch Estimate the power spectral density (PSD) of a signal using Welch’s method.
tfe Transfer function estimate from input and output.

References
Purpose
Convolution and polynomial multiplication.

Syntax
\[ c = \text{conv}(a,b) \]

Description
\( \text{conv}(a,b) \) convolves vectors \( a \) and \( b \). The convolution sum is

\[
c(n + 1) = \sum_{k=0}^{N-1} a(k+1)b(n-k)
\]

where \( N \) is the maximum sequence length. The series is indexed from \( n + 1 \) and \( k + 1 \) instead of the usual \( n \) and \( k \) because MATLAB vectors run from 1 to \( n \) instead of from 0 to \( n-1 \).

The \text{conv} function is part of the standard MATLAB language.

Example
The convolution of \( a = [1 \ 2 \ 3] \) and \( b = [4 \ 5 \ 6] \) is

\[ c = \text{conv}(a,b) \]

\[ c = [4 \ 13 \ 28 \ 27 \ 18] \]

Algorithm
The \text{conv} function is an M-file that uses the \text{filter} primitive. \text{conv} computes the convolution operation as FIR filtering with an appropriate number of zeros appended to the input.

See Also
- \text{conv2} Two-dimensional convolution.
- \text{convmtx} Convolution matrix.
- \text{convn} N-dimensional convolution (see the online MATLAB Function Reference).
- \text{deconv} Deconvolution and polynomial division.
- \text{filter} Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- \text{residuez} z-transform partial fraction expansion.
- \text{xcorr} Cross-correlation function estimate.
conv2

**Purpose**
Two-dimensional convolution.

**Syntax**

\[
C = \text{conv2}(A,B) \\
C = \text{conv2}(A,B,'\text{shape}')
\]

**Description**

\[
C = \text{conv2}(A,B)
\]
computes the two-dimensional convolution of matrices A and B. If one of these matrices describes a two-dimensional FIR filter, the other matrix is filtered in two dimensions.

Each dimension of the output matrix C is equal in size to the sum of the corresponding dimensions of the input matrices minus 1. For [ma,na] = size(A) and [mb,nb] = size(B), then

\[
\text{size}(C) = [ma+mb–1,na+nb–1]
\]

\[
C = \text{conv2}(A,B,'\text{shape}')
\]
returns a subsection of the two-dimensional convolution with size specified by shape, where:

- full returns the full two-dimensional convolution (default)
- same returns the central part of the convolution that is the same size as A
- valid returns only those parts of the convolution that are computed without the zero-padded edges. Using this option, size(C) = [ma–mb+1,na–nb+1] when size(A) > size(B)

**Examples**

In image processing, the Sobel edge-finding operation is a two-dimensional convolution of an input array with the special matrix

\[
s = [1 2 1; 0 0 0; –1 –2 –1];
\]

Given any image, the following line extracts the horizontal edges:

\[
h = \text{conv2}(I,s);
\]

The lines below extract first the vertical edges, then both horizontal and vertical edges combined:

\[
v = \text{conv2}(I,s');
\]

\[
v2 = (\text{sqrt}(h.^2 + v.^2))
\]
### See Also

<table>
<thead>
<tr>
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<tbody>
<tr>
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<td>Convolution and polynomial multiplication.</td>
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<td><code>convn</code></td>
<td>N-dimensional convolution (see the online MATLAB Function Reference).</td>
</tr>
<tr>
<td><code>deconv</code></td>
<td>Deconvolution and polynomial division.</td>
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<tr>
<td><code>filter2</code></td>
<td>Two-dimensional digital filtering.</td>
</tr>
<tr>
<td><code>xcorr</code></td>
<td>Cross-correlation function estimate.</td>
</tr>
<tr>
<td><code>xcorr2</code></td>
<td>Two-dimensional cross-correlation.</td>
</tr>
</tbody>
</table>
**convmtx**

**Purpose**
Convolution matrix.

**Syntax**

\[
A = \text{convmtx}(c,n) \\
A = \text{convmtx}(r,n)
\]

**Description**
A convolution matrix is a matrix, formed from a vector, whose inner product with another vector is the convolution of the two vectors.

\[
A = \text{convmtx}(c,n) \text{ where } c \text{ is a length } m \text{ column vector returns a matrix } A \text{ of size } (m + n-1)\text{-by-}n. \text{ The product of } A \text{ and another column vector } x \text{ of length } n \text{ is the convolution of } c \text{ with } x.
\]

\[
A = \text{convmtx}(r,n) \text{ where } r \text{ is a length } m \text{ row vector returns a matrix } A \text{ of size } n\text{-by-}(m + n-1). \text{ The product of } A \text{ and another row vector } x \text{ of length } n \text{ is the convolution of } r \text{ with } x.
\]

**Example**
Generate a simple convolution matrix:

\[
h = [1 2 3 2 1]; \\
\text{convmtx}(h,7)
\]

\[
\text{ans} = \\
\begin{bmatrix}
1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 2 \\
\end{bmatrix}
\]

Note that `convmtx` handles edge conditions by zero padding.

In practice, it is more efficient to compute convolution using

\[
y = \text{conv}(c,x)
\]

than by using a convolution matrix:

\[
n = \text{length}(x); \\
y = \text{convmtx}(c,n)*x
\]

**Algorithm**
`convmtx` uses the function `toeplitz` to generate the convolution matrix.
### See Also

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><code>conv</code></td>
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<td>N-dimensional convolution (see the online MATLAB Function Reference).</td>
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<td><code>conv2</code></td>
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<tr>
<td><code>dftmtx</code></td>
<td>Discrete Fourier transform matrix.</td>
</tr>
</tbody>
</table>
corrcov

Purpose
Correlation coefficient matrix.

Syntax
C = corrcov(X)
C = corrcov(X,Y)

Description
corrcov returns a matrix of correlation coefficients calculated from an input matrix whose rows are observations and whose columns are variables. If C = cov(X), then corrcov(X) is the matrix whose element (i, j) is

\[
\text{corrcov}(i, j) = \frac{C(i, j)}{\sqrt{C(i, i)C(j, j)}}
\]

C = corrcov(X) is the zeroth lag of the covariance function, that is, the zeroth lag of xcov(x, 'coeff') packed into a square array.

C = corrcov(X,Y) is the same as corrcov([X Y]), that is, it concatenates X and Y in the row direction before its computation.

corrcov removes the mean from each column before calculating the results. See the xcorr function for cross-correlation options.

The corrcov function is part of the standard MATLAB language.

See Also

cov
Covariance matrix.
mean
Average value (see the online MATLAB Function Reference).
median
Median value (see the online MATLAB Function Reference).
std
Standard deviation (see the online MATLAB Function Reference).
xcorr
Cross-correlation function estimate.
xcov
Cross-covariance function estimate (equal to mean-removed cross-correlation).
Purpose
Covariance matrix.

Syntax
\[
c = \text{cov}(x)
\]
\[
c = \text{cov}(x,y)
\]

Description
\text{cov} computes the covariance matrix. If \(x\) is a vector, \(c\) is a scalar containing the variance. For an array where each row is an observation and each column a variable, \(\text{cov}(X)\) is the covariance matrix. \(\text{diag(\text{cov}(X))}\) is a vector of variances for each column, and \(\sqrt{\text{diag(\text{cov}(X))}}\) is a vector of standard deviations.

\(\text{cov}(x)\) is the zeroth lag of the covariance function, that is, the zeroth lag of \(x\text{cov}(x)/(n-1)\) packed into a square array.

\(\text{cov}(x,y)\) where \(x\) and \(y\) are column vectors of equal length is equivalent to \(\text{cov}([x\ y])\), that is, it concatenates \(x\) and \(y\) in the row direction before its computation.

\text{cov} removes the mean from each column before calculating the results.

The \text{cov} function is part of the standard MATLAB language.

Algorithm
\[
[n,p] = \text{size}(x);
\]
\[
x = x-\text{ones}(n,1)\ast(\text{sum}(x)/n);
\]
\[
y = x\ast y/(n-1);
\]

See Also
\text{corrcoef} & Correlation coefficient matrix.
\text{mean} & Average value (see the online MATLAB Function Reference).
\text{median} & Median value (see the online MATLAB Function Reference).
\text{std} & Standard deviation (see the online MATLAB Function Reference).
\text{xcorr} & Cross-correlation function estimate.
\text{xcov} & Cross-covariance function estimate (equal to mean-removed cross-correlation).
cplxpair

Purpose
Group complex numbers into complex conjugate pairs.

Syntax
y = cplxpair(x)
y = cplxpair(x,tol)

Description
y = cplxpair(x) returns x with complex conjugate pairs grouped together.
cplxpair orders the conjugate pairs by increasing real part. Within a pair, the
element with negative imaginary part comes first. The function returns all
purely real values following all the complex pairs.

y = cplxpair(x,tol) includes a tolerance, tol, for determining which
numbers are real and which are paired complex conjugates. By default,
cplxpair uses a tolerance of 100*eps relative to abs(x(i)). cplxpair forces
the complex conjugate pairs to be exact complex conjugates.

The cplxpair function is part of the standard MATLAB language.

Example
Order five poles evenly spaced around the unit circle into complex pairs:

cplxpair(exp(2*pi*sqrt(-1)*(0:4)/5))

ans =
-0.8090  - 0.5878i
-0.8090  + 0.5878i
 0.3090  - 0.9511i
 0.3090  + 0.9511i
 1.0000

Diagnostics
If there is an odd number of complex numbers, or if the complex numbers
cannot be grouped into complex conjugate pairs within the tolerance, cplxpair
generates the error message:

Complex numbers can't be paired.
cremez

**Purpose**
Complex and nonlinear-phase equiripple FIR filter design.

**Syntax**

- \( b = \text{cremez}(n,f,'fresp') \)
- \( b = \text{cremez}(n,f,'fresp',w) \)
- \( b = \text{cremez}(n,f,\{'fresp',p1,p2,...\},w) \)
- \( b = \text{cremez}(n,f,a,w) \)
- \( b = \text{cremez}(...,\{'sym'\}) \)
- \( b = \text{cremez}(...,\{'skip\_stage2'\}) \)
- \( b = \text{cremez}(...,\{'debug'\}) \)
- \( b = \text{cremez}(...,\{lgrid\}) \)
- \([b,\delta,\text{opt}] = \text{cremez}(...)\)

**Description**
cremez allows arbitrary frequency-domain constraints to be specified for the design of a possibly complex FIR filter. The Chebyshev (or minimax) filter error is optimized, producing equiripple FIR filter designs.

- \( b = \text{cremez}(n,f,'fresp') \) returns a length \( n+1 \) FIR filter with the best approximation to the desired frequency response as returned by function \( fresp \). \( f \) is a vector of frequency band edge pairs, specified in the range \(-1\) and \( 1 \), where \( 1 \) corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order, and \( f \) must have even length. The frequency bands span \( f(k) \) to \( f(k+1) \) for \( k \) odd; the intervals \( f(k+1) \) to \( f(k+2) \) for \( k \) odd are “transition bands” or “don’t care” regions during optimization.

- \( b = \text{cremez}(n,f,'fresp',w) \) uses the real, non-negative weights in vector \( w \) to weight the fit in each frequency band. The length of \( w \) is half the length of \( f \), so there is exactly one weight per band.

- \( b = \text{cremez}(n,f,\{'fresp',p1,p2,...\},...) \) supplies optional parameters \( p1, p2, ... \) to the frequency response function \( fresp \). Predefined ‘fresp’ frequency response functions are included for a number of common filter designs, as described below. For all of the predefined frequency response functions, the symmetry option ‘sym’ defaults to ‘even’ if no negative frequencies are contained in \( f \) and \( d = 0 \); otherwise ‘sym’ defaults to ‘none’. (See the ‘sym’ option below for details.) For all of the predefined frequency response functions, \( d \) specifies a group-delay offset such that the filter response
cremez

has a group delay of \( n/2 + d \) in units of the sample interval. Negative values create less delay; positive values create more delay. By default, \( d = 0 \).

- **lowpass, highpass, bandpass, bandstop**
  These functions share a common syntax, exemplified here by 'lowpass':
  
  \[
  b = \text{cremez}(n,f,'\text{lowpass}',...) \quad \text{and} \quad \\
  b = \text{cremez}(n,f,\{'\text{lowpass}',d\},...) 
  \]
  design a linear-phase \( (n/2 + d) \) delay) filter.

- **multiband** designs a linear-phase frequency response filter with arbitrary band amplitudes.
  \[
  b = \text{cremez}(n,f,\{'\text{multiband}',a\},...) \quad \text{and} \quad \\
  b = \text{cremez}(n,f,\{'\text{multiband}',a,d\},...) 
  \]
  specify vector \( a \) containing the desired amplitudes at the band edges in \( f \). The desired amplitude at frequencies between pairs of points \( f(k) \) and \( f(k+1) \) for \( k \) odd is the line segment connecting the points \( (f(k),a(k)) \) and \( (f(k+1),a(k+1)) \).

- **differentiator** designs a linear-phase differentiator. For these designs, zero-frequency must be in a transition band, and band weighting is set to be inversely proportional to frequency.
  \[
  b = \text{cremez}(n,f,\{'\text{differentiator}',Fs\},...) \quad \text{and} \quad \\
  b = \text{cremez}(n,f,\{'\text{differentiator}',Fs,d\},...) 
  \]
  specify the sample rate \( Fs \) used to determine the slope of the differentiator response. If omitted, \( Fs \) defaults to 1.

- **hilbfilt** designs a linear-phase Hilbert transform filter response. For Hilbert designs, zero-frequency must be in a transition band.
  \[
  b = \text{cremez}(n,f,\{'\text{hilbfilt}'\},...) \quad \text{and} \quad \\
  b = \text{cremez}(N,F,\{'\text{hilbfilt}',d\},...) 
  \]
  design a linear-phase \( (n/2 + d) \) delay) Hilbert transform filter.

\[
\begin{align*}
\text{b} & = \text{cremez}(n,f,a,w) \quad \text{is a synonym for} \\
\text{b} & = \text{cremez}(n,f,\{'\text{multiband}',a\},w). \\
\text{b} & = \text{cremez}(\ldots,\{'\text{sym}'\}) \quad \text{imposes a symmetry constraint on the impulse response of the design, where 'sym' may be one of the following:}
\end{align*}
\]
• 'none' indicates no symmetry constraint
  This is the default if any negative band edge frequencies are passed, or if
  'fresp' does not supply a default.
• 'even' indicates a real and even impulse response
  This is the default for highpass, lowpass, bandpass, bandstop, and
  multiband designs.
• 'odd' indicates a real and odd impulse response
  This is the default for Hilbert and differentiator designs.
• 'real' indicates conjugate symmetry for the frequency response

If any 'sym' option other than 'none' is specified, the band edges should only
be specified over positive frequencies; the negative frequency region is filled in
from symmetry. If a 'sym' option is not specified, the 'fresp' function is
queried for a default setting.

\[
\begin{align*}
  b &= \text{cremez}(\ldots,'\text{skip\_stage2}') \quad \text{disables the second-stage optimization} \\
  & \quad \text{algorithm, which executes only when cremez determines that an optimal} \\
  & \quad \text{solution has not been reached by the standard Remez error-exchange.} \\
  & \quad \text{Disabling this algorithm may increase the speed of computation, but may incur} \\
  & \quad \text{a reduction in accuracy. By default, the second-stage optimization is enabled.}
\end{align*}
\]

\[
\begin{align*}
  b &= \text{cremez}(\ldots,'\text{debug}') \quad \text{enables the display of intermediate results during} \\
  & \quad \text{the filter design, where 'debug' may be one of 'trace', 'plots', 'both', or} \\
  & \quad \text{'off'. By default, it is set to 'off'.}
\end{align*}
\]

\[
\begin{align*}
  b &= \text{cremez}(\ldots,\{lgrid\}) \quad \text{uses the integer lgrid to control the density of the} \\
  & \quad \text{frequency grid, which has roughly } 2^{\text{nextpow2}(lgrid*n)} \text{ frequency points.} \\
  & \quad \text{The default value for lgrid is 25. Note that the \{lgrid\} argument must be a} \\
  & \quad \text{1-by-1 cell array.}
\end{align*}
\]

Any combination of the 'sym', 'skip\_stage2', 'debug', and {lgrid} options
may be specified.

\[
\begin{align*}
  [b,delta] &= \text{cremez}(\ldots) \quad \text{returns the maximum ripple height delta.}
\end{align*}
\]
cremez

[b,delta,opt] = cremez(...) returns a structure opt of optional results computed by cremez and contains the following fields:

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>opt.fgrid</td>
<td>Frequency grid vector used for the filter design optimization</td>
</tr>
<tr>
<td>opt.des</td>
<td>Desired frequency response for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.wt</td>
<td>Weighting for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.H</td>
<td>Actual frequency response for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.error</td>
<td>Error at each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.iextr</td>
<td>Vector of indices into opt.fgrid for extremal frequencies</td>
</tr>
<tr>
<td>opt.fextr</td>
<td>Vector of extremal frequencies</td>
</tr>
</tbody>
</table>

**Examples**

**Example 1**

Design a 31-tap, linear-phase, lowpass filter:

```matlab
b = cremez(30,[-1 -0.5 -0.4 0.7 0.8 1], 'lowpass');
freqz(b, 1, 512, 'whole');
```

![Magnitude and Phase Plots](image)

**Normalized Angular Frequency (\(\times \pi\) rads/sample)**

**Magnitude (dB)**

**Phase (degrees)**
Example 2
Design a nonlinear-phase allpass FIR filter.

First select (or create) the function \textit{fresp} that returns the desired frequency response. For this example, \textit{fresp} is the \texttt{allpass.m} function in the \texttt{signal/signal/private} directory which returns the frequency response of a nonlinear-phase allpass filter. Copy \texttt{allpass.m} to another location on the MATLAB path before trying the example.

Before using \texttt{cremez} with \texttt{allpass.m} to generate the filter coefficients, call \texttt{allpass} alone to create the desired response.

```matlab
n = 22;    % Filter order
f = [-1 1]; % Frequency band edges
w = [1 1];  % Weights for optimization
gf = linspace(-1,1,256); % Grid of frequency points
d = allpass(n,f,gf,w);   % Desired frequency response
```

Vector \texttt{d} now contains the complex frequency response that we desire for the FIR filter computed by \texttt{cremez}.

Now compute the FIR filter that best approximates this response:

```matlab
b = cremez(n,f,'allpass',w,'real'); % Approximation
freqz(b,1,256,'whole');
```

---

0.2
0.1
0.0
-0.1
-0.2

Magnitude (dB)

0.2
0.1
0.0
-0.1
-0.2

Normalized Angular Frequency (\times \pi \text{rads/sample})

0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Phase (degrees)

-3000
-2000
-1000
0
0.1
0.2

Normalized Angular Frequency (\times \pi \text{rads/sample})
The `freqz` plot shows the frequency response of the filter computed by `cremez` to approximate the desired response. Check the accuracy of the approximation by overlaying the desired frequency response on the plot.

```matlab
subplot(2,1,1); hold on
plot(pi*(gf+1),20*log10(abs(fftshift(d))),'r--')

subplot(2,1,2); hold on
plot(pi*(gf+1),unwrap(angle(fftshift(d)))*180/pi,'r--')
legend('Approximation','Desired')
```

**Remarks**

User-definable functions may be used, instead of the predefined frequency response functions for `fresp`. The function is called from within `cremez` using the following syntax:

```
[dh,dw] = fresp(n,f,gf,w,p1,p2,...) where
```

- `n` is the filter order.
- `f` is the vector of frequency band edges that appear monotonically between -1 and 1, where 1 is the Nyquist frequency.
- `gf` is a vector of grid points that have been linearly interpolated over each specified frequency band by `cremez`. `gf` determines the frequency grid at which the response function must be evaluated. This is the same data returned by `cremez` in the `fgrid` field of the `opt` structure.
cremez

- $w$ is a vector of real, positive weights, one per band, used during optimization. $w$ is optional in the call to `cremez`; if not specified, it is set to unity weighting before being passed to `'fresp'`.
- $dh$ and $dw$ are the desired complex frequency response and band weight vectors, respectively, evaluated at each frequency in grid $gf$.
- $p1, p2, ...$, are optional parameters that may be passed to `'fresp'`.

Additionally, a preliminary call is made to `'fresp'` to determine the default symmetry property `'sym'`. This call is made using the syntax:

```matlab
sym = fresp('defaults',{n,f,[],w,p1,p2,...})
```

The arguments may be used in determining an appropriate symmetry default as necessary. The function `private/lowpass.m` may be useful as a template for generating new frequency response functions.

**Algorithm**

An extended version of the Remez exchange method is implemented for the complex case. This exchange method obtains the optimal filter when the equiripple nature of the filter is restricted to have $n+2$ extremals. When it does not converge, the algorithm switches to an ascent-descent algorithm that takes over to finish the convergence to the optimal solution. See the references for further details.
Diagnoses

The following diagnostic messages arise from incorrect usage of \texttt{cremez}:

- Not enough input arguments.
- \( F \) must contain an even number of band edge entries.
- Band edges must be monotonically increasing.
- Expecting a string argument.
- Invalid argument \texttt{arg} specified.
- Invalid default symmetry option \texttt{sym} returned from response function \texttt{fresp}. Must be one of 'none', 'real', 'even', or 'odd'.
- Frequency band edges must be in the range \([-1,+1]\) for designs with \texttt{SYM = 'sym'}.
- Frequency band edges must be in the range \([0,+1]\) for designs with \texttt{SYM = 'sym'}.
- Incorrect size of results from response function \texttt{fresp}. Sizes must be the same size as the frequency grid \texttt{GF}.
- Both \(-1\) and \(1\) have been specified as frequencies in \(F\), and the frequency spacing is too close to move either of them toward its neighbor.
- Internal error: Grid frequencies out of range.
- Internal error: domain must be "whole" or "half".
- Internal error: obtained a negative bandwidth.
- Internal error: two extremal frequencies at the same grid point.
- Internal error: d\texttt{Brange} must be > 0.
See Also

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>Window-based finite impulse response filter design – standard response.</td>
</tr>
<tr>
<td>fir2</td>
<td>Window-based finite impulse response filter design – arbitrary response.</td>
</tr>
<tr>
<td>firls</td>
<td>Least square linear-phase FIR filter design.</td>
</tr>
<tr>
<td>remez</td>
<td>Parks-McClellan optimal FIR filter design.</td>
</tr>
<tr>
<td>private/bandpass</td>
<td>Bandpass filter design function.</td>
</tr>
<tr>
<td>private/bandstop</td>
<td>Bandstop filter design function.</td>
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<tr>
<td>private/differentiator</td>
<td>Differentiator filter design function.</td>
</tr>
<tr>
<td>private/highpass</td>
<td>Highpass filter design function.</td>
</tr>
<tr>
<td>private/hilbfilt</td>
<td>Hilbert filter design function.</td>
</tr>
<tr>
<td>private/lowpass</td>
<td>Lowpass filter design function.</td>
</tr>
<tr>
<td>private/multiband</td>
<td>Multiband filter design function.</td>
</tr>
</tbody>
</table>

References


Purpose

Estimate the cross spectral density (CSD) of two signals.

Syntax

\[ Pxy = \text{csd}(x,y) \]
\[ Pxy = \text{csd}(x,y,nfft) \]
\[ [Pxy,f] = \text{csd}(x,y,nfft,Fs) \]
\[ Pxy = \text{csd}(x,y,nfft,Fs,\text{window}) \]
\[ Pxy = \text{csd}(x,y,nfft,Fs,\text{window},\text{noverlap}) \]
\[ Pxy = \text{csd}(x,y,...,'\text{dflag}') \]
\[ [Pxy,Pxdc,f] = \text{csd}(x,y,nfft,Fs,\text{window},\text{noverlap},p) \]
\[ \text{csd}(x,y,...) \]

Description

\[ Pxy = \text{csd}(x,y) \] estimates the cross spectral density of the length \( n \) sequences \( x \) and \( y \) using the Welch method of spectral estimation. \( Pxy = \text{csd}(x,y) \) uses the following default values:

- \( nfft = \min(256,\text{length}(x)) \)
- \( Fs = 2 \)
- \( \text{window} = \text{hanning}(nfft) \)
- \( \text{noverlap} = 0 \)

\( nfft \) specifies the FFT length that \( \text{csd} \) uses. This value determines the frequencies at which the cross spectrum is estimated. \( Fs \) is a scalar that specifies the sampling frequency. \( \text{window} \) specifies a windowing function and the number of samples \( \text{csd} \) uses in its sectioning of the \( x \) and \( y \) vectors. \( \text{noverlap} \) is the number of samples by which the sections overlap. Any arguments omitted from the end of the parameter list use the default values shown above.

If \( x \) and \( y \) are real, \( \text{csd} \) estimates the cross spectral density at positive frequencies only; in this case, the output \( Pxy \) is a column vector of length \( nfft/2 + 1 \) for \( nfft \) even and \( (nfft + 1)/2 \) for \( nfft \) odd. If \( x \) or \( y \) is complex, \( \text{csd} \) estimates the cross spectral density at both positive and negative frequencies and \( Pxy \) has length \( nfft \).

\[ Pxy = \text{csd}(x,y,nfft) \] uses the FFT length \( nfft \) in estimating the cross spectral density of \( x \) and \( y \). Specify \( nfft \) as a power of 2 for fastest execution.

\[ [Pxy,f] = \text{csd}(x,y,nfft,Fs) \] returns a vector \( f \) of frequencies at which the function evaluates the CSD. \( f \) is the same size as \( Pxy \), so \( \text{plot}(f,Pxy) \) plots the
spectrum versus properly scaled frequency. Fs has no effect on the output Pxy; it is a frequency scaling multiplier.

\[ Pxy = \text{csd}(x,y,nfft,Fs,\text{window}) \]

specifies a windowing function and the number of samples per section of the x vector. If you supply a scalar for window, csd uses a Hanning window of that length. The length of the window must be less than or equal to nfft; csd zero pads the sections if the length of the window is less than nfft. csd returns an error if the length of the window is greater than nfft.

\[ Pxy = \text{csd}(x,y,nfft,Fs,\text{window},\text{noverlap}) \]

overlaps the sections of x and y by noverlap samples.

You can use the empty matrix [] to specify the default value for any input argument except x or y. For example,

\[ \text{csd}(x,y,[] ,10000) \]

is equivalent to

\[ \text{csd}(x) \]

but with a sampling frequency of 10,000 Hz instead of the default of 2 Hz.

\[ Pxy = \text{csd}(x,y,...,'\text{dflag}') \]

specifies a detrend option, where dflag is:

- linear, to remove the best straight-line fit from the prewindowed sections of x and y
- mean, to remove the mean from the prewindowed sections of x and y
- none, for no detrending (default)

The dflag parameter must appear last in the list of input arguments. csd recognizes a dflag string no matter how many intermediate arguments are omitted.

\[ [Pxy,Pxyc,f] = \text{csd}(x,y,nfft,Fs,\text{window},\text{noverlap},p) \]

where p is a positive scalar between 0 and 1 returns a vector Pxyc that contains an estimate of the p*100 percent confidence interval for Pxy. Pxyc is a two-column matrix the same length as Pxy. The interval \([Pxyc(:,1),\ Pxyc(:,2)]\) covers the true CSD with probability p. plot(f,[Pxy\ Pxyc]) plots the cross spectrum inside the p*100 percent confidence interval. If unspecified, p defaults to 0.95.
csd\( (x, y, \ldots) \) plots the CSD versus frequency in the current figure window. If the \( p \) parameter is specified, the plot includes the confidence interval.

**Example**

Generate two colored noise signals and plot their CSD with a confidence interval of 95%. Specify a length 1024 FFT, a 500 point triangular window with no overlap, and a sampling frequency of 10 Hz:

\[
\begin{align*}
  h &= \text{fir1}(30, 0.2, \text{boxcar}(31)); \\
  h_1 &= \frac{\text{ones}(1, 10)}{\sqrt{10}}; \\
  r &= \text{randn}(16384, 1); \\
  x &= \text{filter}(h_1, 1, r); \\
  y &= \text{filter}(h, 1, x); \\
  \text{csd}(x, y, 1024, 10000, \text{triang}(500), 0, [])
\end{align*}
\]

Algorithm

csd implements the Welch method of spectral density estimation (see references [1] and [2]):

1. It applies the window specified by the \text{window} vector to each successive detrended section.
2. It transforms each section with an \text{nfft}-point FFT.
3. It forms the periodogram of each section by scaling the product of the transform of the \( y \) section and the conjugate of the transformed \( x \) section.

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It averages the periodograms of the successive overlapping sections to form $P_{xy}$, the cross spectral density of $x$ and $y$. 

The number of sections that `csd` averages is $k$, where $k$ is

$$\text{fix}((\text{length}(x)-\text{noverlap})/(\text{length}(\text{window})-\text{noverlap}))$$

**Diagnostics**

An appropriate diagnostic message is displayed when incorrect arguments to `csd` are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector (either row or column) input.
- Requires inputs X and Y to have the same length.
- Requires confidence parameter to be a scalar between 0 and 1.

**See Also**

- `cohere` Estimate magnitude squared coherence function between two signals.
- `pburg` Power spectrum estimate using the Burg method.
- `pmtm` Power spectrum estimate using the multitaper method (MTM).
- `pmusic` Power spectrum estimate using MUSIC eigenvector method.
- `pwelch` Estimate the power spectral density (PSD) of a signal using Welch's method.
- `pyulear` Power spectrum estimate using Yule-Walker AR method.
- `tfe` Transfer function estimate from input and output.

**References**


**Purpose**  
Chirp z-transform.

**Syntax**  
\[ y = \text{czt}(x, m, w, a) \]  
\[ y = \text{czt}(x) \]

**Description**  
\[ y = \text{czt}(x, m, w, a) \] returns the chirp z-transform of signal \( x \). The chirp z-transform is the z-transform of \( x \) along a spiral contour defined by \( w \) and \( a \). \( m \) is a scalar that specifies the length of the transform, \( w \) is the ratio between points along the z-plane spiral contour of interest, and scalar \( a \) is the complex starting point on that contour. The contour, a spiral or “chirp” in the z-plane, is given by

\[ z = a*(w.^{-(0:m-1)}) \]

\( y = \text{czt}(x) \) uses the following default values:

- \( m = \text{length}(x) \)
- \( w = \exp(j*2*pi/m) \)
- \( a = 1 \)

With these defaults, \( \text{czt} \) returns the z-transform of \( x \) at \( m \) equally spaced points around the unit circle. This is equivalent to the discrete Fourier transform of \( x \), or \( \text{fft}(x) \). The empty matrix \([\] \) specifies the default value for a parameter.

If \( x \) is a matrix, \( \text{czt}(x, m, w, a) \) transforms the columns of \( x \).

**Examples**  
Create a random vector \( x \) of length 1013 and compute its DFT using \( \text{czt} \). This is faster than the \( \text{fft} \) function on the same sequence.

\[ x = \text{randn}(1013,1); \]
\[ y = \text{czt}(x); \]

Use \( \text{czt} \) to zoom in on a narrow-band section (100 to 150 Hz) of a filter’s frequency response. First design the filter:

\[ h = \text{fir1}(30,125/500,\text{boxcar}(31)); \text{ % filter} \]
Establish frequency and CZT parameters:

\[
\begin{align*}
Fs &= 1000; \\
f1 &= 100; \\
f2 &= 150; \quad \text{\% in Hertz} \\
m &= 1024; \\
w &= \exp(-j*2*pi*(f2-f1)/(m*Fs)); \\
a &= \exp(j*2*pi*f1/Fs);
\end{align*}
\]

Compute both the DFT and CZT of the filter:

\[
\begin{align*}
y &= \text{fft}(h,1000); \\
z &= \text{czt}(h,m,w,a);
\end{align*}
\]

Create frequency vectors and compare the results:

\[
\begin{align*}
fy &= (0:length(y)-1)'*1000/length(y); \\
fz &= ((0:length(z)-1)'*(f2-f1)/length(z)) + f1; \\
\text{plot}(fy(1:500),\text{abs}(y(1:500))); \text{axis([1 500 0 1.2])} \\
&\text{title('FFT')} \\
\text{figure} \\
&\text{plot}(fz,\text{abs}(z)); \text{axis([f1 f2 0 1.2])} \\
&\text{title('CZT')}
\end{align*}
\]

Algorithm

czt uses the next power-of-2 length FFT to perform a fast convolution when computing the z-transform on a specified chirp contour [1]. czt can be significantly faster than fft for large, prime-length sequences.

Diagnostics

If \(m, w, \) or \(a\) is not a scalar, czt gives the following error message:

\[
\text{Inputs M, W, and A must be scalars.}
\]
### See Also

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><code>fft</code></td>
<td>One-dimensional fast Fourier transform.</td>
</tr>
<tr>
<td><code>freqz</code></td>
<td>Frequency response of digital filters.</td>
</tr>
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</table>

### References

Purpose
Discrete cosine transform (DCT).

Syntax
- \( y = \text{dct}(x) \)
- \( y = \text{dct}(x, n) \)

Description
\( y = \text{dct}(x) \) returns the unitary discrete cosine transform of \( x \)

\[
y(k) = w(k) \sum_{n=1}^{N} x(n) \cos \left( \frac{\pi (2n-1)(k-1)}{2N} \right), \quad k = 1, \ldots, N
\]

where
\[
w(k) = \begin{cases} 
\frac{1}{\sqrt{N}}, & k = 1 \\
\frac{\sqrt{2}}{\sqrt{N}}, & 2 \leq k \leq N 
\end{cases}
\]

\( N \) is the length of \( x \), and \( x \) and \( y \) are the same size. If \( x \) is a matrix, \( \text{dct} \)
transforms its columns. The series is indexed from \( n = 1 \) and \( k = 1 \) instead of the usual \( n = 0 \) and \( k = 0 \) because MATLAB vectors run from 1 to \( N \) instead of from 0 to \( N-1 \).

\( y = \text{dct}(x, n) \) pads or truncates \( x \) to length \( n \) before transforming.

The DCT is closely related to the discrete Fourier transform. You can often reconstruct a sequence very accurately from only a few DCT coefficients, a useful property for applications requiring data reduction.
Example

Find how many DCT coefficients represent 99% of the energy in a sequence:

\[
\begin{align*}
x &= (1:100) + 50 \cos((1:100) \cdot 2\pi/40); \\
X &= \text{dct}(x); \\
[XX, \text{ind}] &= \text{sort}(\text{abs}(X)); \text{ind} = \text{fliplr}(\text{ind}); \\
i &= 1; \\
\text{while } (\text{norm}([X(\text{ind}(1:i)) \text{ zeros}(1,100-i)]/\text{norm}(X)<.99)) \\
&\quad i = i + 1; \\
\text{end} \\
i &= 3
\end{align*}
\]

See Also

- **fft**: One-dimensional fast Fourier transform.
- **idct**: Inverse discrete cosine transform.
- **dct2**: Two-dimensional DCT (see Image Processing Toolbox User’s Guide).
- **idct2**: Two-dimensional inverse DCT (see Image Processing Toolbox User’s Guide).

References


decimate

**Purpose** Decrease the sampling rate for a sequence (decimation).

**Syntax**

\[
y = \text{decimate}(x,r)
\]
\[
y = \text{decimate}(x,r,n)
\]
\[
y = \text{decimate}(x,r,'\text{fir}')
\]
\[
y = \text{decimate}(x,r,n,'\text{fir}')
\]

**Description** Decimation reduces the original sampling rate for a sequence to a lower rate. It is the opposite of interpolation. The decimation process filters the input data with a lowpass filter and then resamples the resulting smoothed signal at a lower rate.

\[
y = \text{decimate}(x,r)\]
reduces the sample rate of \(x\) by a factor \(r\). The decimated vector \(y\) is \(r\) times shorter in length than the input vector \(x\). By default, \text{decimate} employs an eighth-order lowpass Chebyshev type I filter. It filters the input sequence in both the forward and reverse directions to remove all phase distortion, effectively doubling the filter order.

\[
y = \text{decimate}(x,r,n)\]
uses an order \(n\) Chebyshev filter. Orders above 13 are not recommended because of numerical instability. MATLAB displays a warning in this case.

\[
y = \text{decimate}(x,r,'\text{fir}')\]
uses a 30-point FIR filter, instead of the Chebyshev IIR filter. Here \text{decimate} filters the input sequence in only one direction. This technique conserves memory and is useful for working with long sequences.

\[
y = \text{decimate}(x,r,n,'\text{fir}')\]
uses a length \(n\) FIR filter.

**Example** Decimate a signal by a factor of four:

\[
t = 0:.00025:1;\]
\[
x = \sin(2\pi * 30*t) + \sin(2\pi * 60*t);
\]
\[
y = \text{decimate}(x,4);
\]
View the original and decimated signals:

```matlab
stem(x(1:120)), axis([0 120 -2 2]) % original signal
title('Original Signal')
figure
stem(y(1:30)) % decimated signal
title('Decimated Signal')
```

**Algorithm**

decimate uses decimation algorithms 8.2 and 8.3 from [1]:

1. It designs a lowpass filter. By default, decimate uses a Chebyshev type I filter with normalized cutoff frequency $0.8/r$ and 0.05 dB of passband ripple. For the fir option, decimate designs a lowpass FIR filter with cutoff frequency $1/r$ using fir1.

2. For the FIR filter, decimate applies the filter to the input vector in one direction. In the IIR case, decimate applies the filter in forward and reverse directions with filtfilt.

3. decimate resamples the filtered data by selecting every $r$-th point.

**Diagnostics**

If $r$ is not an integer, decimate gives the following error message:

```
Resampling rate R must be an integer.
```

If $n$ specifies an IIR filter with order greater than 13, decimate gives the following warning:

```
Warning: IIR filters above order 13 may be unreliable.
```
See Also

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>interp</td>
<td>Increase sampling rate by an integer factor (interpolation).</td>
</tr>
<tr>
<td>resample</td>
<td>Change sampling rate by any rational factor.</td>
</tr>
<tr>
<td>spline</td>
<td>Cubic spline interpolation (see the online MATLAB Function Reference).</td>
</tr>
<tr>
<td>upfirdn</td>
<td>Upsample, apply an FIR filter, and downsample.</td>
</tr>
</tbody>
</table>

References

Purpose
Deconvolution and polynomial division.

Syntax
[q,r] = deconv(b,a)

Description
[q,r] = deconv(b,a) deconvolves vector a out of vector b, using long division. The result (quotient) is returned in vector q and the remainder in vector r such that b = conv(q,a) + r.

If a and b are vectors of polynomial coefficients, convolving them is equivalent to polynomial multiplication, and deconvolution is equivalent to polynomial division. The result of dividing b by a is quotient q and remainder r.

The deconv function is part of the standard MATLAB language.

Example
The convolution of a = [1 2 3] and b = [4 5 6] is

\[
c = \text{conv}(a,b)
\]

\[
c = \begin{bmatrix} 4 & 13 & 28 & 27 & 18 \end{bmatrix}
\]

Use deconv to divide b back out:

\[
[q,r] = \text{deconv}(c,a)
\]

\[
q = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}
\]

\[
r = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Algorithm
This function calls filter to compute the deconvolution as the impulse response of an IIR filter.

See Also
conv
Convolution and polynomial multiplication.

filter
Filter data with a recursive (IIR) or nonrecursive (FIR) filter.

residuez
z-transform partial fraction expansion.
demod

Purpose
Demodulation for communications simulation.

Syntax
- `x = demod(y,Fc,Fs,'method')`
- `x = demod(y,Fc,Fs,'method',opt)`
- `x = demod(y,Fc,Fs,'pwm','centered')`
- `[x1,x2] = demod(y,Fc,Fs,'qam')`

Description
`demod` performs demodulation, that is, it obtains the original signal from a modulated version of the signal. `demod` undoes the operation performed by `modulate`.

- `x = demod(y,Fc,Fs,'method')` and
- `x = demod(y,Fc,Fs,'method',opt)` demodulate the real carrier signal `y` with a carrier frequency `Fc` and sampling frequency `Fs`, using one of the options listed below for `method`. (Note that some methods accept an option, `opt`.)

- **amdsb-sc**
  - **Amplitude demodulation, double sideband, suppressed carrier.** Multiplies `y` by a sinusoid of frequency `Fc` and applies a fifth-order Butterworth lowpass filter using `filtfilt`:
    ```
    x = y.*cos(2*pi*Fc*t);
    [b,a] = butter(5,Fc*2/Fs);
    x = filtfilt(b,a,x);
    ```

- **amdsb–tc**
  - **Amplitude demodulation, double sideband, transmitted carrier.** Multiplies `y` by a sinusoid of frequency `Fc`, and applies a fifth-order Butterworth lowpass filter using `filtfilt`:
    ```
    x = y.*cos(2*pi*Fc*t);
    [b,a] = butter(5,Fc*2/Fs);
    x = filtfilt(b,a,x);
    ```
  - If you specify `opt`, `demod` subtracts scalar `opt` from `x`. The default value for `opt` is 0.

- **amssb**
  - **Amplitude demodulation, single sideband.** Multiplies `y` by a sinusoid of frequency `Fc` and applies a fifth-order Butterworth lowpass filter using `filtfilt`:
    ```
    x = y.*cos(2*pi*Fc*t);
    [b,a] = butter(5,Fc*2/Fs);
    x = filtfilt(b,a,x);
    ```
**fm**  
**Frequency demodulation.** Demodulates the FM waveform by modulating the Hilbert transform of \( y \) by a complex exponential of frequency \( -F_c \) Hz and obtains the instantaneous frequency of the result.

**pm**  
**Phase demodulation.** Demodulates the PM waveform by modulating the Hilbert transform of \( y \) by a complex exponential of frequency \( -F_c \) Hz and obtains the instantaneous phase of the result.

**ptm**  
**Pulse-time demodulation.** Finds the pulse times of a pulse-time modulated signal \( y \). For correct demodulation, the pulses cannot overlap. \( x \) is length \( y \) * \( F_c / F_s \).

**pwm**  
**Pulse-width demodulation.** Finds the pulse widths of a pulse-width modulated signal \( y \). \( \text{demod} \) returns in \( x \) a vector whose elements specify the width of each pulse in fractions of a period. The pulses in \( y \) should start at the beginning of each carrier period, that is, they should be left justified.

**qam**  
**Quadrature amplitude demodulation.**  
\[ [x_1, x_2] = \text{demod}(y, F_c, F_s, 'qam') \] multiplies \( y \) by a cosine and a sine of frequency \( F_c \) and applies a fifth-order Butterworth lowpass filter using \( \text{ffiltfilt} \):

\[
\begin{align*}
x_1 &= y \cdot \cos(2\pi F_c t); \\
x_2 &= y \cdot \sin(2\pi F_c t); \\
[b, a] &= \text{butter}(5, F_c*2/F_s); \\
x_1 &= \text{ffiltfilt}(b, a, x1); \\
x_2 &= \text{ffiltfilt}(b, a, x2);
\end{align*}
\]

The default method is 'am'. Except for the 'ptm' and 'pwm' cases, \( x \) is the same size as \( y \).

If \( y \) is a matrix, \( \text{demod} \) demodulates its columns.

\( x = \text{demod}(y, F_c, F_s, 'pwm', 'centered') \) finds the pulse widths assuming they are centered at the beginning of each period. \( x \) is length \( \text{length}(y) \) * \( F_c / F_s \).
demod

See Also

modulate  Modulation for communications simulation.
vco  Voltage controlled oscillator.
## Purpose
Discrete Fourier transform matrix.

## Syntax
A = dftmtx(n)

## Description
A discrete Fourier transform matrix is a complex matrix of values around the unit circle, whose matrix product with a vector computes the discrete Fourier transform of the vector.

\[ A = dftmtx(n) \]

returns the \( n \)-by-\( n \) complex matrix \( A \) that, when multiplied into a length \( n \) column vector \( x \):

\[ y = A\times x \]

computes the discrete Fourier transform of \( x \).

The inverse discrete Fourier transform matrix is

\[ A^i = \frac{\text{conj}(dftmtx(n))}{n} \]

## Example
In practice, the discrete Fourier transform is computed more efficiently and uses less memory with an FFT algorithm

\[ x = 1:256; \]
\[ y1 = \text{fft}(x); \]

than by using the Fourier transform matrix:

\[ n = \text{length}(x); \]
\[ y2 = x\times dftmtx(n); \]
\[ \text{norm}(y1-y2) \]
\[ \text{ans} = 1.8297e-009 \]

## Algorithm
dftmtx uses an outer product to generate the transform matrix.

## See Also
- convmtx
  - Convolution matrix.
- fft
  - One-dimensional fast Fourier transform.
diric

Purpose
Dirichlet or periodic sinc function.

Syntax
y = diric(x,n)

Description
y = diric(x,n) returns a vector or array y the same size as x. The elements of y are the Dirichlet function of the elements of x. n must be a positive integer.

The Dirichlet function, or periodic sinc function, is

\[
\text{diric}(x, n) = \begin{cases} 
\frac{x}{2\pi (n-1)} & x = 0, \pm 2\pi, \pm 4\pi, \ldots \\
\frac{\sin(nx/2)}{n\sin(x/2)} & \text{else}
\end{cases}
\]

for any nonzero integer n. This function has period \(2\pi\) for \(n\) odd and period \(4\pi\) for \(n\) even. Its peak value is 1, and its minimum value is -1 for \(n\) even. The magnitude of this function is \((1/n)\) times the magnitude of the discrete-time Fourier transform of the \(n\)-point rectangular window.

Diagnostics
If \(n\) is not a positive integer, \textit{diric} gives the following error message:

Requires \(n\) to be a positive integer.

See Also
\begin{itemize}
\item \textit{cos} Cosine of vector/matrix elements (see the online MATLAB Function Reference).
\item \textit{gauspuls} Gaussian-modulated sinusoidal pulse generator.
\item \textit{pulstran} Pulse train generator.
\item \textit{rectpuls} Sampled aperiodic rectangle generator.
\item \textit{sawtooth} Sawtooth or triangle wave generator.
\item \textit{sin} Sine of vector/matrix elements (see the online MATLAB Function Reference).
\item \textit{sinc} Sinc or \(\sin(\pi t)/\pi t\) function.
\item \textit{square} Square wave generator.
\item \textit{tripuls} Sampled aperiodic triangle generator.
\end{itemize}
**Purpose**
Discrete prolate spheroidal sequences (Slepian sequences).

**Syntax**

\[ [e,v] = dpss(n,nw) \]

\[ [e,v] = dpss(n,nw,k) \]

\[ [e,v] = dpss(n,nw,[k1 k2]) \]

\[ [e,v] = dpss(n,nw,'spline') \]

\[ [e,v] = dpss(n,nw,'spline',Ni) \]

\[ [e,v] = dpss(n,nw,'linear') \]

\[ [e,v] = dpss(n,nw,'linear',Ni) \]

\[ [e,v] = dpss(...,'trace') \]

\[ [e,v] = dpss(...,'int','trace') \]

**Description**

\[ [e,v] = dpss(n,nw) \] generates the first \(2 \times nw\) discrete prolate spheroidal sequences (DPSS) of length \(n\) in the columns of \(e\), and their corresponding concentrations in vector \(v\). They are also generated in the DPSS MAT-file database dpss.mat. \(nw\) must be less than \(n/2\).

\[ [e,v] = dpss(n,nw,k) \] returns the \(k\) most band-limited sequences of the \(2 \times nw\) discrete prolate spheroidal sequences calculated. \(k\) must be between 0 and \(2 \times nw\).

\[ [e,v] = dpss(n,nw,[k1 k2]) \] returns the \(k1\)-th through the \(k2\)-th sequences from the \(2 \times nw\) discrete prolate spheroidal sequences calculated, where \(1 \leq k1 \leq k2 \leq (2 \times nw)\).

For all of the above forms,

- The Slepian sequences are calculated directly.
- The sequences are generated in the frequency band \(|\omega| \leq (2\pi W)\), where \(W = nw/n\) is the half-bandwidth and \(\omega\) is in radians.
- \(e(:,1)\) is the length \(n\) signal most concentrated in the frequency band \(|\omega| \leq (2\pi W)\) radians, \(e(:,2)\) is the signal orthogonal to \(e(:,1)\) that is most concentrated in this band, \(e(:,3)\) is the signal orthogonal to both \(e(:,1)\) and \(e(:,2)\) that is most concentrated in this band, etc.
- For multitaper spectral analysis, typical choices for \(nw\) are 2, 5/2, 3, 7/2, or 4.

\[ [e,v] = dpss(n,nw,'spline') \] uses spline interpolation to compute \(e\) and \(v\) from the sequences in dpss.mat with length closest to \(n\).
[e,v] = dpss(n,nw,'spline',Ni) interpolates from existing length Ni sequences.

[e,v] = dpss(n,nw,'linear') and

[e,v] = dpss(n,nw,'linear',Ni) use linear interpolation, which is much faster but less accurate than spline interpolation. 'linear' requires Ni > n.

[e,v] = dpss(...,'trace') and

[e,v] = dpss(...,'int','trace') use a trailing 'trace' argument to find out which method DPSS uses, where 'int' is either 'spline' or 'linear'.

See Also

dpssclear Remove discrete prolate spheroidal sequences from database.
dpssdir Discrete prolate spheroidal sequences database directory.
dpssload Load discrete prolate spheroidal sequences from database.
dpsssave Save discrete prolate spheroidal sequences in database.
pmtm Power spectrum estimate using the multitaper method (MTM).

References

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<td><strong>Syntax</strong></td>
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<td><strong>Description</strong></td>
<td><code>dpssclear(n,nw)</code> removes sequences with length $n$ and time-bandwidth product $nw$ from the DPSS MAT-file database <code>dpss.mat</code>.</td>
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</table>
**Purpose**

Discrete prolate spheroidal sequences database directory.

**Syntax**

```matlab
dpssdir
dpssdir(n)
dpssdir(nw,'nw')
dpssdir(n,nw)
index = dpssdir
```

**Description**

`dpssdir` manages the database directory that contains the generated DPSS samples in the DPSS MAT-file database `dpss.mat`.

`dpssdir` lists the directory of saved sequences in `dpss.mat`.

`dpssdir(n)` lists the sequences saved with length `n`.

`dpssdir(nw,'nw')` lists the sequences saved with time-bandwidth product `nw`.

`dpssdir(n,nw)` lists the sequences saved with length `n` and time-bandwidth product `nw`.

```matlab
index = dpssdir
```

`index = dpssdir` is a structure array describing the DPSS database. Pass `n` and `nw` options as for the no output case to get a filtered `index`.

**See Also**

- `dpss` Discrete prolate spheroidal sequences (Slepian sequences).
- `dpssclear` Remove discrete prolate spheroidal sequences from database.
- `dpssload` Load discrete prolate spheroidal sequences from database.
- `dpsssave` Save discrete prolate spheroidal sequences in database.
**Purpose**
Load discrete prolate spheroidal sequences from database.

**Syntax**
\[
[e,v] = dpssload(n,nw)
\]

**Description**
\[
[e,v] = dpssload(n,nw)
\] loads all sequences with length \( n \) and time-bandwidth product \( nw \) in the columns of \( e \) and their corresponding concentrations in vector \( v \) from the DPSS MAT-file database \( dpss.mat \).

**See Also**
- **dpss**
  Discrete prolate spheroidal sequences (Slepian sequences).
- **dpssclear**
  Remove discrete prolate spheroidal sequences from database.
- **dpssdir**
  Discrete prolate spheroidal sequences database directory.
- **dpsssave**
  Save discrete prolate spheroidal sequences in database.
**dpsssave**

**Purpose**  
Save discrete prolate spheroidal sequences in database.

**Syntax**  
dpsssave(nw,e,v)  
status = dpsssave(nw,e,v)

**Description**  
dpsssave(nw,e,v) saves the sequences in the columns of e and their corresponding concentrations in vector v in the DPSS MAT-file database dpss.mat.

- It is not necessary to specify sequence length, because the length of the sequence is determined by the number of rows of e.
- nw is the time-bandwidth product that was specified when the sequence was created using dpss.

status = dpsssave(nw,e,v) returns 0 if the save was successful and 1 if there was an error.

**See Also**  
dpss  
Discrete prolate spheroidal sequences (Slepian sequences).

dpssclear  
Remove discrete prolate spheroidal sequences from database.

dpssdir  
Discrete prolate spheroidal sequences database directory.

dpssload  
Load discrete prolate spheroidal sequences from database.
**Purpose**
Elliptic (Cauer) filter design.

**Syntax**

\[
[b,a] = \text{ellip}(n,Rp,Rs,Wn)
\]

- \[b,a\] = \text{ellip}(n,Rp,Rs,Wn,'ftype')
- \[b,a\] = \text{ellip}(n,Rp,Rs,Wn,'s')
- \[b,a\] = \text{ellip}(n,Rp,Rs,Wn,'ftype','s')
- \[z,p,k\] = \text{ellip}(...)
- \[A,B,C,D\] = \text{ellip}(...)

**Description**

`ellip` designs lowpass, bandpass, highpass, and bandstop digital and analog elliptic filters. Elliptic filters offer steeper rolloff characteristics than Butterworth or Chebyshev filters, but are equiripple in both the pass- and stopbands. In general, elliptic filters meet given performance specifications with the lowest order of any filter type.

**Digital Domain**

\[[b,a] = \text{ellip}(n,Rp,Rs,Wn)\]
designs an order \(n\) lowpass digital elliptic filter with cutoff frequency \(Wn\), \(Rp\) dB of ripple in the passband, and a stopband \(Rs\) dB down from the peak value in the passband. It returns the filter coefficients in the length \(n + 1\) row vectors \(b\) and \(a\), with coefficients in descending powers of \(z\):

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}
\]

The cutoff frequency is the edge of the passband, at which the magnitude response of the filter is \(-Rp\) dB. For `ellip`, the cutoff frequency \(Wn\) is a number between 0 and 1, where 1 corresponds to half the sample frequency (Nyquist frequency). Smaller values of passband ripple \(Rp\) and larger values of stopband attenuation \(Rs\) both lead to wider transition widths (shallower rolloff characteristics).

If \(Wn\) is a two-element vector, \(Wn = [w1 \ w2]\), `ellip` returns an order \(2*n\) bandpass filter with passband \(w1 < \omega < w2\).

\[[b,a] = \text{ellip}(n,Rp,Rs,Wn,'ftype')\] designs a highpass or bandstop filter, where `ftype` is:
ellip

- high for a highpass digital filter with cutoff frequency \( W_n \)
- stop for an order \( 2n \) bandstop digital filter if \( W_n \) is a two-element vector, \( W_n = [w_1 \ w_2] \)

The stopband is \( w_1 < \omega < w_2 \).

With different numbers of output arguments, ellip directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

\[
[z, p, k] = \text{ellip}(n, Rp, Rs, Wn) \text{ or }
\]

\[
[z, p, k] = \text{ellip}(n, Rp, Rs, Wn, 'ftype') \text{ returns the zeros and poles in length n column vectors } z \text{ and } p \text{ and the gain in the scalar } k.
\]

To obtain state-space form, use four output arguments:

\[
[A, B, C, D] = \text{ellip}(n, Rp, Rs, Wn) \text{ or }
\]

\[
[A, B, C, D] = \text{ellip}(n, Rp, Rs, Wn, 'ftype') \text{ where } A, B, C, \text{ and } D \text{ are }
\]

\[
x[n + 1] = Ax[n] + Bu[n]
\]

\[
y[n] = Cx[n] + Du[n]
\]

and \( u \) is the input, \( x \) is the state vector, and \( y \) is the output.

**Analog Domain**

\[
[b, a] = \text{ellip}(n, Rp, Rs, Wn, 's') \text{ designs an order } n \text{ lowpass analog elliptic filter with cutoff frequency } W_n \text{ and returns the filter coefficients in the length } n + 1 \text{ row vectors } b \text{ and } a, \text{ in descending powers of } s:
\]

\[
H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \cdots + b(n + 1)}{s^n + a(2)s^{n-1} + \cdots + a(n + 1)}
\]

The cutoff frequency is the edge of the passband, at which the magnitude response of the filter is \(-Rp \) dB. For ellip, the cutoff frequency \( W_n \) must be greater than 0.

If \( W_n \) is a two-element vector with \( w_1 < w_2 \), then \( \text{ellip}(n, Rp, Rs, Wn, 's') \) returns an order \( 2n \) bandpass analog filter with passband \( w_1 < \omega < w_2 \).
ellip

[b,a] = ellip(n,Rp,Rs,Wn,'ftype','s') designs a highpass or bandstop filter, where ftype is:

- **high** for a highpass analog filter with cutoff frequency Wn
- **stop** for an order 2*n bandstop analog filter. Wn is a two-element vector, [w1 w2], specifying the stopband w1 < ω < w2.

With different numbers of output arguments, ellip directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

[z,p,k] = ellip(n,Rp,Rs,Wn,'s') or

[z,p,k] = ellip(n,Rp,Rs,Wn,'ftype','s') returns the zeros and poles in length n or 2*n column vectors z and p and the gain in the scalar k.

To obtain state-space form, use four output arguments:

[A,B,C,D] = ellip(n,Rp,Rs,Wn,'s') or

[A,B,C,D] = ellip(n,Rp,Rs,Wn,'ftype','s') where A, B, C, and D are

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

and u is the input, x is the state vector, and y is the output.

**Examples**

For data sampled at 1000 Hz, design a sixth-order lowpass elliptic filter with a cutoff frequency of 300 Hz, 3 dB of ripple in the passband, and 50 dB of attenuation in the stopband:

[b,a] = ellip(6,3,50,300/500);
The filter's frequency response is

\[ \text{freqz(b,a,512,1000)} \]
\[ \text{title('n=6 Lowpass Elliptic Filter')} \]
Design a 20th-order bandpass elliptic filter with a passband from 100 to 200 Hz and plot its impulse response:

```matlab
n = 10; Rp = 0.5; Rs = 20;
Wn = [100 200]/500;
[b,a] = ellip(n,Rp,Rs,Wn);
[y,t] = impz(b,a,101); stem(t,y)
title('Impulse Response of n=10 Elliptic Filter')
```

Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

The design of elliptic filters is the most difficult and computationally intensive of the Butterworth, Chebyshev type I and II, and elliptic designs. `ellip` uses a five-step algorithm:

1. It finds the lowpass analog prototype poles, zeros, and gain using the `ellipap` function.
2. It converts the poles, zeros, and gain into state-space form.
3. It transforms the lowpass filter to a bandpass, highpass, or bandstop filter with the desired cutoff frequencies using a state-space transformation.
ellip

4 For digital filter design, ellip uses bilinear to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at \( \omega_n \) or \( \omega_1 \) and \( \omega_2 \).

5 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

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<thead>
<tr>
<th>Function</th>
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<td>ellipord</td>
<td>Elliptic filter order selection.</td>
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Purpose

Elliptic analog lowpass filter prototype.

Syntax

\[ [z,p,k] = \text{ellipap}(n,Rp,Rs) \]

Description

\[ [z,p,k] = \text{ellipap}(n,Rp,Rs) \]
returns the zeros, poles, and gain of an order \( n \)
elliptic analog lowpass filter prototype, with \( Rp \) dB of ripple in the passband, 
and a stopband \( Rs \) dB down from the peak value in the passband. The zeros and 
poles are returned in length \( n \) column vectors \( z \) and \( p \) and the gain in scalar \( k \). 
If \( n \) is odd, \( z \) is length \( n - 1 \). The transfer function is

\[
H(s) = \frac{z(s)}{p(s)} = k \cdot \frac{(s - \text{z}(1))(s - \text{z}(2)) \cdots (s - \text{z}(n))}{(s - \text{p}(1))(s - \text{p}(2)) \cdots (s - \text{p}(n))}
\]

Elliptic filters are equiripple in both the passband and stopband. They offer 
steeper rolloff characteristics than Butterworth and Chebyshev filters, but 
they are equiripple in both the passband and the stopband. Of the four classical 
filter types, elliptic filters usually meet a given set of filter performance 
specifications with the lowest filter order.

\text{ellip} \quad \text{sets the cutoff frequency } \omega_0 \quad \text{of the elliptic filter to 1 for a normalized} 
\text{result. The cutoff frequency is the frequency at which the passband ends and} 
\text{the filter has a magnitude response of } 10^{-\text{Rp}/20}.

Algorithm

\text{ellipap} \quad \text{uses the algorithm outlined in [1]. It employs the M-file } \text{ellipk} \quad \text{to} 
calculate the complete elliptic integral of the first kind and the M-file } \text{ellipj} \quad \text{to} 
calculate Jacoby elliptic functions.

See Also

\begin{align*}
\text{besselap} & \quad \text{Bessel analog lowpass filter prototype.} \\
\text{buttap} & \quad \text{Butterworth analog lowpass filter prototype.} \\
\text{cheb1ap} & \quad \text{Chebyshev type I analog lowpass filter prototype.} \\
\text{cheb2ap} & \quad \text{Chebyshev type II analog lowpass filter prototype.} \\
\text{ellip} & \quad \text{Elliptic (Cauer) filter design.}
\end{align*}

References

**ellipord**

**Purpose**
Elliptic filter order selection.

**Syntax**

\[ [n, Wn] = ellipord(Wp, Ws, Rp, Rs) \]

\[ [n, Wn] = ellipord(Wp, Ws, Rp, Rs, 's') \]

**Description**

`ellipord` selects the minimum order digital or analog elliptic filter required to meet a set of lowpass filter design specifications:

- **Wp** Passband corner frequency. \( Wp \), the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

- **Ws** Stopband corner frequency. \( Ws \) is in the same units as \( Wp \); it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

- **Rp** Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is \( 0 < w < Wp \).

- **Rs** Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is \( Ws < w < 1 \).

**Digital Domain**

\[ [n, Wn] = ellipord(Wp, Ws, Rp, Rs) \] returns the order \( n \) of the lowest order elliptic filter that loses no more than \( Rp \) dB in the passband and has at least \( Rs \) dB of attenuation in the stopband. The passband runs from 0 to \( Wp \) and the stopband extends from \( Ws \) to 1, the Nyquist frequency. `ellipord` also returns \( Wn \), the cutoff frequency that allows `ellip` to achieve the given specifications.

Use `ellipord` for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, \( Wp \) is greater than \( Ws \). For bandpass and bandstop filters, \( Wp \) and \( Ws \) are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, `ellipord` returns \( Wn \) as a two-element row vector for input to `ellip`.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.
Analog Domain

\[ [n, Wn] = \text{ellipord}(Wp, Ws, Rp, Rs, 's') \]

finds the minimum order \( n \) and cutoff frequencies \( Wn \) for an analog filter. In this case the frequencies in \( Wp \) and \( Ws \) are in radians per second and may be greater than 1.

Use \text{ellipord} for lowpass, highpass, bandpass, and bandstop filters as described under “Digital Domain.”
**Examples**

For 1000 Hz data, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz and at least 15 dB of attenuation from 150 Hz to the Nyquist frequency:

\[
\begin{align*}
W_p &= 100/500; \quad W_s = 150/500; \\
R_p &= 3; \quad R_s = 15; \\
[n,W_n] &= \text{ellipord}(W_p,W_s,R_p,R_s)
\end{align*}
\]

\[
\begin{align*}
n &= 2 \\
W_n &= 0.2000 \\
[b,a] &= \text{ellip}(n,R_p,R_s,W_n); \\
freqz(b,a,512,1000); \\
title('n=2 Elliptic Lowpass Filter')
\end{align*}
\]
Now design a bandpass filter with a passband from 90 Hz to 200 Hz, less than 3 dB of ripple throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

```matlab
Wp = [90 200]/500; Ws = [50 250]/500;
Rp = 3; Rs = 30;
[n,Wn] = ellipord(Wp,Ws,Rp,Rs)

n = 3
Wn =
    0.1800    0.4000

[b,a] = ellip(n,Rp,Rs,Wn);
freqz(b,a,512,1000);
title('n=3 Elliptic Bandpass Filter')
```

Algorithm

ellipord uses the elliptic lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both the analog and digital cases. For the digital case, it converts the frequency
ellipord

parameters to the s-domain before the order and natural frequency estimation process, then converts them back to the z-domain.

ellipord initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.

See Also

- buttord: Butterworth filter order selection.
- cheb1ord: Chebyshev type I filter order selection.
- cheb2ord: Chebyshev type II filter order selection.
- ellip: Elliptic (Cauer) filter design.

References

### Purpose
One-dimensional fast Fourier transform.

### Syntax
- `y = fft(x)`
- `y = fft(x, n)`

### Description
The `fft` function computes the discrete Fourier transform of a vector or matrix. This function implements the transform given by

\[
X(k + 1) = \sum_{n=0}^{N-1} x(n + 1) W_n^{kn}
\]

where \( W_n = e^{i(2\pi/N)} \) and \( N = \text{length}(x) \). Note that the series is indexed as \( n + 1 \) and \( k + 1 \) instead of the usual \( n \) and \( k \) because MATLAB vectors run from 1 to \( N \) instead of from 0 to \( N-1 \).

- `y = fft(x)` is the discrete Fourier transform of vector \( x \), computed with a fast Fourier transform (FFT) algorithm. If \( x \) is a matrix, \( y \) is the FFT of each column of the matrix. The `fft` function employs a radix-2 fast Fourier transform algorithm if the length of the sequence is a power of two, and a slower algorithm if it is not; see the “Algorithm” section for details.

- `y = fft(x, n)` is the \( n \)-point FFT. If the length of \( x \) is less than \( n \), `fft` pads \( x \) with trailing zeros to length \( n \). If the length of \( x \) is greater than \( n \), `fft` truncates the sequence \( x \). If \( x \) is an array, `fft` adjusts the length of the columns in the same manner.

The `fft` function is part of the standard MATLAB language.

### Example
A common use of the Fourier transform is to find the frequency components of a time-domain signal buried in noise. Consider data sampled at 1000 Hz. Form
a signal consisting of 50 Hz and 120 Hz sinusoids and corrupt the signal with zero-mean random noise:

```matlab
    t = 0:0.001:0.6;
    x = sin(2*pi*50*t) + sin(2*pi*120*t);
    y = x + 2*randn(1,length(t));
    plot(y(1:50))
```

It is difficult to identify the frequency components by studying the original signal. Convert to the frequency domain by taking the discrete Fourier transform of the noisy signal $y$ using a 512-point fast Fourier transform (FFT):

```matlab
    Y = fft(y,512);
```

The power spectral density, a measurement of the energy at various frequencies, is

```matlab
    Pyy = Y.*conj(Y) / 512;
```
Graph the first 256 points (the other 256 points are symmetric) on a meaningful frequency axis:

\[
f = 1000 \cdot (0:255)/512;
\]

\[
\text{plot}(f, \text{Pyy}(1:256))
\]

See the \texttt{pwelch} function for details on calculating spectral density.

Sometimes it is useful to normalize the output of \texttt{fft} so that a unit sinusoid in the time domain corresponds to unit amplitude in the frequency domain. To produce a normalized discrete-time Fourier transform in this manner, use

\[
Pn = \frac{\text{abs} (\text{fft}(x)) \cdot 2}{\text{length}(x)}
\]

**Algorithm**

\texttt{fft} is a built-in MATLAB function. When the sequence length is a power of two, \texttt{fft} uses a high-speed radix-2 fast Fourier transform algorithm. The radix-2 FFT routine is optimized to perform a real FFT if the input sequence is purely real; otherwise it computes the complex FFT. This causes a real power-of-two FFT to be about 40\% faster than a complex FFT of the same length.

When the sequence length is not an exact power of two, a separate algorithm finds the prime factors of the sequence length and computes the mixed-radix discrete Fourier transforms of the shorter sequences.
fft

The execution time for fft depends on the sequence length. If the length of a sequence has many prime factors, the function computes the FFT quickly; if the length has few prime factors, execution is slower. For sequences whose lengths are prime numbers, fft uses the raw (and slow) DFT algorithm. For this reason it is usually better to use power-of-two FFTs, if this is supported by your application. For example, on one machine a 4096-point real FFT takes 2.1 seconds and a complex FFT of the same length takes 3.7 seconds. The FFTs of neighboring sequences of length 4095 and 4097, however, take 7 seconds and 58 seconds, respectively.

Suppose a sequence x of N points is obtained at a sample frequency of \( f_s \). Then, for up to the Nyquist frequency, or point \( n = N/2 + 1 \), the relationship between the actual frequency and the index \( k \) into \( x \) (out of \( N \) possible indices) is

\[
f = (k - 1) \times f_s \times \frac{1}{N}
\]

See Also

- dct: Discrete cosine transform (DCT).
- dftmtx: Discrete Fourier transform matrix.
- fft2: Two-dimensional fast Fourier transform.
- fftshift: Rearrange the outputs of fft and fft2.
- filter: Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- ifft: One-dimensional inverse fast Fourier transform.
- pwelch: Estimate the power spectral density (PSD) of a signal using Welch's method.
Purpose  Two-dimensional fast Fourier transform.

Syntax  

\[
Y = \text{fft2}(X) \\
Y = \text{fft2}(X,m,n)
\]

Description  \(Y = \text{fft2}(X)\) performs a two-dimensional FFT, producing a result \(Y\) the same size as \(X\). If \(X\) is a vector, \(Y\) has the same orientation as \(X\).

\(Y = \text{fft2}(X,m,n)\) truncates or zero pads \(X\), if necessary, to create an \(m\)-by-\(n\) array before performing the FFT. The result \(Y\) is also \(m\)-by-\(n\).

The \texttt{fft2} function is part of the standard MATLAB language.

Algorithm  \(\text{fft2}(x)\) is simply

\[
\text{fft}(\text{fft}(x).').'
\]

This computes the one-dimensional \texttt{fft} of each column of \(x\), then of each row of the result. The time required to compute \(\text{fft2}(x)\) depends on the number of prime factors in \([m,n] = \text{size}(x)\). \texttt{fft2} is fastest when \(m\) and \(n\) are powers of 2.

See Also  

\texttt{fft}  One-dimensional fast Fourier transform.
\texttt{fftshift}  Rearrange the outputs of \texttt{fft} and \texttt{fft2}.
\texttt{ifft}  One-dimensional inverse fast Fourier transform.
\texttt{ifft2}  Two-dimensional inverse fast Fourier transform.
**Purpose**

FFT-based FIR filtering using the overlap-add method.

**Syntax**

```matlab
y = fftfilt(b,x)
```

```matlab
y = fftfilt(b,x,n)
```

**Description**

`fftfilt` filters data using the efficient FFT-based method of overlap-add, a frequency domain filtering technique that works only for FIR filters.

```matlab
y = fftfilt(b,x)
```

filters the data in vector `x` with the filter described by coefficient vector `b`. It returns the data vector `y`. The operation performed by `fftfilt` is described in the time domain by the difference equation

\[ y(n) = b(1)x(n) + b(2)x(n-1) + \cdots + b(nb+1)x(n-nb) \]

An equivalent representation is the z-transform or frequency domain description

\[ Y(z) = (b(1) + b(2)z^{-1} + \cdots + b(nb+1)z^{-nb})X(z) \]

By default, `fftfilt` chooses an FFT length and data block length that guarantee efficient execution time.

```matlab
y = fftfilt(b,x,n)
```

uses an FFT length of `nfft = 2^{\text{nextpow2}(n)}` and a data block length of `nfft - \text{length}(b) + 1`.

`fftfilt` works for both real and complex inputs.

**Example**

Show that the results from `fftfilt` and `filter` are identical:

```matlab
b = [1 2 3 4];
x = [1 zeros(1,99)'];
ans =
norm(fftfilt(b,x) - filter(b,1,x))
```

```matlab
ans =
9.5914e-15
```

**Algorithm**

`fftfilt` uses `fft` to implement the overlap-add method [1], a technique that combines successive frequency domain filtered blocks of an input sequence. `fftfilt` breaks an input sequence `x` into length `L` data blocks:

```
x L 2L 3L \cdots \text{ceil}(nx/L)L
```

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and convolves each block with the filter \( b \) by

\[
y = \text{ifft}(\text{fft}(x(i:i+L-1),nfft).*\text{fft}(b,nfft));
\]

where \( nfft \) is the FFT length. \texttt{fftfilt} overlaps successive output sections by \( nb-1 \) points, where \( nb \) is the length of the filter, and sums them:

\[
L \quad nb-1 \\
\overline{2L} \quad nb-1 \\
\overline{3L} \quad nb-1 \\
\ldots
\]

\texttt{fftfilt} chooses the key parameters \( L \) and \( nfft \) in different ways, depending on whether you supply an FFT length \( n \) and on the lengths of the filter and signal. If you do not specify a value for \( n \) (which determines FFT length), \texttt{fftfilt} chooses these key parameters automatically:

- If \( \text{length}(x) > \text{length}(b) \), \texttt{fftfilt} chooses values that minimize the number of blocks times the number of flops per FFT.
- If \( \text{length}(b) \geq \text{length}(x) \), \texttt{fftfilt} uses a single FFT of length

\[
2^{\text{nextpow2}}(\text{length}(b) + \text{length}(x) - 1)
\]

This essentially computes

\[
y = \text{ifft}(\text{fft}(B,nfft).*\text{fft}(X,nfft))
\]

If you supply a value for \( n \), \texttt{fftfilt} chooses an FFT length, \( nfft \), of \( 2^{\text{nextpow2}}(n) \) and a data block length of \( nfft - \text{length}(b) + 1 \). If \( n \) is less than \( \text{length}(b) \), \texttt{fftfilt} sets \( n \) to \( \text{length}(b) \).

### See Also

- \texttt{conv} Convolution and polynomial multiplication.
- \texttt{filter} Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- \texttt{filtfilt} Zero-phase digital filtering.

### References

Purpose
Rearrange the outputs of the FFT functions.

Syntax

\[ y = \text{fftshift}(x) \]

Description
\[ y = \text{fftshift}(x) \] rearranges the outputs of \texttt{fft} and \texttt{fft2} by moving the zero frequency component to the center of the spectrum, which is sometimes a more convenient form.

For vectors, \texttt{fftshift(x)} returns a vector with the left and right halves swapped.

For arrays, \texttt{fftshift(x)} swaps quadrants one and three with quadrants two and four.

The \texttt{fftshift} function is part of the standard MATLAB language.

Example
For any array \( X \),

\[
Y = \text{fft2}(x)
\]

has \( Y(1,1) = \text{sum(sum}(X)) \); the DC component of the signal is in the upper-left corner of the two-dimensional FFT. For

\[
Z = \text{fftshift}(Y)
\]

the DC component is near the center of the matrix.

See Also
- \texttt{fft} One-dimensional fast Fourier transform.
- \texttt{fft2} Two-dimensional fast Fourier transform.
Purpose
Filter data with a recursive (IIR) or nonrecursive (FIR) filter.

Syntax
\[ y = \text{filter}(b,a,x) \]
\[ [y,zf] = \text{filter}(b,a,x) \]
\[ [...] = \text{filter}(b,a,x,zi) \]
\[ [...] = \text{filter}(b,a,x,zi,dim) \]

Description
filter filters real or complex data using a digital filter. The filter realization is the transposed direct form II structure\(^1\), which can handle both FIR and IIR filters.

If \( a(1) \neq 1 \), filter normalizes the filter coefficients by \( a(1) \). If \( a(1) = 0 \), the input is in error.

\[ y = \text{filter}(b,a,x) \] filters the data in vector \( x \) with the filter described by coefficient vectors \( a \) and \( b \) to create the filtered data vector \( y \). When \( x \) is a matrix, filter operates on the columns of \( x \). When \( x \) is an \( N \)-dimensional array, filter operates on the first non-singleton dimension.

\[ [y,zf] = \text{filter}(b,a,x) \] returns the final values of the states in the vector \( zf \).

\[ [...] = \text{filter}(b,a,x,zi) \] specifies initial state conditions in the vector \( zi \).

The size of the initial/final condition vector is \( \max(\text{length}(b),\text{length}(a))-1 \). \( zi \) or \( zf \) can also be an array of such vectors, one for each column of \( x \) if \( x \) is a matrix. If \( x \) is a multidimensional array, filter works across the first nonsingleton dimension of \( x \) by default.

\[ [...] = \text{filter}(b,a,x,zi,dim) \] works across the dimension \( dim \) of \( x \). Set \( zi \) to empty to get the default initial conditions.

The \text{filter} function is part of the standard MATLAB language.
Example

Find and graph the 100-point unit impulse response of a digital filter:

```matlab
x = [1 zeros(1,100)];
[b,a] = butter(12,400/1000);
y = filter(b,a,x);
stem(y)
```

Algorithm

filter is implemented as a transposed direct form II structure

where \( n-1 \) is the filter order.
The operation of filter at sample \( m \) is given by the time domain difference equations for \( y \) and the states \( z_i \):

\[
\begin{align*}
y(m) &= b(1)x(m) + z_1(m - 1) - a(1)y(m) \\
z_1(m) &= b(2)x(m) + z_2(m - 1) - a(2)y(m) \\
& \quad \vdots \\
z_{n-2}(m) &= b(n-1)x(m) + z_{n-1}(m - 1) - a(n-1)y(m) \\
z_{n-1}(m) &= b(n)x(m) - a(n)y(m)
\end{align*}
\]

You can use \texttt{filtic} to generate the state vector \( z_i(0) \) from past inputs and outputs.

The input-output description of this filtering operation in the \( z \)-transform domain is a rational transfer function:

\[
Y(z) = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na+1)z^{-na}} X(z)
\]

**Diagnostics**

If \( a(1) = 0 \), filter gives the following error message:

First denominator coefficient must be nonzero.

If the length of the initial condition vector is not the greater of \( na \) and \( nb \), filter gives the following error message:

Initial condition vector has incorrect dimensions.

**See Also**

- \texttt{fftfilt} FFT-based FIR filtering using the overlap-add method.
- \texttt{filter2} Two-dimensional digital filtering.
- \texttt{filtfilt} Zero-phase digital filtering.
- \texttt{filtic} Make initial conditions for filter function.

**References**

Two-dimensional digital filtering.

Y = filter2(B,X)
Y = filter2(B,X,'shape')

Y = filter2(B,X) filters the two-dimensional data in X with the two-dimensional FIR filter in the matrix B. The result, Y, is computed using two-dimensional convolution and is the same size as X.

Y = filter2(B,X,'shape') returns Y computed with size specified by shape:

• same returns the central part of the convolution that is the same size as X (default).
• full returns the full two-dimensional convolution, size(Y) > size(X).
• valid returns only those parts of the convolution that are computed without the zero-padded edges, size(Y) < size(X).

The filter2 function is part of the standard MATLAB language.

The filter2 function uses conv2 to compute the full two-dimensional convolution of the FIR filter with the input matrix. By default, filter2 extracts and returns the central part of the convolution that is the same size as the input matrix. Use the shape parameter to specify an alternate part of the convolution for return.

conv2 Two-dimensional convolution.
filter Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
Purpose
Zero-phase digital filtering.

Syntax
y = filtfilt(b,a,x)

Description
y = filtfilt(b,a,x) performs zero-phase digital filtering by processing the input data in both the forward and reverse directions (see problem 5.39 in [1]). After filtering in the forward direction, it reverses the filtered sequence and runs it back through the filter. The resulting sequence has precisely zero-phase distortion and double the filter order. filtfilt minimizes start-up and ending transients by matching initial conditions, and works for both real and complex inputs.

Note that filtfilt should not be used with differentiator and Hilbert FIR filters, since the operation of these filters depends heavily on their phase response.

Algorithm
filtfilt is an M-file that uses the filter function. In addition to the forward-reverse filtering, it attempts to minimize startup transients by adjusting initial conditions to match the DC component of the signal and by prepending several filter lengths of a flipped, reflected copy of the input signal.

See Also
fftfilt FFT-based FIR filtering using the overlap-add method.
filter Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
filter2 Two-dimensional digital filtering.

References
**Purpose**
Find initial conditions for a transposed direct form II filter implementation.

**Syntax**

\[
z = \text{filtic}(b, a, y, x) \\
z = \text{filtic}(b, a, y)
\]

**Description**

\[
z = \text{filtic}(b, a, y, x) \text{ finds the initial conditions } z \text{ for the delays in the transposed direct form II filter implementation given past outputs } y \text{ and inputs } x. \text{ The vectors } b \text{ and } a \text{ represent the numerator and denominator coefficients, respectively, of the filter's transfer function.}
\]

The vectors \(x\) and \(y\) contain the most recent input or output first, and oldest input or output last:

\[
x = \{x(-1), x(-2), x(-3), \ldots, x(-nb), \ldots\} \\
y = \{y(-1), y(-2), y(-3), \ldots, y(-na), \ldots\}
\]

where \(nb\) is \(\text{length}(b)-1\) (the numerator order) and \(na\) is \(\text{length}(a)-1\) (the denominator order). If \(\text{length}(x)\) is less than \(nb\), \text{filtic} pads it with zeros to length \(nb\); if \(\text{length}(y)\) is less than \(na\), \text{filtic} pads it with zeros to length \(na\). Elements of \(x\) beyond \(x(nb-1)\) and elements of \(y\) beyond \(y(na-1)\) are unnecessary so \text{filtic} ignores them.

Output \(z\) is a column vector of length equal to the larger of \(nb\) and \(na\). \(z\) describes the state of the delays given past inputs \(x\) and past outputs \(y\).

\[
z = \text{filtic}(b, a, y) \text{ assumes that the input } x \text{ is 0 in the past.}
\]

The transposed direct form II structure is

![Transposed Direct Form II Filter](image)

where \(n-1\) is the filter order.

\text{filtic} works for both real and complex inputs.
Algorithm  filtic performs a reverse difference equation to obtain the delay states \( z \).

Diagnostics  If any of the input arguments \( y \), \( x \), \( b \), or \( a \) is not a vector (that is, if any argument is a scalar or array), filtic gives the following error message:

   Requires vector inputs.

See Also  filter  Filter data with a recursive (IIR) or nonrecursive (FIR) filter.

   filtfilt  Zero-phase digital filtering.

Purpose
Window-based finite impulse response filter design – standard response.

Syntax
- `b = fir1(n,Wn)`
- `b = fir1(n,Wn,'ftype')`  
- `b = fir1(n,Wn,window)`
- `b = fir1(n,Wn,'ftype',window)`
- `b = fir1(...,'noscale')`

Description
`fir1` implements the classical method of windowed linear-phase FIR digital filter design [1]. It designs filters in standard lowpass, bandpass, highpass, and bandpass configurations. (For windowed filters with arbitrary frequency response, use `fir2`.)

`b = fir1(n,Wn)` returns row vector `b` containing the `n+1` coefficients of an order `n` lowpass FIR filter. This is a Hamming-windowed, linear-phase filter with cutoff frequency `Wn`. The output filter coefficients, `b`, are ordered in descending powers of `z`:

\[ B(z) = b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n} \]

`Wn`, the cutoff frequency, is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

If `Wn` is a two-element vector, `Wn = [w1 w2]`, `fir1` returns a bandpass filter with passband `w1 < \omega < w2`.

If `Wn` is a multi-element vector, `Wn = [w1 w2 w3 w4 w5 ... wn]`, `fir1` returns an order `n` multiband filter with bands `0 < \omega < w1, w1 < \omega < w2, ..., wn < \omega < 1`.

By default, the filter is scaled so that the center of the first passband has magnitude exactly 1 after windowing.

`b = fir1(n,Wn,'ftype')` specifies a filter type, where `ftype` is:

- `high` for a highpass filter with cutoff frequency `Wn`
- `stop` for a bandstop filter, if `Wn = [w1 w2]`
  - The stopband is `w1 < \omega < w2`.
- `'DC-1'` to make the first band of a multiband filter a passband
- `'DC-0'` to make the first band of a multiband filter a stopband


**Algorithm**

fir1 uses the window method of FIR filter design [1]. If \( w(n) \) denotes a window, where \( 1 \leq n \leq N \), and the impulse response of the ideal filter is \( h(n) \), where \( h(n) \) is the inverse Fourier transform of the ideal frequency response, then the windowed digital filter coefficients are given by

\[
b(n) = w(n)h(n), \quad 1 \leq n \leq N
\]
**Examples**

**Example 1**
Design a 48th-order FIR bandpass filter with passband $0.35 \leq w \leq 0.65$:

```matlab
b = fir1(48,[0.35 0.65]);
freqz(b,1,512)
```

**Example 2**
The `chirp.mat` file contains a signal, $y$, that has most of its power above $F_s/4$, or half the Nyquist frequency. Design a 34th-order FIR highpass filter to attenuate the components of the signal below $F_s/4$. Use a cutoff frequency of 0.48 and a Chebyshev window with 30 dB of ripple:

```matlab
load chirp % loads y and Fs
b = fir1(34,0.48,'high',chebwin(35,30));
yfilt = filter(b,1,y);
```
Check the spectra of the original and filtered signals.

\[
[Py,fy] = \text{pburg}(y,10,512,Fs);
[Pyfilt,fyfilt] = \text{pburg}(yfilt,10,512,Fs);
\]
\[
\text{plot}(fy,10*\text{log10}(Py),':',fyfilt,10*\text{log10}(Pyfilt)); \text{grid}
\]
\[
ylabel('Magnitude (dB)'); \text{xlabel('Frequency (Hz)')};
\]
\[
\text{legend('Chirp','Filtered Chirp')}
\]

Play the two signals using \text{sound}.

\[
\text{sound}(y,Fs);
\]
\[
\text{sound}(yfilt,Fs);
\]

\textbf{Diagnostics} If \text{n} is odd and you specify a bandstop or highpass filter, \text{fir1} gives the following warning message:

For highpass and bandstop filters, \text{N} must be even. Order is being increased by 1.
### See Also

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### References

**Purpose**

Window-based finite impulse response filter design – arbitrary response.

**Syntax**

```
b = fir2(n,f,m)
b = fir2(n,f,m,window)
b = fir2(n,f,m,npt)
b = fir2(n,f,m,npt,window)
b = fir2(n,f,m,npt,lap)
b = fir2(n,f,m,npt,lap,window)
```

**Description**

`fir2` designs windowed digital FIR filters with arbitrarily shaped frequency response. (For standard lowpass, bandpass, highpass, and bandstop configurations, use `fir1`.)

`b = fir2(n,f,m)` returns row vector `b` containing the \( n+1 \) coefficients of an order \( n \) FIR filter. The frequency-magnitude characteristics of this filter match those given by vectors `f` and `m`:

- `f` is a vector of frequency points in the range from 0 to 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The first point of `f` must be 0 and the last point 1. The frequency points must be in increasing order.
- `m` is a vector containing the desired magnitude response at the points specified in `f`.
- `f` and `m` must be the same length.
- Duplicate frequency points are allowed, corresponding to steps in the frequency response.

Use `plot(f,m)` to view the filter shape.

The output filter coefficients, `b`, are ordered in descending powers of \( z \):

```
b(z) = b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}
```

`b = fir2(n,f,m,window)` uses the window specified in column vector `window` for the filter design. The vector `window` must be \( n+1 \) elements long. If no window is specified, `fir2` employs a Hamming window.
b = fir2(n,f,m,npt) and
b = fir2(n,f,m,npt, window) specify the number of points npt for the grid
onto which fir2 interpolates the frequency response, with or without a window
specification.

b = fir2(n,f,m,npt,lap) and
b = fir2(n,f,m,npt,lap,window) specify the size of the region, lap, that
fir2 inserts around duplicate frequency points, with or without a window
specification.

See the “Algorithm” section for more on npt and lap.

Algorithm

The desired frequency response is interpolated onto a dense, evenly spaced grid
of length npt. npt is 512 by default. If two successive values of f are the same,
a region of lap points is set up around this frequency to provide a smooth but
steep transition in the requested frequency response. By default, lap is 25. The
filter coefficients are obtained by applying an inverse fast Fourier transform to
the grid and multiplying by a window; by default, this is a Hamming window.
**Example**

Design a 30th-order lowpass filter and overplot the desired frequency response with the actual frequency response:

```matlab
f = [0 0.6 0.6 1]; m = [1 1 0 0];
b = fir2(30,f,m);
[h,w] = freqz(b,1,128);
plot(f,m,w/pi,abs(h))
```

**See Also**

- **butter**: Butterworth analog and digital filter design.
- **cheby1**: Chebyshev type I filter design (passband ripple).
- **cheby2**: Chebyshev type II filter design (stopband ripple).
- **ellip**: Elliptic (Cauer) filter design.
- **fir1**: Window-based finite impulse response filter design - standard response.
- **maxflat**: Generalized digital Butterworth filter design.
- **remez**: Parks-McClellan optimal FIR filter design.
- **yulewalk**: Recursive digital filter design.
**Purpose**
Constrained least square FIR filter design for multiband filters.

**Syntax**
```
b = fircls(n,f,amp,up,lo)
fircls(n,f,amp,up,lo,'design_flag')
```

**Description**
`b = fircls(n,f,amp,up,lo)` generates a length \( n+1 \) linear phase FIR filter \( b \). The frequency-magnitude characteristics of this filter match those given by vectors \( f \) and \( amp \):

- \( f \) is a vector of transition frequencies in the range from 0 to 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The first point of \( f \) must be 0 and the last point 1. The frequency points must be in increasing order.
- \( amp \) is a vector describing the piecewise constant desired amplitude of the frequency response. The length of \( amp \) is equal to the number of bands in the response and should be equal to \( \text{length}(f)-1 \).
- \( up \) and \( lo \) are vectors with the same length as \( amp \). They define the upper and lower bounds for the frequency response in each band.

`fircls(n,f,amp,up,lo,'design_flag')` enables you to monitor the filter design, where `design_flag` can be:

- `trace`, for a textual display of the design error at each iteration step.
- `plots`, for a collection of plots showing the filter's full-band magnitude response and a zoomed view of the magnitude response in each sub-band. All plots are updated at each iteration step.
- `both`, for both the textual display and plots.
Example

Design an order 50 bandpass filter:

```matlab
n = 50;
f = [0 0.4 0.8 1];
amp = [0 1 0];
up = [0.02 1.02 0.01];
lo = [-0.02 0.98 -0.01];
b = fircls(n,f,amp,up,lo,'plots'); % plots magnitude response
```

NOTE

Normally, the lower value in the stopband will be specified as negative. By setting `lo` equal to 0 in the stopbands, a nonnegative frequency response amplitude can be obtained. Such filters can be spectrally factored to obtain minimum phase filters.

Algorithm

The algorithm is a multiple exchange algorithm that uses Lagrange multipliers and Kuhn-Tucker conditions on each iteration.
See Also

- `fircls1`: Constrained least square filter design for lowpass and highpass linear phase FIR filters.
- `firls`: Least square linear-phase FIR filter design.
- `remez`: Parks-McClellan optimal FIR filter design.

References


**Purpose**  
Constrained least square filter design for lowpass and highpass linear phase FIR filters.

**Syntax**

```
b = fircls1(n,wo,dp,ds)
b = fircls1(n,wo,dp,ds,'high')
b = fircls1(n,wo,dp,ds,wt)
b = fircls1(n,wo,dp,ds,wt,'high')
b = fircls1(n,wo,dp,ds,wp,ws,k)
b = fircls1(n,wo,dp,ds,wp,ws,k,'high')
b = fircls1(n,wo,dp,ds,...,'design_flag')
```

**Description**

`b = fircls1(n,wo,dp,ds)` generates a lowpass FIR filter `b`. `n+1` is the filter length, `wo` is the normalized cutoff frequency in the range between 0 and 1 (where 1 corresponds to half the sampling frequency, that is, the Nyquist frequency), `dp` is the maximum passband deviation from 1 (passband ripple), and `ds` is the maximum stopband deviation from 0 (stopband ripple).

`b = fircls1(n,wo,dp,ds,'high')` generates a highpass FIR filter `b`.

`b = fircls1(n,wo,dp,ds,wt)` and  
`b = fircls1(n,wo,dp,ds,wt,'high')` specify a frequency `wt` above which (for `wt>wo`) or below which (for `wt<wo`) the filter is guaranteed to meet the given band criterion. This will help you design a filter that meets a passband or stopband edge requirement. There are four cases:

- **Lowpass**:
  - `0<wt<wo<1`: the amplitude of the filter is within `dp` of 1 over the frequency range `0<ω<wt`.
  - `0<wo<wt<1`: the amplitude of the filter is within `ds` of 0 over the frequency range `wt<ω<1`.

- **Highpass**:
  - `0<wt<wo<1`: the amplitude of the filter is within `ds` of 0 over the frequency range `0<ω<wt`.
  - `0<wo<wt<1`: the amplitude of the filter is within `dp` of 1 over the frequency range `wt<ω<1`.

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\[
b = \text{fircls1}(n, wo, dp, ds, wp, ws, k)
\]
generates a lowpass FIR filter \(b\) with a weighted function. \(n\) is the filter length, \(wo\) is the normalized cutoff frequency, \(dp\) is the maximum passband deviation from 1 (passband ripple), and \(ds\) is the maximum stopband deviation from 0 (stopband ripple). \(wp\) is the passband edge of the L2 weight function, and \(ws\) is the stopband edge of the L2 weight function, where \(wp < wo < ws\).

\[
b = \text{fircls1}(n, wo, dp, ds, wp, ws, k, \text{'high'})
\]
generates a highpass FIR filter \(b\) with a weighted function, where \(ws < wo < wp\).

\[
b = \text{fircls1}(n, wo, dp, ds, \ldots, \text{design_flag})
\]
enables you to monitor the filter design, where \(\text{design_flag}\) can be:
- \text{trace}, for a textual display of the design table used in the design
- \text{plots}, for plots of the filter's magnitude, group delay, and zeros and poles
- both, for both the textual display and plots

**NOTE**

In the design of very narrow band filters with small \(dp\) and \(ds\), there may not exist a filter of the given length that meets the specifications.
**Example**

Design an order 55 lowpass filter with a cutoff frequency at 0.3:

\[
\begin{align*}
n &= 55; \ wo &= 0.3; \\
dp &= 0.02; \ ds &= 0.008; \\
b &= \text{fircls1}(n, wo, dp, ds, 'plots'); \ % \text{plot magnitude response}
\end{align*}
\]

Algorithm

The algorithm is a multiple exchange algorithm that uses Lagrange multipliers and Kuhn-Tucker conditions on each iteration.

See Also

- **fircls**: Constrained least square FIR filter design for multiband filters.
- **firls**: Least square linear-phase FIR filter design.
- **remez**: Parks-McClellan optimal FIR filter design.

References


**firls**

**Purpose**
Least square linear-phase FIR filter design.

**Syntax**

```matlab
b = firls(n,f,a)
b = firls(n,f,a,w)
b = firls(n,f,a,'ftype')
b = firls(n,f,a,w,'ftype')
```

**Description**

firls designs a linear-phase FIR filter that minimizes the weighted, integrated squared error between an ideal piecewise linear function and the magnitude response of the filter over a set of desired frequency bands.

\[
b = firls(n,f,a) \text{ returns row vector } b \text{ containing the } n+1 \text{ coefficients of the order } n \text{ FIR filter whose frequency-amplitude characteristics approximately match those given by vectors } f \text{ and } a. \text{ The output filter coefficients, or “taps,” in } b \text{ obey the symmetry relation}
\]

\[
b(k) = b(n + 2 - k), \quad k = 1, \ldots, n + 1
\]

These are type I (n odd) and type II (n even) linear-phase filters. Vectors \( f \) and \( a \) specify the frequency-amplitude characteristics of the filter:

- \( f \) is a vector of pairs of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order. Duplicate frequency points are allowed and, in fact, can be used to design a filter exactly the same as those returned by the fir1 and fir2 functions with a rectangular or boxcar window.
- \( a \) is a vector containing the desired amplitude at the points specified in \( f \).

The desired amplitude function at frequencies between pairs of points \( (f(k), f(k+1)) \) for \( k \) odd is the line segment connecting the points \( (f(k), a(k)) \) and \( (f(k+1), a(k+1)) \).

The desired amplitude function at frequencies between pairs of points \( (f(k), f(k+1)) \) for \( k \) even is unspecified. These are transition or “don’t care” regions.

- \( f \) and \( a \) are the same length. This length must be an even number.
The relationship between the `f` and `a` vectors in defining a desired amplitude response is

\[ b = \text{firls}(n,f,a,w) \]

uses the weights in vector `w` to weight the fit in each frequency band. The length of `w` is half the length of `f` and `a`, so there is exactly one weight per band.

\[ b = \text{firls}(n,f,a,{'\texttt{ftype}'}) \]

and

\[ b = \text{firls}(n,f,a,w,{'\texttt{ftype}'}) \]

specify a filter type, where `ftype` is

- **hilbert** for linear-phase filters with odd symmetry (type III and type IV)
  
  The output coefficients in `b` obey the relation \( b(k) = -b(n + 2 - k) \), \( k = 1, ..., n + 1 \). This class of filters includes the Hilbert transformer, which has a desired amplitude of \( 1 \) across the entire band.

- **differentiator** for type III and type IV filters, using a special weighting technique

  For nonzero amplitude bands, the integrated squared error has a weight of \((1/f)^2\) so that the error at low frequencies is much smaller than at high frequencies. For FIR differentiators, which have an amplitude characteristic proportional to frequency, the filters minimize the relative integrated squared error (the integral of the square of the ratio of the error to the desired amplitude).
**Examples**

Design an order 255 lowpass filter with transition band:

```matlab
b = firls(255,[0 0.25 0.3 1],[1 1 0 0]);
```

Design a 31 coefficient differentiator:

```matlab
b = firls(30,[0 0.9],[0 0.9],'differentiator');
```

Design a 24th-order anti-symmetric filter with piecewise linear passbands and plot the desired and actual frequency response:

```matlab
F = [0 0.3 0.4 0.6 0.7 0.9];
A = [0 1 0 0 0.5 0.5];
b = firls(24,F,A,'hilbert');
for i=1:2:6,
    plot([F(i) F(i+1)],[A(i) A(i+1)],'- -'), hold on
end
[H,f] = freqz(b,1,512,2);
plot(f,abs(H)), grid on, hold off
```

**Algorithm**

Reference [1] describes the theoretical approach that `firls` takes. The function solves a system of linear equations involving an inner product matrix of size roughly \( n/2 \) using MATLAB's \( \backslash \) operator.
firls

This function designs type I, II, III, and IV linear-phase filters. Type I and II are the defaults for \( n \) even and odd respectively, while the 'hilbert' and 'differentiator' flags produce type III (\( n \) even) and IV (\( n \) odd) filters. The various filter types have different symmetries and constraints on their frequency responses (see [2] for details).

<table>
<thead>
<tr>
<th>Linear Phase Filter Type</th>
<th>Filter Order ( n )</th>
<th>Symmetry of Coefficients</th>
<th>Response ( H(f) ), ( f = 0 )</th>
<th>Response ( H(f) ), ( f = 1 ) (Nyquist)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>Even</td>
<td>even: ( b(k) = b(n + 2 - k) ), ( k = 1, \ldots, n + 1 )</td>
<td>No restriction</td>
<td>No restriction</td>
</tr>
<tr>
<td>Type II</td>
<td>Odd</td>
<td>No restriction</td>
<td>( H(1) = 0 )</td>
<td></td>
</tr>
<tr>
<td>Type III</td>
<td>Even</td>
<td>odd: ( b(k) = -b(n + 2 - k) ), ( k = 1, \ldots, n + 1 )</td>
<td>( H(0) = 0 )</td>
<td>( H(1) = 0 )</td>
</tr>
<tr>
<td>Type IV</td>
<td>Odd</td>
<td>( H(0) = 0 )</td>
<td></td>
<td>No restriction</td>
</tr>
</tbody>
</table>

**Diagnostics**

An appropriate diagnostic message is displayed when incorrect arguments are used:

- \( F \) must be even length.
- \( F \) and \( A \) must be equal lengths.
- Requires symmetry to be 'hilbert' or 'differentiator'.
- Requires one weight per band.
- Frequencies in \( F \) must be nondecreasing.
- Frequencies in \( F \) must be in range [0,1].

A more serious warning message is

Warning: Matrix is close to singular or badly scaled.

This tends to happen when the filter length times the transition width grows large. In this case, the filter coefficients \( b \) might not represent the desired filter. You can check the filter by looking at its frequency response.
See Also

- **fir1**  
  Window-based finite impulse response filter design – standard response.

- **fir2**  
  Window-based finite impulse response filter design – arbitrary response.

- **firrcos**  
  Raised cosine FIR filter design.

- **remez**  
  Parks-McClellan optimal FIR filter design.

References


**Purpose**

Raised cosine FIR filter design.

\[
b = \text{firrcos}(n,F0,df,Fs)
\]

\[
b = \text{firrcos}(n,F0,df)
\]

\[
b = \text{firrcos}(n,F0,r,Fs,'rolloff')
\]

\[
b = \text{firrcos}(...,'type')
\]

\[
b = \text{firrcos}(...,'type',delay)
\]

\[
b = \text{firrcos}(...,'type',delay,\text{window})
\]

\[
[b,a] = \text{firrcos}(...)
\]

**Description**

\text{firrcos}(n,F0,df,Fs)\) returns an order \(n\) lowpass linear-phase FIR filter with a raised cosine transition band. The filter has cutoff frequency \(F0\), transition bandwidth \(df\), and sampling frequency \(Fs\), all in Hertz. \(df\) must be small enough so that \(F0 \pm \frac{df}{2}\) is between 0 and \(Fs/2\). The coefficients in \(b\) are normalized so that the nominal passband gain is always equal to one.

\text{firrcos}(n,F0,df)\) uses a default sampling frequency of \(Fs = 2\).

\(b = \text{firrcos}(n,F0,r,Fs,'rolloff')\) interprets the third argument, \(r\), as the rolloff factor instead of the transition bandwidth, \(df\). \(r\) must be in the range \([0,1]\).

\(b = \text{firrcos}(...,'type')\) designs either a normal raised cosine filter or a square root raised cosine filter depending on the \textit{type} specification, which can be

- \textit{normal}, for a regular raised cosine filter. This is the default, and is also in effect when the \textit{type} argument is left empty, \([]\).
- \textit{sqrt}, for a square root raised cosine filter.

\(b = \text{firrcos}(...,'type',delay)\) specifies an integer delay in the range \([0,n+1]\). The default is \(n/2\) for even \(n\) and \((n+1)/2\) for odd \(n\).

\(b = \text{firrcos}(...,'type',delay,\text{window})\) applies a length \(n+1\) window to the designed filter to reduce the ripple in the frequency response. \text{window} must be a \(n+1\) long column vector. If no window is specified, a boxcar (rectangular) window is used. Care must be exercised when using a window with a delay other than the default.

\([b,a] = \text{firrcos}(...))\) always returns \(a = 1\).
Design an order 20 raised cosine FIR filter with cutoff frequency 0.25 of the Nyquist frequency and a transition bandwidth of 0.25:

```matlab
h = firrcos(20,0.25,0.25);
freqz(h,1,'Fs',2,'phase','no')
```

See Also

- **fir1**: Window-based finite impulse response filter design – standard response.
- **fir2**: Window-based finite impulse response filter design – arbitrary response.
- **firls**: Least square linear-phase FIR filter design.
- **remez**: Parks-McClellan optimal FIR filter design.
**Purpose**

Frequency response of analog filters.

**Syntax**

\[
\begin{align*}
    h &= \text{freqs}(b,a,w) \\
    [h,w] &= \text{freqs}(b,a) \\
    [h,w] &= \text{freqs}(b,a,n) \\
    \text{freqs}(b,a)
\end{align*}
\]

**Description**

freqs returns the complex frequency response \( H(jw) \) (Laplace transform) of an analog filter:

\[
H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^{nb} + b(2)s^{nb-1} + \cdots + b(nb+1)}{a(1)s^{na} + a(2)s^{na-1} + \cdots + a(na+1)}
\]

given the numerator and denominator coefficients in vectors \( b \) and \( a \).

\( h = \text{freqs}(b,a,w) \) returns the complex frequency response of the analog filter specified by coefficient vectors \( b \) and \( a \). freqs evaluates the frequency response along the imaginary axis in the complex plane at the frequencies specified in real vector \( w \).

\( [h,w] = \text{freqs}(b,a) \) automatically picks a set of 200 frequency points \( w \) on which to compute the frequency response \( h \).

\( [h,w] = \text{freqs}(b,a,n) \) picks \( n \) frequencies on which to compute the frequency response \( h \).

freqs with no output arguments plots the magnitude and phase response versus frequency in the current figure window.

freqs works only for real input systems and positive frequencies.
**Example**

Find and graph the frequency response of the transfer function given by

\[
H(s) = \frac{0.2s^2 + 0.3s + 1}{s^2 + 0.4s + 1}
\]

```matlab
a = [1 0.4 1];
b = [0.2 0.3 1];
w = logspace(-1,1);
freqs(b,a,w)
```

You can also create the plot with

```matlab
h = freqs(b,a,w);
mag = abs(h);
phase = angle(h);
subplot(2,1,1), loglog(w,mag)
subplot(2,1,2), semilogx(w,phase)
```

To convert to Hertz, degrees, and decibels, use

```matlab
f = w/(2*pi);
mag = 20*log10(mag);
phase = phase*180/pi;
```
**Algorithm**

freqs evaluates the polynomials at each frequency point, then divides the numerator response by the denominator response:

```matlab
s = i*w;
h = polyval(b,s)./polyval(a,s);
```

**See Also**

- **abs** - Absolute value (magnitude).
- **angle** - Phase angle.
- **freqz** - Frequency response of digital filters.
- **invfreqs** - Continuous-time (analog) filter identification from frequency data.
- **logspace** - Generate logarithmically spaced vectors (see the online MATLAB Function Reference).
- **polyval** - Polynomial evaluation (see the online MATLAB Function Reference).
freqspace

Purpose

Frequency spacing for frequency response.

Syntax

- `f = freqspace(n)`
- `f = freqspace(n,'whole')`  
- `[f1,f2] = freqspace(n)`  
- `[f1,f2] = freqspace([m n])`  
- `[x1,y1] = freqspace(n,'meshgrid')`  
- `[x1,y1] = freqspace([m n],'meshgrid')`

Description

freqspace returns the implied frequency range for equally spaced frequency responses. This is useful when creating frequency vectors for use with `freqz`.

- `f = freqspace(n)` returns the frequency vector `f` assuming `n` evenly spaced points around the unit circle. For `n` even or odd, `f` is `(0:2/n:1)`. For `n` even, `freqspace` returns `(n + 2)/2` points. For `n` odd, it returns `(n + 1)/2` points.

- `f = freqspace(n,'whole')` returns `n` evenly spaced points around the whole unit circle. In this case, `f` is `0:2/n:2*(n–1)/n`.

- `[f1,f2] = freqspace(n)` returns the two-dimensional frequency vectors `f1` and `f2` for an `n`-by-`n` matrix. For `n` odd, both `f1` and `f2` are `[-1 + 1/n:2/n:1–1/n]`. For `n` even, both `f1` and `f2` are `[-1:2/n:1–2/n]`.

- `[f1,f2] = freqspace([m n])` returns the two-dimensional frequency vectors `f1` and `f2` for an `m`-by-`n` matrix.

- `[x1,y1] = freqspace(n,'meshgrid')` and `[x1,y1] = freqspace([m n],'meshgrid')` are equivalent to

  - `f1,f2 = freqspace(...);`
  - `[x1,y1] = meshgrid(f1,f2);`

See the online MATLAB Function Reference for details on the `meshgrid` function.

See Also

- `freqz` Frequency response of digital filters.
- `invfreqz` Discrete-time filter identification from frequency data.
**Purpose**
Frequency response of digital filters.

**Syntax**
- \([h,w] = \text{freqz}(b,a,n)\)
- \([h,f] = \text{freqz}(b,a,n,Fs)\)
- \([h,w] = \text{freqz}(b,a,n,'whole')\)
- \([h,f] = \text{freqz}(b,a,n,'whole',Fs)\)
- \(h = \text{freqz}(b,a,w)\)
- \(h = \text{freqz}(b,a,f,Fs)\)
- \(\text{freqz}(...)\)

**Description**
\text{freqz} returns the complex frequency response \(H(e^{j\omega})\) of a digital filter, given the (real or complex) numerator and denominator coefficients in vectors \(b\) and \(a\).

\([h,w] = \text{freqz}(b,a,n)\) returns the \(n\)-point complex frequency response of the digital filter

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na}}
\]
evaluated at \(z = e^{j\omega}\) given the coefficient vectors \(b\) and \(a\). \text{freqz} returns both \(h\), the complex frequency response, and \(w\), a vector containing the \(n\) frequency points in units of rads/sample. \text{freqz} evaluates the frequency response at \(n\) points equally spaced around the upper half of the unit circle, so \(w\) contains \(n\) points between 0 and \(\pi\).

It is best, although not necessary, to choose a value for \(n\) that is an exact power of two, because this allows fast computation using an FFT algorithm. If you do not specify a value for \(n\), it defaults to 512.

\([h,f] = \text{freqz}(b,a,n,Fs)\) specifies a positive sampling frequency \(Fs\), in Hertz. The default for \(Fs\) is 1. It returns a vector \(f\) containing the actual frequency points between 0 and \(Fs/2\) (the Nyquist frequency) at which it calculated the frequency response. \(f\) is of length \(n\).

\([h,w] = \text{freqz}(b,a,n,'whole')\) uses \(n\) points around the whole unit circle, so \(w\) has range \([0,2\pi)\).

\([h,f] = \text{freqz}(b,a,n,'whole',Fs)\) uses \(n\) points around the whole unit circle, so \(f\) has range \([0,Fs)\).
h = freqz(b,a,w) returns the frequency response at the frequencies in vector w (specified in rads/sample).

h = freqz(b,a,f,Fs) returns the frequency response at the frequencies in vector f (specified in Hz).

freqz(...) with no output arguments plots the magnitude and phase response versus frequency in the current figure window.

Examples

Plot the magnitude and phase response of an FIR filter.

\[
b = \text{fir1}(80,0.5,\text{kaiser}(81,8));
\]

\[
freqz(b,1);
\]

Algorithm

freqz uses an FFT algorithm when argument n is present. It computes the frequency response as the ratio of the transformed numerator and denominator coefficients, padded with zeros to the desired length:

\[
h = \text{fft}(b,n)./\text{fft}(a,n)
\]

If n is not a power of two, the FFT algorithm is not as efficient and may cause long computation times.

When a frequency vector w or f is present, or if n is less than max(length(b),length(a)), freqz evaluates the polynomials at each frequency point using Horner's method of polynomial evaluation and then divides the numerator response by the denominator response.
<table>
<thead>
<tr>
<th>See Also</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>abs</td>
<td>Absolute value (magnitude).</td>
</tr>
<tr>
<td>angle</td>
<td>Phase angle.</td>
</tr>
<tr>
<td>fft</td>
<td>One-dimensional fast Fourier transform.</td>
</tr>
<tr>
<td>filter</td>
<td>Filter data with a recursive (IIR) or nonrecursive (FIR) filter.</td>
</tr>
<tr>
<td>freqs</td>
<td>Frequency response of analog filters.</td>
</tr>
<tr>
<td>impz</td>
<td>Impulse response of digital filters.</td>
</tr>
<tr>
<td>invfreqz</td>
<td>Discrete-time filter identification from frequency data.</td>
</tr>
<tr>
<td>logspace</td>
<td>Generate logarithmically spaced vectors (see the online MATLAB Function Reference).</td>
</tr>
</tbody>
</table>
gauspuls

**Purpose**
Gaussian-modulated sinusoidal pulse generator.

**Syntax**

```
yi = gauspuls(t,fc,bw)
yi = gauspuls(t,fc,bw,bwr)
[yi,yq] = gauspuls(…)
[yi,yq,ye] = gauspuls(…)
tc = gauspuls('cutoff',fc,bw,bwr,tpe)
```

**Description**

gauspuls generates Gaussian-modulated sinusoidal pulses.

- `yi = gauspuls(t,fc,bw)` returns a unity-amplitude Gaussian RF pulse at the times indicated in array `t`, with a center frequency `fc` in Hertz and a fractional bandwidth `bw`, which must be greater than 0. The default value for `fc` is 1000 Hz and for `bw` is 0.5.

- `yi = gauspuls(t,fc,bw,bwr)` returns a unity-amplitude Gaussian RF pulse with a fractional bandwidth of `bw` as measured at a level of `bwr` dB with respect to the normalized signal peak. The fractional bandwidth reference level `bwr` must be less than 0, because it indicates a reference level less than the peak (unity) envelope amplitude. The default value for `bwr` is -6 dB.

- `[yi,yq] = gauspuls(…)` returns both the in-phase and quadrature pulses.

- `[yi,yq,ye] = gauspuls(…)` returns the RF signal envelope.

- `tc = gauspuls('cutoff',fc,bw,bwr,tpe)` returns the cutoff time `tc` (greater than or equal to 0) at which the trailing pulse envelope falls below `tpe` dB with respect to the peak envelope amplitude. The trailing pulse envelope level `tpe` must be less than 0, because it indicates a reference level less than the peak (unity) envelope amplitude. The default value for `tpe` is -60 dB.

**Remarks**

Default values are substituted for empty or omitted trailing input arguments.
Example

Plot a 50 kHz Gaussian RF pulse with 60% bandwidth, sampled at a rate of 1 MHz. Truncate the pulse where the envelope falls 40 dB below the peak:

```
tc = gauspuls('cutoff',50e3,0.6,[],-40);
t = –tc : 1e–6 : tc;
yi = gauspuls(t,50e3,0.6);
plot(t,yi)
```

See Also

- `chirp` - Swept-frequency cosine generator.
- `cos` - Cosine of vector/matrix elements (see the online MATLAB Function Reference).
- `diric` - Dirichlet or periodic sinc function.
- `pulstran` - Pulse train generator.
- `rectpuls` - Sampled aperiodic rectangle generator.
- `sawtooth` - Sawtooth or triangle wave generator.
- `sin` - Sine of vector/matrix elements (see the online MATLAB Function Reference).
- `sinc` - Sinc or sin(\(\pi t\))/\(\pi t\) function.
- `square` - Square wave generator.
- `tripuls` - Sampled aperiodic triangle generator.
Purpose

Average filter delay (group delay).

Syntax

[gd,w] = grpdelay(b,a,n)
[gd,f] = grpdelay(b,a,n,Fs)
[gd,w] = grpdelay(b,a,n,'whole')
[gd,f] = grpdelay(b,a,n,'whole',Fs)
gd = grpdelay(b,a,w)
gd = grpdelay(b,a,f,Fs)
grpdelay(b,a)

Description

The group delay of a filter is a measure of the average delay of the filter as a function of frequency. It is the negative first derivative of the phase response of the filter. If the complex frequency response of a filter is \( H(e^{j\omega}) \), then the group delay is

\[
\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}
\]

where \( \omega \) is frequency and \( \theta \) is the phase angle of \( H(e^{j\omega}) \).

\[ [gd,w] = grpdelay(b,a,n) \]
returns the \( n \)-point group delay, \( \tau_g(\omega) \), of the digital filter

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1)+b(2)z^{-1}+\cdots+b(nb+1)z^{-nb}}{a(1)+a(2)z^{-1}+\cdots+a(na+1)z^{-na}}
\]

given the numerator and denominator coefficients in vectors \( b \) and \( a \). \( grpdelay \)
returns both \( gd \), the group delay, and \( w \), a vector containing the \( n \) frequency
points in radians. \( grpdelay \) evaluates the group delay at \( n \) points equally
spaced around the upper half of the unit circle, so \( w \) contains \( n \) points between
0 and \( \pi \). A value for \( n \) that is an exact power of two allows fast computation
using an FFT algorithm.

\[ [gd,f] = grpdelay(b,a,n,Fs) \]
specifies a positive sampling frequency \( Fs \) in Hertz. It returns a length \( n \) vector \( f \) containing the actual frequency points at which the group delay is calculated, also in Hertz. \( f \) contains \( n \) points between 0 and \( Fs/2 \).
[gd, w] = grpdelay(b, a, n, 'whole') and
[gd, f] = grpdelay(b, a, n, 'whole', Fs) use n points around the whole unit
circle (from 0 to 2π, or from 0 to Fs).

gd = grpdelay(b, a, w) and
gd = grpdelay(b, a, f, Fs) return the group delay evaluated at the points in w
(in radians) or f (in Hertz), respectively, where Fs is the sampling frequency in
Hertz.

grpdelay with no output arguments plots the group delay versus frequency in
the current figure window.

grpdelay works for both real and complex input systems.

Examples

Plot the group delay of Butterworth filter \(b(z)/a(z)\):

\[
[b, a] = \text{butter}(6, 0.2);
\]

\[
\text{grpdelay}(b, a, 128)
\]
Plot both the group and phase delays of a system on the same graph:

```matlab
gd = grpdelay(b,a,512);
gd(1) = []; % avoid NaNs
[h,w] = freqz(b,a,512); h(1) = []; w(1) = [];
pd = -unwrap(angle(h))./w;
plot(w,gd,w,pd,:)
xlabel('Frequency (rads/sec)'); grid;
legend('Group Delay','Phase Delay');
```

**Algorithm**
grpdelay multiplies the filter coefficients by a unit ramp. After Fourier transformation, this process corresponds to differentiation.

**See Also**
- `cceps`: Complex cepstral analysis.
- `fft`: One-dimensional fast Fourier transform.
- `hilbert`: Hilbert transform.
- `icceps`: Inverse complex cepstrum.
- `rceps`: Real cepstrum and minimum phase reconstruction.
**Purpose**
Hamming window.

**Syntax**

```matlab
w = hamming(n)
w = hamming(n,sflag)
```

**Description**

`w = hamming(n)` returns an `n`-point symmetrically sampled Hamming window in the column vector `w`. `n` should be a nonnegative integer. The coefficients of a Hamming window are

\[
w[k+1] = 0.54 - 0.46\cos\left(\frac{2\pi k}{n-1}\right), \quad k = 0, \ldots, n - 1
\]

`w = hamming(n,sflag)` returns an `n`-point Hamming window using the window sampling specified by `sflag`, which can be either 'periodic' or 'symmetric' (the default). When 'periodic' is specified, `hamming` computes a length `n+1` window and returns the first `n` points.

**Diagnostics**

An error message is displayed when incorrect arguments are used:

- Order cannot be less than zero.
- Sampling must be either 'symmetric' or 'periodic'.

A warning message is displayed for noninteger `n`:

- Warning: Rounding order to nearest integer.

**See Also**

- `bartlett` — Bartlett window.
- `blackman` — Blackman window.
- `boxcar` — Rectangular window.
- `chebwin` — Chebyshev window.
- `hanning` — Hanning window.
- `kaiser` — Kaiser window.
- `triang` — Triangular window.

**References**

hanning

Purpose

Hanning window.

Syntax

\[ \text{w} = \text{hanning}(\text{n}) \]

\[ \text{w} = \text{hanning}(\text{n}, \text{sflag}) \]

Description

\[ \text{w} = \text{hanning}(\text{n}) \] returns an \( n \)-point symmetrically sampled Hanning window in the column vector \( w \). \( n \) should be a nonnegative integer. The coefficients of a Hanning window are

\[ w[k] = 0.5 \left(1 - \cos \left(\frac{2\pi k}{n+1}\right)\right), \quad k = 1, \ldots, n \]

\[ \text{w} = \text{hanning}(\text{n}, \text{sflag}) \] returns an \( n \)-point Hanning window using the window sampling specified by \( \text{sflag} \), which can be either 'periodic' or 'symmetric' (the default). When 'periodic' is specified, \text{hanning} computes a length \( n+1 \) window and returns the first \( n \) points.

Diagnostics

An error message is displayed when incorrect arguments are used:

- Order cannot be less than zero.
- Sampling must be either 'symmetric' or 'periodic'.

A warning message is displayed for noninteger \( n \):

Warning: Rounding order to nearest integer.

See Also

\text{bartlett} \quad \text{Bartlett window.}
\text{blackman} \quad \text{Blackman window.}
\text{boxcar} \quad \text{Rectangular window.}
\text{chebwin} \quad \text{Chebyshev window.}
\text{hamming} \quad \text{Hamming window.}
\text{kaiser} \quad \text{Kaiser window.}
\text{triang} \quad \text{Triangular window.}

References

Purpose
Hilbert transform.

Syntax
\[
y = \text{hilbert}(x)
\]

Description
\[
y = \text{hilbert}(x)\] returns a complex helical sequence, sometimes called the analytic signal, from a real data sequence. The analytic signal has a real part, which is the original data, and an imaginary part, which contains the Hilbert transform. The imaginary part is a version of the original real sequence with a 90° phase shift. Sines are therefore transformed to cosines and vice versa. The Hilbert transformed series has the same amplitude and frequency content as the original real data and includes phase information that depends on the phase of the original data.

If \( x \) is a matrix, \( y = \text{hilbert}(x) \) operates columnwise on the matrix, finding the Hilbert transform of each column.

The Hilbert transform is useful in calculating instantaneous attributes of a time series, especially the amplitude and frequency. The instantaneous amplitude is the amplitude of the complex Hilbert transform; the instantaneous frequency is the time rate of change of the instantaneous phase angle. For a pure sinusoid, the instantaneous amplitude and frequency are constant. The instantaneous phase, however, is a sawtooth, reflecting the way in which the local phase angle varies linearly over a single cycle. For mixtures of sinusoids, the attributes are short term, or local, averages spanning no more than two or three points.

Reference [1] describes the Kolmogorov method for minimum phase reconstruction, which involves taking the Hilbert transform of the logarithm of the spectrum of a time series. The toolbox function `rceps` performs this reconstruction.

Algorithm
The analytic signal for a sequence \( x \) has a one-sided Fourier transform, that is, negative frequencies are 0. To approximate the analytic signal, \text{hilbert} calculates the FFT of the input sequence, replaces those FFT coefficients that correspond to negative frequencies with zeros, and calculates the inverse FFT of the result.

In detail, \text{hilbert} uses a four-step algorithm:
hilbert

1 It calculates the FFT of the input sequence, storing the result in a vector $y$. Before transforming, it zero pads the input sequence so its length $n$ is the closest power of two, if necessary. This ensures the most efficient FFT computation.

2 It creates a vector $h$ whose elements $h(i)$ have the values
   - 1 for $i = 1, (n/2) + 1$
   - 2 for $i = 2, 3, \ldots, (n/2)$
   - 0 for $i = (n/2) + 2, \ldots, n$

3 It calculates the element-wise product of $y$ and $h$.

4 It calculates the inverse FFT of the sequence obtained in step 3 and returns the first $n$ elements of the result.

If the input data $x$ is a matrix, hilbert operates in a similar manner, extending each step above to handle the matrix case.

See Also

- fft One-dimensional fast Fourier transform.
- ifft One-dimensional inverse fast Fourier transform.
- rceps Real cepstrum and minimum phase reconstruction.

References

icceps

**Purpose**
Inverse complex cepstrum.

**Syntax**

```matlab
x = icceps(xhat,nd)
```

**Description**

`x = icceps(xhat,nd)` returns the inverse complex cepstrum of the (assumed real) sequence `xhat`, removing `nd` samples of delay. If `xhat` was obtained with `cceps(x)`, then the amount of delay that was added to `x` was the element of `round(unwrap(angle(fft(x)))/pi)` corresponding to π radians.

**See Also**

- `cceps` Complex cepstral analysis.
- `hilbert` Hilbert transform.
- `rceps` Real cepstrum and minimum phase reconstruction.
- `unwrap` Unwrap phase angles.

**References**

Purpose
Inverse discrete cosine transform.

Syntax
x = idct(y)
x = idct(y,n)

Description
The inverse discrete cosine transform reconstructs a sequence from its discrete cosine transform (DCT) coefficients. The idct function is the inverse of the dct function.

\[ x(n) = \sum_{k=1}^{N} w(k)y(k)\cos\left(\frac{(2n-1)(k-1)}{2N}\right), \quad n = 1, \ldots, N \]

where

\[ w(k) = \begin{cases} 
\frac{1}{\sqrt{N}}, & k = 1 \\
\frac{2}{\sqrt{N}}, & 2 \leq k \leq N 
\end{cases} \]

and \( N = \text{length}(x) \), which is the same as \( \text{length}(y) \). The series is indexed from \( n = 1 \) and \( k = 1 \) instead of the usual \( n = 0 \) and \( k = 0 \) because MATLAB vectors run from 1 to \( N \) instead of from 0 to \( N-1 \).

\( x = \text{idct}(y,n) \) appends zeros or truncates the vector \( y \) to length \( n \) before transforming.

If \( y \) is a matrix, \( \text{idct} \) transforms its columns.

See Also
- dct    Discrete cosine transform (DCT).
- dct2   Two-dimensional DCT (see Image Processing Toolbox User’s Guide).
- idct2  Two-dimensional inverse DCT (see Image Processing Toolbox User’s Guide).
- ifft   One-dimensional inverse fast Fourier transform.
References


Purpose

One-dimensional inverse fast Fourier transform.

Syntax

\[
y = \text{ifft}(x) \\
y = \text{ifft}(x,n)
\]

Description

\text{ifft} computes the inverse Fourier transform of a vector or array. This function implements the inverse transform given by

\[
x(n + 1) = \frac{1}{N} \sum_{k=0}^{N-1} X(k + 1) W_n^{-kn}
\]

where \(W_n = e^{-j(2\pi/N)}\) and \(N = \text{length}(x)\). Note that the series is indexed as \(n + 1\) and \(k + 1\) instead of the usual \(n\) and \(k\) because MATLAB vectors run from 1 to \(N\) instead of from 0 to \(N-1\).

\(y = \text{ifft}(x)\) is the inverse Fourier transform of vector \(x\). If \(x\) is an array, \(y\) is the inverse FFT of each column of the matrix.

\(y = \text{ifft}(x,n)\) is the \(n\)-point inverse FFT. If the length of \(x\) is less than \(n\), \text{ifft} pads \(x\) with trailing zeros to length \(n\). If the length of \(x\) is greater than \(n\), \text{ifft} truncates the sequence \(x\). When \(x\) is an array, \text{ifft} adjusts the length of the columns in the same manner.

The \text{ifft} function is part of the standard MATLAB language.

Algorithm

The \text{ifft} function is an M-file. The algorithm for \text{ifft} is the same as that for \text{fft}, except for a sign change and a scale factor of \(n = \text{length}(x)\). The execution time is fastest when \(n\) is a power of two and slowest when \(n\) is a large prime.

See Also

\text{fft} \quad \text{One-dimensional fast Fourier transform.}
\text{fft2} \quad \text{Two-dimensional fast Fourier transform.}
\text{fftshift} \quad \text{Rearrange the outputs of \text{fft} and \text{fft2}.}
\text{ifft2} \quad \text{Two-dimensional inverse fast Fourier transform.}
### Purpose
Two-dimensional inverse fast Fourier transform.

### Syntax

\[
Y = \text{ifft2}(X) \\
Y = \text{ifft2}(X,m,n)
\]

### Description

\(Y = \text{ifft2}(X)\) returns the two-dimensional inverse fast Fourier transform (FFT) of the array \(X\). If \(X\) is a vector, \(Y\) has the same orientation as \(X\).

\(Y = \text{ifft2}(X,m,n)\) truncates or zero pads \(X\), if necessary, to create an \(m\)-by-\(n\) array before performing the inverse FFT. The result \(Y\) is also \(m\)-by-\(n\).

For any \(X\), \(\text{ifft2}(\text{fft2}(X))\) equals \(X\) to within roundoff error. If \(X\) is real, \(\text{ifft2}(\text{fft2}(X))\) may have small imaginary parts.

The \text{ifft2} function is part of the standard MATLAB language.

### Algorithm
The algorithm for \text{ifft2} is the same as that for \text{fft2}, except for a sign change and scale factors of \([m \ n]= \text{size}(X)\). The execution time is fastest when \(m\) and \(n\) are powers of two and slowest when they are large primes.

### See Also

- \text{fft}
  One-dimensional fast Fourier transform.
- \text{fft2}
  Two-dimensional fast Fourier transform.
- \text{fftn}
  \(N\)-dimensional fast Fourier transform (see the online MATLAB Function Reference).
- \text{fftshift}
  Rearrange the outputs of \text{fft} and \text{fft2}.
- \text{ifft}
  One-dimensional inverse fast Fourier transform.
- \text{ifftn}
  \(N\)-dimensional inverse fast Fourier transform (see the online MATLAB Function Reference).
**Purpose**

Impulse invariance method of analog-to-digital filter conversion.

**Syntax**

- `[bz,az] = impinvar(b,a,Fs)`
- `[bz,az] = impinvar(b,a)`
- `[bz,az] = impinvar(b,a,Fs,tol)`

**Description**

- `[bz,az] = impinvar(b,a,Fs)` creates a digital filter with numerator and denominator coefficients `bz` and `az`, respectively, whose impulse response is equal to the impulse response of the analog filter with coefficients `b` and `a`, scaled by `1/Fs`.
- `[bz,az] = impinvar(b,a)` uses the default value of 1 Hz for `Fs`.
- `[bz,az] = impinvar(b,a,Fs,tol)` uses the tolerance specified by `tol` to determine whether poles are repeated. A larger tolerance increases the likelihood that `impinvar` will consider nearby poles to be repeated. The default is 0.001, or 0.1% of a pole's magnitude. Note that the accuracy of the pole values is still limited to the accuracy obtainable by the `roots` function.

**Example**

Convert an analog lowpass filter to a digital filter using `impinvar` with a sampling frequency of 10 Hz:

- `[b,a] = butter(4,0.3,'s');`
- `[bz,az] = impinvar(b,a,10)`

```matlab
bz =
1.0e-006 *
 -0.0000  0.1324  0.5192  0.1273  0
az =
 1.0000  -3.9216  5.7679  -3.7709  0.9246
```

**Algorithm**

`impinvar` performs the impulse-invariant method of analog-to-digital transfer function conversion discussed in reference [1]:

1. It finds the partial fraction expansion of the system represented by `b` and `a`.
2. It replaces the poles `p` by the poles `exp(p/Fs)`.
It finds the transfer function coefficients of the system from the residues from step 1 and the poles from step 2.

**See Also**

- `bilinear` Bilinear transformation method of analog-to-digital filter conversion.
- `lp2bp` Lowpass to bandpass analog filter transformation.
- `lp2bs` Lowpass to bandstop analog filter transformation.
- `lp2hp` Lowpass to highpass analog filter transformation.
- `lp2lp` Lowpass to lowpass analog filter transformation.

**References**

impz

**Purpose**

Impulse response of digital filters.

**Syntax**

\[
[h,t] = \text{impz}(b,a) \\
[h,t] = \text{impz}(b,a,n) \\
[h,t] = \text{impz}(b,a,n,Fs) \\
\text{impz}(b,a) \\
\text{impz}(...) 
\]

**Description**

\[
[h,t] = \text{impz}(b,a) \text{ computes the impulse response of the filter with numerator coefficients } b \text{ and denominator coefficients } a. \text{impz chooses the number of samples and returns the response in column vector } h \text{ and times (or sample intervals) in column vector } t \text{ (where } t = (0:n-1)' \text{ and } n \text{ is the computed impulse response length). }
\]

\[
[h,t] = \text{impz}(b,a,n) \text{ computes } n \text{ samples of the impulse response. If } n \text{ is a vector of integers, } \text{impz computes the impulse response at those integer locations where } 0 \text{ is the starting point of the filter.}
\]

\[
[h,t] = \text{impz}(b,a,n,Fs) \text{ computes } n \text{ samples and scales } t \text{ so that samples are spaced } 1/Fs \text{ units apart. Fs is } 1 \text{ by default. Leave } n \text{ empty, } [], \text{ to let } \text{impz select the number of samples.}
\]

\text{impz with no output arguments plots the impulse response in the current figure window using } \text{stem}(t,h).

\text{impz works for both real and complex input systems.}
**Example**

Plot the first 50 samples of the impulse response of a fourth-order lowpass elliptic filter with cutoff frequency of 0.4 times the Nyquist frequency:

```matlab
[b,a] = ellip(4,0.5,20,0.4);
impz(b,a,50)
```

**Algorithm**

impz filters a length n impulse sequence using

```matlab
filter(b,a,[1 zeros(1,n–1)])
```

To compute n in the auto-length case, impz either uses n = length(b) for the FIR case or first finds the poles using p = roots(a), if length(a) is greater than 1.

If the filter is unstable, n is chosen to be the point at which the term from the largest pole reaches 10^{-6} times its original value.

If the filter is stable, n is chosen to be the point at which the term due to the largest amplitude pole is 5*10^{-5} of its original amplitude.

If the filter is oscillatory (poles on the unit circle only), impz computes five periods of the slowest oscillation.

If the filter has both oscillatory and damped terms, n is chosen to equal five periods of the slowest oscillation or the point at which the term due to the
impz

largest (nonunity) amplitude pole is $5 \times 10^{-5}$ of its original amplitude, whichever is greater.

impz also allows for delay in the numerator polynomial, which it adds to the resulting $n$.

See Also

- **impulse**: Unit impulse response (see Control System Toolbox User’s Guide).
- **stem**: Plot discrete sequence data (see the online MATLAB Function Reference).
**Purpose**
Increase sampling rate by an integer factor (interpolation).

**Syntax**

\[ y = \text{interp}(x,r) \]
\[ y = \text{interp}(x,r,l,alpha) \]
\[ [y,b] = \text{interp}(x,r,l,alpha) \]

**Description**
Interpolation increases the original sampling rate for a sequence to a higher rate. \text{interp} performs lowpass interpolation by inserting zeros into the original sequence and then applying a special lowpass filter.

\[ y = \text{interp}(x,r) \] increases the sampling rate of \( x \) by a factor of \( r \). The interpolated vector \( y \) is \( r \) times longer than the original input \( x \).

\[ y = \text{interp}(x,r,l,alpha) \] specifies \( l \) (filter length) and \( alpha \) (cut-off frequency). The default value for \( l \) is 4 and the default value for \( alpha \) is 0.5.

\[ [y,b] = \text{interp}(x,r,l,alpha) \] returns vector \( b \) containing the filter coefficients used for the interpolation.

**Example**
Interpolate a signal by a factor of four:

```matlab
% time vector
x = sin(2*pi*30*t) + sin(2*pi*60*t);
y = interp(x,4);
stem(x(1:30)); title('Original Signal');
figure
stem(y(1:120)); title('Interpolated Signal');
```

![Original Signal](image1.jpg)

![Interpolated Signal](image2.jpg)
interp

Algorithm

interp uses the lowpass interpolation Algorithm 8.1 described in [1]:

1. It expands the input vector to the correct length by inserting zeros between the original data values.
2. It designs a special symmetric FIR filter that allows the original data to pass through unchanged and interpolates between so that the mean-square errors between the interpolated points and their ideal values are minimized.
3. It applies the filter to the input vector to produce the interpolated output vector.

The length of the FIR lowpass interpolating filter is \(2 \times l \times r + 1\). The number of original sample values used for interpolation is \(2 \times 1\). Ordinarily, \(l\) should be less than or equal to 10. The original signal is assumed to be band limited with normalized cutoff frequency \(0 \leq \alpha \leq 1\), where 1 is half the original sampling frequency (the Nyquist frequency). The default value for \(l\) is 4 and the default value for \(\alpha\) is 0.5.

Diagnostics

If \(r\) is not an integer, interp gives the following error message:

Resampling rate \(R\) must be an integer.

See Also

- decimate: Decrease the sampling rate for a sequence (decimation).
- interp1: One-dimensional data interpolation (table lookup) (see the online MATLAB Function Reference).
- resample: Change sampling rate by any rational factor.
- spline: Cubic spline interpolation (see the online MATLAB Function Reference).
- upfirdn: Upsample, apply an FIR filter, and downsample.

References

Purpose

Interpolation FIR filter design.

Syntax

\[ b = \text{intfilt}(r, l, \alpha) \]
\[ b = \text{intfilt}(r, n, 'Lagrange') \]

Description

\[ b = \text{intfilt}(r, l, \alpha) \] designs a linear phase FIR filter that performs ideal bandlimited interpolation using the nearest \(2 \times l\) nonzero samples, when used on a sequence interleaved with \(r-1\) consecutive zeros every \(r\) samples. It assumes an original bandlimitedness of \(\alpha\) times the Nyquist frequency. The returned filter is identical to that used by \text{interp}.

\[ b = \text{intfilt}(r, n, 'Lagrange') \] or \[ b = \text{intfilt}(r, n, 'L') \] designs an FIR filter that performs \(n\)th-order Lagrange polynomial interpolation on a sequence interleaved with \(r-1\) consecutive zeros every \(r\) samples. \(b\) has length \((n + 1) \times r\) for \(n\) even, and length \((n + 1) \times r-1\) for \(n\) odd.

Both types of filters are basically lowpass and are intended for interpolation and decimation.

Examples

Design a digital interpolation filter to upsample a signal by four, using the bandlimited method:

\[
\begin{align*}
\alpha &= 0.5; \quad \% \text{"bandlimitedness" factor} \\
h1 &= \text{intfilt}(4, 2, \alpha); \quad \% \text{bandlimited interpolation}
\end{align*}
\]

The filter \(h1\) works best when the original signal is bandlimited to \(\alpha\) times the Nyquist frequency. Create a bandlimited noise signal:

\[
\begin{align*}
\text{randn('seed',0)} \\
x &= \text{filter(fir1(40,0.5),1,randn(200,1))}; \quad \% \text{bandlimit}
\end{align*}
\]

Now zero pad the signal with three zeros between every sample. The resulting sequence is four times the length of \(x\):

\[
\begin{align*}
xr &= \text{reshape}([x \text{ zeros}(\text{length}(x),3)]',4*\text{length}(x),1); \\
\end{align*}
\]

Interpolate using the \text{filter} command:

\[
y = \text{filter}(h1,1,xr);
\]
y is an interpolated version of x, delayed by seven samples (the group-delay of the filter). Zoom in on a section to see this:

```matlab
plot(100:200, y(100:200), 7+(101:4:196), x(26:49), 'o')
```

intfilt's other type of filter performs Lagrange polynomial interpolation of the original signal. For example, first-order polynomial interpolation is just linear interpolation, which is accomplished with a triangular filter:

```matlab
h2 = intfilt(4, 1, 'l')   % Lagrange interpolation
h2 =
   0.2500   0.5000   0.7500   1.0000   0.7500   0.5000  0.2500
```

**Algorithm**
The bandlimited method uses `firls` to design an interpolation FIR equivalent to that presented in [1]. The polynomial method uses Lagrange's polynomial interpolation formula on equally spaced samples to construct the appropriate filter.

**See Also**
- `decimate`: Decrease the sampling rate for a sequence (decimation).
- `interp`: Increase sampling rate by an integer factor (interpolation).
- `resample`: Change sampling rate by any rational factor.

**References**
**invfreqs**

**Purpose**
Continuous-time (analog) filter identification from frequency data.

**Syntax**

```plaintext
[b,a] = invfreqs(h,w,nb,na)
[b,a] = invfreqs(h,w,nb,na,wt)
[b,a] = invfreqs(h,w,nb,na,wt,iter)
[b,a] = invfreqs(h,w,nb,na,wt,iter,tol)
[b,a] = invfreqs(h,w,nb,na,wt,iter,tol,'trace')
[b,a] = invfreqs(h,w,'complex',nb,na,...)
```

**Description**

`invfreqs` is the inverse operation of `freqs`; it finds a continuous-time transfer function that corresponds to a given complex frequency response. From a laboratory analysis standpoint, `invfreqs` is useful in converting magnitude and phase data into transfer functions.

`[b,a] = invfreqs(h,w,nb,na)` returns the real numerator and denominator coefficient vectors `b` and `a` of the transfer function

\[
H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^{nb} + b(2)s^{nb-1} + \cdots + b(nb+1)}{a(1)s^{na} + a(2)s^{na-1} + \cdots + a(na+1)}
\]

whose complex frequency response is given in vector `h` at the frequency points specified in vector `w`. Scalars `nb` and `na` specify the desired orders of the numerator and denominator polynomials.

Frequency is specified in radians between 0 and π, and the length of `h` must be the same as the length of `w`. `invfreqs` uses `conj(h)` at –w to ensure the proper frequency domain symmetry for a real filter.

`[b,a] = invfreqs(h,w,nb,na,wt)` weights the fit-errors versus frequency. `wt` is a vector of weighting factors the same length as `w`.

`invfreqs(h,w,nb,na,wt,iter)` and

`invfreqs(h,w,nb,na,wt,iter,tol)` provide a superior algorithm that guarantees stability of the resulting linear system and searches for the best fit using a numerical, iterative scheme. The `iter` parameter tells `invfreqs` to end the iteration when the solution has converged, or after `iter` iterations, whichever comes first. `invfreqs` defines convergence as occurring when the
invfreqs

norm of the (modified) gradient vector is less than tol. tol is an optional parameter that defaults to 0.01. To obtain a weight vector of all ones, use

invfreqs(h,w,nb,na,[],iter,tol)

invfreqs(h,w,nb,na,wt,iter,tol,'trace') displays a textual progress report of the iteration.

invfreqs(h,w,'complex',nb,na,...) creates a complex filter. In this case no symmetry is enforced, and the frequency is specified in radians between -π and π.

Remarks

When building higher order models using high frequencies, it is important to scale the frequencies, dividing by a factor such as half the highest frequency present in w, so as to obtain well conditioned values of a and b. This corresponds to a rescaling of time.

Examples

Convert a simple transfer function to frequency response data and then back to the original filter coefficients:

\[
\begin{align*}
a &= [1 2 3 2 1 4]; \\
b &= [1 2 3 2 3]; \\
[h,w] &= \text{freqs}(b,a,64); \\
[bb,aa] &= \text{invfreqs}(h,w,4,5)
\end{align*}
\]

\[
\begin{align*}
bb &= \\
&= \begin{bmatrix} 1.0000 & 2.0000 & 3.0000 & 2.0000 & 3.0000 \\
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
aa &= \\
&= \begin{bmatrix} 1.0000 & 2.0000 & 3.0000 & 2.0000 & 1.0000 & 4.0000 \\
\end{bmatrix}
\end{align*}
\]
Notice that \( b_b \) and \( a_a \) are equivalent to \( b \) and \( a \), respectively. However, \( a_a \) has poles in the left half-plane and thus the system is unstable. Use \texttt{invfreqs}'s iterative algorithm to find a stable approximation to the system:

\[
[b_{bb}, a_{aa}] = \text{invfreqs}(h,w,4,5,[],30)
\]

\[
\begin{align*}
b_{bb} &= 0.6816 & 2.1015 & 2.6694 & 0.9113 & -0.1218 \\
a_{aa} &= 1.0000 & 3.4676 & 7.4060 & 6.2102 & 2.5413 & 0.0001
\end{align*}
\]

Suppose you have two vectors, \( \text{mag} \) and \( \text{phase} \), that contain magnitude and phase data gathered in a laboratory, and a third vector \( w \) of frequencies. You can convert the data into a continuous-time transfer function using \texttt{invfreqs}:

\[
[b,a] = \text{invfreqs}(\text{mag}.*\exp(j*\text{phase}),w,2,3);
\]

**Algorithm**

By default, \texttt{invfreqs} uses an equation error method to identify the best model from the data. This finds \( b \) and \( a \) in

\[
\min_{b,a} \sum_{k=1}^{n} wt(k) |h(k)A(w(k)) - B(w(k))|^2
\]

by creating a system of linear equations and solving them with MATLAB's \( \backslash \) operator. Here \( A(w(k)) \) and \( B(w(k)) \) are the Fourier transforms of the polynomials \( a \) and \( b \), respectively, at the frequency \( w(k) \), and \( n \) is the number of frequency points (the length of \( h \) and \( w \)). This algorithm is based on Levi [1]. Several variants have been suggested in the literature, where the weighting function \( wt \) gives less attention to high frequencies.

The superior ("output-error") algorithm uses the damped Gauss-Newton method for iterative search [2], with the output of the first algorithm as the initial estimate. This solves the direct problem of minimizing the weighted sum of the squared error between the actual and the desired frequency response points:

\[
\min_{b,a} \sum_{k=1}^{n} wt(k) \left| h(k) - \frac{B(w(k))}{A(w(k))} \right|^2
\]
### invfreqs

**See Also**

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**References**


Purpose
Discrete-time filter identification from frequency data.

Syntax
[b,a] = invfreqz(h,w,nb,na)
[b,a] = invfreqz(h,w,nb,na,wt)
[b,a] = invfreqz(h,w,nb,na,wt,iter)
[b,a] = invfreqz(h,w,nb,na,wt,iter,tol)
[b,a] = invfreqz(h,w,nb,na,wt,iter,tol,'trace')
[b,a] = invfreqz(h,w,'complex',nb,na,...)

Description
invfreqz is the inverse operation of freqz; it finds a discrete-time transfer function that corresponds to a given complex frequency response. From a laboratory analysis standpoint, invfreqz can be used to convert magnitude and phase data into transfer functions.

[b,a] = invfreqz(h,w,nb,na) returns the real numerator and denominator coefficients in vectors b and a of the transfer function

\[ H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na}} \]

whose complex frequency response is given in vector h at the frequency points specified in vector w. Scalars nb and na specify the desired orders of the numerator and denominator polynomials.

Frequency is specified in radians between 0 and π, and the length of h must be the same as the length of w. invfreqz uses conj(h) at -w to ensure the proper frequency domain symmetry for a real filter.

[b,a] = invfreqz(h,w,nb,na,wt) weights the fit-errors versus frequency. wt is a vector of weighting factors the same length as w.

invfreqz(h,w,nb,na,wt,iter) and

invfreqz(h,w,nb,na,wt,iter,tol) provide a superior algorithm that guarantees stability of the resulting linear system and searches for the best fit using a numerical, iterative scheme. The iter parameter tells invfreqz to end the iteration when the solution has converged, or after iter iterations, whichever comes first. invfreqz defines convergence as occurring when the
norm of the (modified) gradient vector is less than \( \text{tol} \). \( \text{tol} \) is an optional parameter that defaults to 0.01. To obtain a weight vector of all ones, use

```matlab
close(h,w,nb,na,[],iter,tol)
```

`invfreqz(h,w,nb,na,wt,iter,tol,'trace')` displays a textual progress report of the iteration.

`invfreqz(h,w,'complex',nb,na,...)` creates a complex filter. In this case no symmetry is enforced, and the frequency is specified in radians between \(-\pi\) and \(\pi\).

**Example**

Convert a simple transfer function to frequency response data and then back to the original filter coefficients:

```matlab
a = [1 2 3 2 1 4]; b = [1 2 3 2 3];
[h,w] = freqz(b,a,64);
[bb,aa] = invfreqz(h,w,4,5)
```

\[
\text{bb} = \\
\begin{bmatrix}
1.0000 & 2.0000 & 3.0000 & 2.0000 & 3.0000
\end{bmatrix}
\]

\[
\text{aa} = \\
\begin{bmatrix}
1.0000 & 2.0000 & 3.0000 & 2.0000 & 1.0000 & 4.0000
\end{bmatrix}
\]

Notice that \( \text{bb} \) and \( \text{aa} \) are equivalent to \( \text{b} \) and \( \text{a} \), respectively. However, \( \text{aa} \) has poles outside the unit circle and thus the system is unstable. Use `invfreqz`’s iterative algorithm to find a stable approximation to the system:

```matlab
[bbb,aaa] = invfreqz(h,w,4,5,[],30)
```

\[
\text{bbb} = \\
\begin{bmatrix}
0.2427 & 0.2788 & 0.0069 & 0.0971 & 0.1980
\end{bmatrix}
\]

\[
\text{aaa} = \\
\begin{bmatrix}
1.0000 & -0.8944 & 0.6954 & 0.9997 & -0.8933 & 0.6949
\end{bmatrix}
\]

**Algorithm**

By default, `invfreqz` uses an equation error method to identify the best model from the data. This finds \( b \) and \( a \) in
by creating a system of linear equations and solving them with MATLAB's \ operator. Here \( A(w(k)) \) and \( B(w(k)) \) are the Fourier transforms of the polynomials \( a \) and \( b \), respectively, at the frequency \( w(k) \), and \( n \) is the number of frequency points (the length of \( h \) and \( w \)). This algorithm is based on Levi [1].

The superior ("output-error") algorithm uses the damped Gauss-Newton method for iterative search [2], with the output of the first algorithm as the initial estimate. This solves the direct problem of minimizing the weighted sum of the squared error between the actual and the desired frequency response points:

\[
\min_{b,a} \sum_{k=1}^{n} w_t(k) \left| h(k) - \frac{B(w(k))}{A(w(k))} \right|^2
\]

**See Also**

- freqs: Frequency response of analog filters.
- invfreqz: Continuous-time (analog) filter identification from frequency data.
- prony: Prony's method for time domain IIR filter design.

**References**


Kaiser window.

Syntax

\[
w = \text{kaiser}(n, \beta)\]

Description

\(w = \text{kaiser}(n, \beta)\) returns an \(n\)-point Kaiser \(I_0 - \sinh\) window in the column vector \(w\). \(\beta\) is the Kaiser window \(\beta\) parameter that affects the sidelobe attenuation of the Fourier transform of the window.

To obtain a Kaiser window that designs an FIR filter with sidelobe height \(-\alpha\) dB, use the following \(\beta\):

\[
\beta = \begin{cases} 
0.1102(\alpha - 8.7), & \alpha > 50 \\
0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\
0, & \alpha < 21 
\end{cases}
\]

Increasing \(\beta\) widens the mainlobe and decreases the amplitude of the sidelobes (increases the attenuation).

See Also

- bartlett
- blackman
- boxcar
- chebwin
- hamming
- hanning
- kaiserord
- triang

References


kaiserord

Purpose
Estimate parameters for an FIR filter design with Kaiser window.

Syntax

\[
\begin{align*}
\text{[}n, Wn, \beta, \text{ftype}\text{]} &= \text{kaiserord}(f, a, \text{dev}) \\
\text{[}n, Wn, \beta, \text{ftype}\text{]} &= \text{kaiserord}(f, a, \text{dev}, Fs) \\
c &= \text{kaiserord}(f, a, \text{dev}, Fs, 'cell')
\end{align*}
\]

Description
kaiserord returns a filter order \( n \) and \( \beta \) parameter to specify a Kaiser window for use with the fir1 function. Given a set of specifications in the frequency domain, kaiserord estimates the minimum FIR filter order that will approximately meet the specifications. kaiserord converts the given filter specifications into passband and stopband ripples and converts cutoff frequencies into the form needed for windowed FIR filter design.

**NOTE** If the band ripples are specified as unequal, the smallest one is used, since the Kaiser window method is constrained to give filters with equal ripple heights in all the passbands and stopbands.

\[
\text{[}n, Wn, \beta, \text{ftype}\text{]} = \text{kaiserord}(f, a, \text{dev})
\]
finds the approximate order \( n \), normalized frequency band edges \( Wn \), and weights that meet input specifications \( f \), \( a \), and \( \text{dev} \). \( f \) is a vector of band edges and \( a \) is a vector specifying the desired amplitude on the bands defined by \( f \). The length of \( f \) is twice the length of \( a \), minus 2. Together, \( f \) and \( a \) define a desired piecewise constant response function. \( \text{dev} \) is a vector the same size as \( a \) that specifies the maximum allowable error or deviation between the frequency response of the output filter and its desired amplitude, for each band.

fir1 can use the resulting order \( n \), frequency vector \( Wn \), multiband magnitude type \( \text{ftype} \), and the Kaiser window parameter \( \beta \). The \( \text{ftype} \) string is intended for use with fir1; it is equal to 'high' for a highpass filter and 'stop' for a bandstop filter. For multiband filters, it can be equal to 'dc–0' when the first band is a stopband (starting at \( f = 0 \)) or 'dc–1' when the first band is a passband.

To design a filter \( b \) that approximately meets the specifications given by kaiser parameters \( f \), \( a \), and \( \text{dev} \):

\[
b = \text{fir1}(n, Wn, \text{kaiser}(n+1, \beta), \text{ftype}, '\text{noscale}')
\]

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[n, Wn, beta, ftype] = kaiserord(f, a, dev, Fs) specifies a sampling frequency Fs. If not present, Fs defaults to 2 Hz, implying a Nyquist frequency of 1 Hz. You can therefore specify band edges scaled to a particular application's sampling frequency.

c = kaiserord(f, a, dev, Fs, 'cell') is a cell-array whose elements are the parameters to fir1.

NOTE In some cases, kaiserord underestimates or overestimates the order n. If the filter does not meet the specifications, try a higher order such as n+1, n+2, and so on, or a lower order.

Results are inaccurate if the cutoff frequencies are near 0 or the Nyquist frequency, or if dev is large (greater than 10%).

Algorithm

kaiserord uses empirically derived formulas for estimating the orders of lowpass filters, as well as differentiators and Hilbert transformers. Estimates for multiband filters (such as bandpass filters) are derived from the lowpass design formulas.

The design formulas that underlie the Kaiser window and its application to FIR filter design are

\[
\beta = \begin{cases} 
0.1102(\alpha - 8.7), & \alpha > 50 \\
0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\
0, & \alpha < 21 
\end{cases}
\]

where \( \alpha = -20 \log_{10} \delta \) is the stopband attenuation expressed in decibels (recall that \( \delta_p = \delta_s \) is required). The design formula is:

\[
n = \frac{\alpha - 7.95}{2.285(\Delta \omega)}
\]

where \( n \) is the filter order and \( \Delta \omega \) is the width of the smallest transition region.
Examples

Design a lowpass filter with passband from 0 to 1 kHz and stopband from 1500 Hz to 4 kHz. Specify passband ripple of 5% and stopband attenuation of 40 dB:

```matlab
fsamp = 8000;
fcuts = [1000 1500];
mags = [1 0];
devs = [0.05 0.01];
[n,Wn,beta,ftype] = kaiserord(fcuts,mags,devs,fsamp);
hh = fir1(n,Wn,ftype,kaiser(n+1,beta),'noscale');
freqz(hh)
```
Design an odd-length bandpass filter (note that odd length means even order, so the input to `fir1` must be an even integer):

```matlab
fsamp = 8000;
fcuts = [1000 1300 2210 2410];
mags = [0 1 0];
devs = [0.01 0.05 0.01];
[n,Wn,beta,ftype] = kaiserord(fcuts,mags,devs,fsamp);
n = n + rem(n,2);
hh = fir1(n,Wn,ftype,kaiser(n+1,beta),'noscale');
[H,f] = freqz(hh,1,1024,fsamp);
plot(f,abs(H)), grid on
```

Design a lowpass filter with a passband cutoff of 1500 Hz, a stopband cutoff of 2000 Hz, passband ripple of 0.01, stopband ripple of 0.1, and a sampling frequency of 8000 Hz:

```matlab
[n,Wn,beta,ftype] = kaiserord([1500 2000],[1 0],[0.01 0.1],8000);
b = fir1(n,Wn,ftype,kaiser(n+1,beta),'noscale');
```

This is equivalent to

```matlab
c = kaiserord([1500 2000],[1 0],[0.01 0.1],8000,'cell');
b = fir1(c{:});
```
Remarks

Be careful to distinguish between the meanings of filter length and filter order. The filter length is the number of impulse response samples in the FIR filter. Generally, the impulse response is indexed from \( n = 0 \) to \( n = L - 1 \), where \( L \) is the filter length. The filter order is the highest power in a z-transform representation of the filter. For an FIR transfer function, this representation is a polynomial in \( z \), where the highest power is \( z^{L-1} \) and the lowest power is \( z^0 \). The filter order is 1 less than the length (\( L - 1 \)) and is also equal to the number of zeros of the \( z \) polynomial.

Diagnostics

The lengths of the frequency, magnitude, and deviation vectors must match; otherwise, kaiserord gives the following error message:

- Requires \( M \) and \( \text{DEV} \) to be the same length.
- Length of \( F \) must be \( 2 \times \text{length}(M)-2 \).

When magnitudes are not specified correctly, kaiserord gives the following error messages:

- Stopbands must be zero.
- All passbands must have the same height.

If the band edges are not strictly increasing, kaiserord gives the following error message:

- Bandedges must be strictly increasing.

See Also

fir1
Window-based finite impulse response filter design – standard response.
kaiser
Kaiser window.
remezord
Parks-McClellan optimal FIR filter order estimation.

References


latc2tf

Purpose

Lattice filter to transfer function conversion.

Syntax

[num,den] = latc2tf(k,v)
[num,den] = latc2tf(k,'iir')
num = latc2tf(k,'fir')
num = latc2tf(k)

Description

[num,den] = latc2tf(k,v) finds the transfer function numerator num and
denominator den from the IIR lattice coefficients k and ladder coefficients v.

[num,den] = latc2tf(k,'iir') assumes that k is associated with an all-pole
IIR lattice filter.

num = latc2tf(k,'fir') and

num = latc2tf(k) find the transfer function numerators from the FIR lattice
coefficients specified by k.

See Also

latcfilt Lattice and lattice-ladder filter implementation.
tf2latc Conversion of transfer function to lattice filter.
Purpose
Lattice and lattice-ladder filter implementation.

Syntax
[f,g] = latcfilt(k,x)
[f,g] = latcfilt(k,v,x)
[f,g] = latcfilt(k,1,x)
[f,g,zf] = latcfilt(...,'ic',zi)

Description
[f,g] = latcfilt(k,x) filters x with the FIR lattice coefficients in vector k. f
is the forward lattice filter result and g is the backward filter result.
If k and x are vectors, the result is a (signal) vector.
Matrix arguments are permitted under the following rules:

• If x is a matrix and k is a vector, each column of x is processed through the
  lattice filter specified by k.
• If x is a vector and k is a matrix, each column of k is used to filter x, and a
  signal matrix is returned.
• If x and k are both matrices with the same number of columns, then the i-th
  column of k is used to filter the i-th column of x. A signal matrix is returned.

[f,g] = latcfilt(k,v,x) filters x with the IIR lattice coefficients k and
ladder coefficients v. k and v must be vectors, while x may be a signal matrix.

[f,g] = latcfilt(k,1,x) filters x with the IIR all-pole lattice specified by k.
k and x may be vectors or matrices according to the rules given for the FIR
lattice.

[f,g,zf] = latcfilt(...,'ic',zi) accepts a length-k vector zi specifying
the initial condition of the lattice states. Output zf is a length-k vector
specifying the final condition of the lattice states.

See Also
filter Filter data with a recursive (IIR) or nonrecursive
(FIR) filter.
latc2tf Lattice filter to transfer function conversion.
tf2latc Conversion of transfer function to lattice filter.
**levinson**

**Purpose**
Levinson-Durbin recursion.

**Syntax**
```
a = levinson(r,p)
```

**Description**
The Levinson-Durbin recursion is an algorithm for finding an all-pole IIR filter with a prescribed deterministic autocorrelation sequence. It has applications in filter design, coding, and spectral estimation. The filter that `levinson` produces is minimum phase.

`a = levinson(r,p)` finds the coefficients of an $p$th-order autoregressive linear process which has $r$ as its autocorrelation sequence. $r$ is a real or complex deterministic autocorrelation sequence (a vector), and $p$ is the order of denominator polynomial $A(z)$; that is, $a = [1 \ a(2) \ ... \ a(p+1)]$. The filter coefficients are ordered in descending powers of $z$:

$$H(z) = \frac{1}{A(z)} = \frac{1}{1 + a(2)z^{-1} + \cdots + a(p+1)z^{-p}}$$

**Algorithm**
`levinson` solves the symmetric Toeplitz system of linear equations.

$$
\begin{bmatrix}
    r(1) & r(2)^* & \cdots & r(p)^* \\
    r(2) & r(1) & \cdots & r(p-1)^* \\
    \vdots & \vdots & \ddots & \vdots \\
    r(p) & \cdots & r(2) & r(1)
\end{bmatrix}
\begin{bmatrix}
    a(2) \\
    a(3) \\
    \vdots \\
    a(p+1)
\end{bmatrix}
= 
\begin{bmatrix}
    -r(2) \\
    -r(3) \\
    \vdots \\
    -r(p+1)
\end{bmatrix}
$$

where $r = [r(1) ... r(p+1)]$ is the input autocorrelation vector, and $r(i)^*$ denotes the complex conjugate of $r(i)$. The algorithm requires $O(p^2)$ flops and is thus much more efficient than the MATLAB `\` command for large $p$. However, the `levinson` function uses `\` for low orders to provide the fastest possible execution.

**See Also**
`lpc` Linear prediction coefficients.
`prony` Prony’s method for time domain IIR filter design.
`rlevinson` Reverse Levinson-Durbin recursion.
`stmcb` Linear model using Steiglitz-McBride iteration.
References

**lp2bp**

**Purpose**
Lowpass to bandpass analog filter transformation.

**Syntax**

\[
[bt, at] = \text{lp2bp}(b, a, Wo, Bw)
\]

\[
[A, B, C, D, Wo, Bw]
\]

**Description**

lp2bp transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into bandpass filters with desired bandwidth and center frequency. The transformation is one step in the digital filter design process for the butter, cheby1, cheby2, and ellip functions.

lp2bp can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

**Transfer Function Form (Polynomial)**

\[
[bt, at] = \text{lp2bp}(b, a, Wo, Bw)
\]

transforms an analog lowpass filter prototype given by polynomial coefficients into a bandpass filter with center frequency \( Wo \) and bandwidth \( Bw \). Row vectors \( b \) and \( a \) specify the coefficients of the numerator and denominator of the prototype in descending powers of \( s \):

\[
\frac{b(s)}{a(s)} = \frac{b(1)s^{nb} + \cdots + b(nb)s + b(nb + 1)}{a(1)s^{na} + \cdots + a(na)s + a(na + 1)}
\]

Scalars \( Wo \) and \( Bw \) specify the center frequency and bandwidth in units of radians/second. For a filter with lower band edge \( w1 \) and upper band edge \( w2 \), use

\[
Wo = \text{sqrt}(w1\times w2)
\]

and

\[
Bw = w2 - w1
\]

lp2bp returns the frequency transformed filter in row vectors \( bt \) and \( at \).

**State-Space Form**

\[
[At, Bt, Ct, Dt] = \text{lp2bp}(A, B, C, D, Wo, Bw)
\]

converts the continuous-time state-space lowpass filter prototype in matrices \( A, B, C, D \):

\[
\dot{x} = Ax + Bu
\]

\[
y =Cx + Du
\]
into a bandpass filter with center frequency $\omega_0$ and bandwidth $B_w$. For a filter with lower band edge $w_1$ and upper band edge $w_2$, use $\omega_0 = \sqrt{w_1 \cdot w_2}$ and $B_w = w_2 - w_1$.

The bandpass filter is returned in matrices $A_t$, $B_t$, $C_t$, $D_t$.

**Algorithm**

`lp2bp` is a highly accurate state-space formulation of the classic analog filter frequency transformation. Consider the state-space system:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

where $u$ is the input, $x$ is the state vector, and $y$ is the output. The Laplace transform of the first equation (assuming zero initial conditions) is

\[
sX(s) = AX(s) + BU(s)
\]

Now if a bandpass filter is to have center frequency $\omega_0$ and bandwidth $B_w$, the standard s-domain transformation is

\[
s = Q(p^2 + 1)/p
\]

where $Q = \omega_0/B_w$ and $\rho = s/\omega_0$. Substituting this for $s$ in the Laplace transformed state-space equation, and considering the operator $p$ as $d/dt$:

\[
Q\ddot{x} + Qx = \dot{Ax} + B\dot{u}
\]

or

\[
Q\ddot{x} - \dot{Ax} - B\dot{u} = -Qx
\]

Now define

\[
Q\dot{\omega} = -Qx
\]

which, when substituted, leads to

\[
Q\ddot{x} = Ax + Q\omega + Bu
\]
The last two equations give equations of state. Write them in standard form and multiply the differential equations by $\omega_0$ to recover the time/frequency scaling represented by $p$ and find state matrices for the bandpass filter:

$$Q = Wo/Bw; \quad [ma,na] = \text{size}(A);$$
$$At = Wo*[A/Q \text{ eye}(ma,na);-\text{eye}(ma,na) \text{ zeros}(ma,na)];$$
$$Bt = Wo*[B/Q; \text{ zeros}(ma,nb)];$$
$$Ct = [C \text{ zeros}(mc,ma)];$$
$$Dt = d;$$

If the input to `lp2bp` is in transfer function form, the function transforms it into state-space form before applying this algorithm.

**See Also**

- `bilinear` Bilinear transformation method of analog-to-digital filter conversion.
- `impinvar` Impulse invariance method of analog-to-digital filter conversion.
- `lp2bs` Lowpass to bandstop analog filter transformation.
- `lp2hp` Lowpass to highpass analog filter transformation.
- `lp2lp` Lowpass to lowpass analog filter transformation.
Purpose

Lowpass to bandstop analog filter transformation.

Syntax

\[ [bt, at] = \text{lp2bs}(b,a,Wo,Bw) \]
\[ [At,Bt,Ct,Dt] = \text{lp2bs}(A,B,C,D,Wo,Bw) \]

Description

\text{lp2bs} transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into bandstop filters with desired bandwidth and center frequency. The transformation is one step in the digital filter design process for the \text{butter}, \text{cheby1}, \text{cheby2}, and \text{ellip} functions.

\text{lp2bs} can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

\[ [bt, at] = \text{lp2bs}(b,a,Wo,Bw) \]
transforms an analog lowpass filter prototype given by polynomial coefficients into a bandstop filter with center frequency \( Wo \) and bandwidth \( Bw \). Row vectors \( b \) and \( a \) specify the coefficients of the numerator and denominator of the prototype in descending powers of \( s \):

\[
\frac{b(s)}{a(s)} = \frac{b(1)s^{nb} + \cdots + b(nb)s + b(nb + 1)}{a(1)s^{na} + \cdots + a(na)s + a(na + 1)}
\]

Scalars \( Wo \) and \( Bw \) specify the center frequency and bandwidth in units of radians/second. For a filter with lower band edge \( w1 \) and upper band edge \( w2 \), use \( Wo = \sqrt{w1 \cdot w2} \) and \( Bw = w2 - w1 \).

\text{lp2bs} returns the frequency transformed filter in row vectors \( bt \) and \( at \).

State-Space Form

\[ [At,Bt,Ct,Dt] = \text{lp2bs}(A,B,C,D,Wo,Bw) \]
converts the continuous-time state-space lowpass filter prototype in matrices \( A, B, C, D \):

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]
into a bandstop filter with center frequency $\omega_0$ and bandwidth $B_w$. For a filter with lower band edge $w_1$ and upper band edge $w_2$, use $\omega_0 = \sqrt{w_1 \cdot w_2}$ and $B_w = w_2 - w_1$.

The bandstop filter is returned in matrices $A_t$, $B_t$, $C_t$, $D_t$.

**Algorithm**

$lp2bs$ is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a bandstop filter is to have center frequency $\omega_0$ and bandwidth $B_w$, the standard s-domain transformation is

$$s = \frac{p}{Q(p^2 + 1)}$$

where $Q = \frac{\omega_0}{B_w}$ and $p = s/\omega_0$. The state-space version of this transformation is

$$Q = \frac{\omega_0}{B_w};$$

$$A_t = [\omega_0/Q \cdot inv(A) \ \omega_0 \cdot eye(ma); \ -\omega_0 \cdot eye(ma) \ \text{zeros}(ma)];$$

$$B_t = -[\omega_0/Q \cdot (A \ B); \ \text{zeros}(ma,nb)];$$

$$C_t = [C/A \ \text{zeros}(mc,ma)];$$

$$D_t = D - C/A \cdot B;$$

See $lp2bp$ for a derivation of the bandpass version of this transformation.

**See Also**

- **bilinear** Bilinear transformation method of analog-to-digital filter conversion.
- **impinvar** Impulse invariance method of analog-to-digital filter conversion.
- **lp2bp** Lowpass to bandpass analog filter transformation.
- **lp2hp** Lowpass to highpass analog filter transformation.
- **lp2lp** Lowpass to lowpass analog filter transformation.
**lp2hp**

**Purpose**
Lowpass to highpass analog filter transformation.

**Syntax**

\[
[bt, at] = \text{lp2hp}(b, a, Wo) \\
[At, Bt, Ct, Dt] = \text{lp2hp}(A, B, C, D, Wo)
\]

**Description**

1p2hp transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into highpass filters with desired cutoff frequency. The transformation is one step in the digital filter design process for the butter, cheby1, cheby2, and ellip functions.

The lp2hp function can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

**Transfer Function Form (Polynomial)**

\[
[bt, at] = \text{lp2hp}(b, a, Wo)
\]
transforms an analog lowpass filter prototype given by polynomial coefficients into a highpass filter with cutoff frequency \(Wo\).

Row vectors \(b\) and \(a\) specify the coefficients of the numerator and denominator of the prototype in descending powers of \(s\):

\[
\begin{align*}
b(s) &= b(1)s^{nb} + \cdots + b(nb)s + b(nb + 1) \\
a(s) &= a(1)s^{na} + \cdots + a(na)s + a(na + 1)
\end{align*}
\]

Scalar \(Wo\) specifies the cutoff frequency in units of radians/second. The frequency transformed filter is returned in row vectors \(bt\) and \(at\).

**State-Space Form**

\[
[At, Bt, Ct, Dt] = \text{lp2hp}(A, B, C, D, Wo)
\]
converts the continuous-time state-space lowpass filter prototype in matrices \(A, B, C, D\):

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]
into a highpass filter with cutoff frequency \(Wo\). The highpass filter is returned in matrices \(At, Bt, Ct, Dt\).
Algorithm

lp2hp is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a highpass filter is to have cutoff frequency \( \omega_0 \), the standard s-domain transformation is

\[
    s = \frac{\omega_0}{p}
\]

The state-space version of this transformation is

\[
    A_t = Wo * inv(A);
    B_t = -Wo*(A\B);
    C_t = C/A;
    D_t = D - C/A*B;
\]

See \( \text{lp2bp} \) for a derivation of the bandpass version of this transformation.

See Also

- \( \text{bilinear} \)  
  Bilinear transformation method of analog-to-digital filter conversion.
- \( \text{impinvar} \)  
  Impulse invariance method of analog-to-digital filter conversion.
- \( \text{lp2bp} \)  
  Lowpass to bandpass analog filter transformation.
- \( \text{lp2bs} \)  
  Lowpass to bandstop analog filter transformation.
- \( \text{lp2lp} \)  
  Lowpass to lowpass analog filter transformation.
Purpose

Lowpass to lowpass analog filter transformation.

Syntax

\[ [b_t, a_t] = \text{lp2lp}(b, a, Wo) \]
\[ [A_t, B_t, C_t, D_t] = \text{lp2lp}(A, B, C, D, Wo) \]

Description

\text{lp2lp} transforms an analog lowpass filter prototype with a cutoff frequency of 1 rad/sec into a lowpass filter with any specified cutoff frequency. The transformation is one step in the digital filter design process for the \text{butter}, \text{cheby1}, \text{cheby2}, and \text{ellip} functions.

The \text{lp2lp} function can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

\[ [b_t, a_t] = \text{lp2lp}(b, a, Wo) \] transforms an analog lowpass filter prototype given by polynomial coefficients into a lowpass filter with cutoff frequency \( Wo \). Row vectors \( b \) and \( a \) specify the coefficients of the numerator and denominator of the prototype in descending powers of \( s \):

\[
\frac{b(s)}{a(s)} = \frac{b(1)s^{nb} + \cdots + b(nb)s + b(nb + 1)}{a(1)s^{na} + \cdots + a(na)s + a(na + 1)}
\]

Scalar \( Wo \) specifies the cutoff frequency in units of radians/second. \text{lp2lp} returns the frequency transformed filter in row vectors \( b_t \) and \( a_t \).

State-Space Form

\[ [A_t, B_t, C_t, D_t] = \text{lp2lp}(A, B, C, D, Wo) \] converts the continuous-time state-space lowpass filter prototype in matrices \( A, B, C, D \):

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

into a lowpass filter with cutoff frequency \( Wo \). \text{lp2lp} returns the lowpass filter in matrices \( A_t, B_t, C_t, D_t \).
1p2lp is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a lowpass filter is to have cutoff frequency $\omega_0$, the standard $s$-domain transformation is

$$s = \frac{p}{\omega_0}$$

The state-space version of this transformation is

$$A_t = \omega_0 A;$$
$$B_t = \omega_0 B;$$
$$C_t = C;$$
$$D_t = D;$$

See 1p2bp for a derivation of the bandpass version of this transformation.

**See Also**

- **bilinear**: Bilinear transformation method of analog-to-digital filter conversion.
- **impinvar**: Impulse invariance method of analog-to-digital filter conversion.
- **1p2bp**: Lowpass to bandpass analog filter transformation.
- **1p2bs**: Lowpass to bandstop analog filter transformation.
- **1p2hp**: Lowpass to highpass analog filter transformation.
**Purpose**
Linear prediction coefficients.

**Syntax**
\[ a = \text{lpc}(x,p) \]

**Description**
`lpc` determines the coefficients of a forward linear predictor by minimizing the prediction error in the least-squares sense. It has applications in filter design and speech coding.

\[ a = \text{lpc}(x,p) \]
finds the coefficients of a \( p \)th-order linear predictor (FIR filter) that predicts the current value of the real time series \( x \) based on past samples:

\[ x(n) = -a(2)x(n-1) - a(3)x(n-2) - \cdots - a(p+1)x(n-p) \]

\( p \) is the order of the prediction polynomial, \( a = [1 \ a(2) \ldots \ a(p+1)] \).

If \( p \) is unspecified, `lpc` uses as a default \( p = \text{length}(x) - 1 \). If \( x \) is a matrix containing a separate signal in each column, `lpc` returns a model estimate for each column in the rows of matrix \( a \).

**Example**
Estimate a data series using a 3rd-order forward predictor, and compare to the original signal.

First, create the signal data as the output of an autoregressive process driven by white noise. Use the last 4096 samples of the AR process output to avoid start-up transients.

\[
\begin{align*}
\text{noise} &= \text{randn}(50000,1); & \text{Normalized white Gaussian noise} \\
\text{x} &= \text{filter}(1,[1 \ 1/2 \ 1/3 \ 1/4],\text{noise}); \\
\text{x} &= \text{x}(45904:50000);
\end{align*}
\]

Compute the predictor coefficients, estimated signal, prediction error, and autocorrelation sequence of the prediction error.

\[
\begin{align*}
\text{a} &= \text{lpc}(\text{x},3); \\
\text{est}_x &= \text{filter}([0 \ -a(2:end)],1,\text{x}); & \text{Estimated signal} \\
\text{e} &= \text{x} - \text{est}_x; & \text{Prediction error} \\
[\text{acs},\text{lags}] &= \text{xcorr}(|\text{e}|,'\text{coeff'}); & \text{ACS of prediction error}
\end{align*}
\]
The prediction error, $e(n)$, can be viewed as the output of the prediction error filter $A(z)$ shown below, where $H(z)$ is the optimal linear predictor, $x(n)$ is the input signal, and $\hat{x}(n)$ is the predicted signal.

```
plot(1:97, x(4001:4097), 1:97, est_x(4001:4097), '--');
title('Original Signal vs. LPC Estimate');
xlabel('Samples'); ylabel('Amplitude'); grid;
legend('Original Signal', 'LPC Estimate')
```

Look at the autocorrelation of the prediction error.

```
plot(lags, acs);
title('Autocorrelation of the Prediction Error');
xlabel('Lags'); ylabel('Normalized Value'); grid;
```
The prediction error is approximately white Gaussian noise, as expected for a 3rd-order AR input process.

Algorithm

`lpc` uses the autocorrelation method of autoregressive (AR) modeling to find the filter coefficients. The generated filter might not model the process exactly even if the data sequence is truly an AR process of the correct order. This is because the autocorrelation method implicitly windows the data, that is, it assumes that signal samples beyond the length of \( x \) are 0.

`lpc` computes the least-squares solution to

\[
Xa = b
\]

where

\[
X = \begin{bmatrix}
    x(1) & 0 & \cdots & 0 \\
    x(2) & x(1) & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    x(m) & \cdots & x(1) & 0 \\
    0 & x(m) & \cdots & x(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & x(m)
\end{bmatrix}, \quad a = \begin{bmatrix}
    1 \\
    a(2) \\
    \vdots \\
    a(p+1)
\end{bmatrix}, \quad b = \begin{bmatrix}
    1 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
\]
and m is the length of x. Solving the least-squares problem via the normal equations

\[ X^H X a = X^H b \]

leads to the Yule-Walker equations

\[
\begin{bmatrix}
  r(1) & r(2)^* & \cdots & r(p)^*
  r(2) & r(1) & \ddots & \vdots
  \vdots & \ddots & \ddots & \vdots
  r(p) & \cdots & r(2) & r(1)
\end{bmatrix}
\begin{bmatrix}
a(2) \\
a(3) \\
\vdots \\
a(p+1)
\end{bmatrix}
= 
\begin{bmatrix}
-r(2) \\
-r(3) \\
\vdots \\
-r(p+1)
\end{bmatrix}
\]

where \( r = [r(1) r(2) \ldots r(p+1)] \) is an autocorrelation estimate for x computed using xcorr. The Yule-Walker equations are solved in \( O(p^2) \) flops by the Levinson-Durbin algorithm (see levinson).

### See Also

- aryule: Compute an estimate of AR model parameters using the Yule-Walker method.
- levinson: Levinson-Durbin recursion.
- prony: Prony’s method for time domain IIR filter design.
- stmcb: Linear model using Steiglitz-McBride iteration.

### References

**Purpose**
Generalized digital Butterworth filter design.

**Syntax**

```matlab
[b,a,] = maxflat(nb,na,Wn)
b = maxflat(nb,'sym',Wn)
[b,a,b1,b2] = maxflat(nb,na,Wn)
[... ] = maxflat(nb,na,Wn,'design_flag')
```

**Description**

`[b,a,] = maxflat(nb,na,Wn)` is a lowpass Butterworth filter with numerator and denominator coefficients `b` and `a` of orders `nb` and `na` respectively. `Wn` is the cutoff frequency at which the magnitude response of the filter is equal to \(1/\sqrt{2}\) (approx. -3 dB). `Wn` must be between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

`b = maxflat(nb,'sym',Wn)` is a symmetric FIR Butterworth filter. `nb` must be even, and `Wn` is restricted to a subinterval of \([0,1]\). The function raises an error if `Wn` is specified outside of this subinterval.

`[b,a,b1,b2] = maxflat(nb,na,Wn)` returns two polynomials `b1` and `b2` whose product is equal to the numerator polynomial `b` (that is, `b = conv(b1,b2)`). `b1` contains all the zeros at \(z = -1\), and `b2` contains all the other zeros.

`[... ] = maxflat(nb,na,Wn,'design_flag')` enables you to monitor the filter design, where `design_flag` is
- `trace`, for a textual display of the design table used in the design
- `plots`, for plots of the filter's magnitude, group delay, and zeros and poles
- `both`, for both the textual display and plots
Examples

```matlab
nb = 10; na = 2; Wn = 0.2*pi;
[b,a,b1,b2] = maxflat(nb,na,Wn,'plots')
```

Algorithm
The method consists of the use of formulae, polynomial root finding, and a
transformation of polynomial roots.

See Also
- **butter**: Butterworth analog and digital filter design.
- **filter**: Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- **freqz**: Frequency response of digital filters.

References
Purpose

One-dimensional median filtering.

Syntax

\[
y = \text{medfilt1}(x,n) \\
y = \text{medfilt1}(x,n,\text{blksz})
\]

Description

\[y = \text{medfilt1}(x,n)\] applies an order \(n\), one-dimensional median filter to vector \(x\). \(y\) is the same length as \(x\); the function treats the signal as if it is 0 beyond the end points.

For \(n\) odd, \(y(k)\) is the median of \(x(k-(n-1)/2:k+(n-1)/2)\).

For \(n\) even, \(y(k)\) is the median of \(x(k-n/2), x(k-(n/2)+1), ..., x(k+(n/2)-1)\). In this case, \text{medfilt1} sorts the numbers, then takes the average of the \((n-1)/2\) and \(((n-1)/2)+1\) elements.

The default for \(n\) is 3.

\[y = \text{medfilt1}(x,n,\text{blksz})\] uses a for-loop to compute \(\text{blksz}\) (block size) output samples at a time. Use \(\text{blksz} < \text{length}(x)\) if you are low on memory, since \text{medfilt1} uses a working matrix of size \(n\)-by-\(\text{blksz}\). By default, \(\text{blksz} = \text{length}(x)\); this is the fastest execution if you have sufficient memory.

If \(x\) is a matrix, \text{medfilt1} median filters its columns using

\[y(:,i) = \text{medfilt1}(x(:,i),n,\text{blksz})\]

in a loop over the columns of \(x\).

See Also

\begin{align*}
\text{filter} & : \text{Filter data with a recursive (IIR) or nonrecursive (FIR) filter.} \\
\text{medfilt2} & : \text{Two-dimensional median filtering (see Image Processing Toolbox User's Guide).} \\
\text{median} & : \text{Median value (see the online MATLAB Function Reference).}
\end{align*}

References

modulate

**Purpose**
Modulation for communications simulation.

**Syntax**
\[
y = \text{modulate}(x,Fc,Fs,’method’) \\
y = \text{modulate}(x,Fc,Fs,’method’,opt) \\
[y,t] = \text{modulate}(x,Fc,Fs)
\]

**Description**
\[
y = \text{modulate}(x,Fc,Fs,’method’) \quad \text{and} \quad y = \text{modulate}(x,Fc,Fs,’method’,opt)
\]

\[
y = \text{modulate}(x,Fc,Fs,’method’,opt) \quad \text{modulate the real message signal } x \quad \text{with a carrier frequency } Fc \quad \text{and sampling frequency } Fs, \quad \text{using one of the options listed below for } method. \quad \text{Note that some methods accept an option, } opt.
\]

- **amdsb–sc**: **Amplitude modulation, double sideband, suppressed carrier.**
  Multiplies \( x \) by a sinusoid of frequency \( Fc \):
  \[
y = x.*\cos(2*pi*Fc*t)
\]

- **amdsb–tc**: **Amplitude modulation, double sideband, transmitted carrier.**
  Subtracts scalar \( opt \) from \( x \) and multiplies the result by a sinusoid of frequency \( Fc \):
  \[
y = (x–opt).*\cos(2*pi*Fc*t)
\]
  If the \( opt \) parameter is not present, modulate uses a default of \( \min(\min(x)) \) so that the message signal \( (x–opt) \) is entirely non-negative and has a minimum value of 0.

- **amssb**: **Amplitude modulation, single sideband.**
  Multiplies \( x \) by a sinusoid of frequency \( Fc \) and adds the result to the Hilbert transform of \( x \) multiplied by a phase shifted sinusoid of frequency \( Fc \):
  \[
y = x.*\cos(2*pi*Fc*t)+\text{imag(hilbert(x))}.*\sin(2*pi*Fc*t)
\]
  This effectively removes the upper sideband.
**modulate**

**fm**  
**Frequency modulation.** Creates a sinusoid with instantaneous frequency that varies with the message signal x:

\[ y = \cos(2\pi F_c t + \text{opt} \cdot \text{cumsum}(x)) \]

cumsum is a rectangular approximation to the integral of x. modulate uses opt as the constant of frequency modulation. If opt is not present, modulate uses a default of

\[ \text{opt} = (F_c/F_s)*2\pi/(\max(\max(x))) \]

so the maximum frequency excursion from Fc is Fc Hz.

**pm**  
**Phase modulation.** Creates a sinusoid of frequency Fc whose phase varies with the message signal x:

\[ y = \cos(2\pi F_c t + \text{opt} \cdot x) \]

modulate uses opt as the constant of phase modulation. If opt is not present, modulate uses a default of

\[ \text{opt} = \pi/(\max(\max(x))) \]

so the maximum phase excursion is \pi radians.

**pwm**  
**Pulse-width modulation.** Creates a pulse-width modulated signal from the pulse widths in x. The elements of x must be between 0 and 1, specifying the width of each pulse in fractions of a period. The pulses start at the beginning of each period, that is, they are left justified.

\[
\text{modulate}(x,F_c,F_s,'\text{pwm}','\text{centered}')
\]
yields pulses centered at the beginning of each period. y is length

\[ \text{length}(x)*F_s/F_c \]

**ptm**  
**Pulse time modulation.** Creates a pulse time modulated signal from the pulse times in x. The elements of x must be between 0 and 1, specifying the left edge of each pulse in fractions of a period. opt is a scalar between 0 and 1 that specifies the length of each pulse in fractions of a period. The default for opt is 0.1. y is length

\[ \text{length}(x)*F_s/F_c \]

**qam**  
**Quadrature amplitude modulation.** Creates a quadrature amplitude modulated signal from signals x and opt:

\[ y = x.*\cos(2\pi F_c t) + \text{opt}.*\sin(2\pi F_c t) \]

opt must be the same size as x.
If you do not specify `method`, then `modulate` assumes `am`. Except for the `pwm` and `ptm` cases, `y` is the same size as `x`.

If `x` is an array, `modulate` modulates its columns.

`[y,t] = modulate(x,Fc,Fs)` returns the internal time vector `t` that `modulate` uses in its computations.

**See Also**

- **demod** Demodulation for communications simulation.
- **vco** Voltage controlled oscillator.
**Purpose**

Power spectrum estimate using the Burg method.

**Syntax**

- `Pxx = pburg(x,p,nfft)`
- `[Pxx,freq] = pburg(x,p,nfft)`
- `[Pxx,freq] = pburg(x,p,nfft,Fs)`
- `[Pxx,freq] = pburg(x,p,nfft,Fs,'range')`
- `pburg(...)`
- `pburg(...,'squared')`

**Description**

`pburg` estimates the power spectral density (PSD) of the signal vector `x[n]` using the Burg method. This method fits an autoregressive (AR) model to the signal by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. The spectral estimate returned by `pburg` is the magnitude squared frequency response of this AR model. The correct choice of the model order `p` is important.

- `Pxx = pburg(x,p,nfft)` returns `Pxx`, the power spectrum estimate. `x` is the input signal, `p` is the model order for the all-pole filter, and `nfft` is the FFT length (defaults to 256 if not specified). `Pxx` has length `(nfft/2+1)` for `nfft` even, `(nfft+1)/2` for `nfft` odd, and `nfft` if `x` is complex.

- `[Pxx,freq] = pburg(x,p,nfft)` returns `Pxx`, the power spectrum estimate, and `freq`, a vector of frequencies at which the PSD was estimated. If the input signal is real-valued, the range of `freq` is `[0,π)`. If the input signal is complex, the range of `freq` is `[0,2π)`.

- `[Pxx,freq] = pburg(x,p,nfft,Fs)` uses the signal's sampling frequency, `Fs`, to scale both the PSD vector (`Pxx`) and the frequency vector (`freq`). `Pxx` is scaled by `1/Fs`. If the input signal is real-valued, the range of `freq` is `[0,Fs/2)`. If the input signal is complex, the range of `freq` is `[0,Fs)`.

- `[Pxx,freq] = pburg(x,p,nfft,Fs,'range')` specifies the range of frequency values to include in `freq`. `range` can be:
  - `half`, to compute the PSD over the range `[0,Fs/2)` for real `x`, and `[0,Fs)` for complex `x`. If `Fs` is left blank, `[]`, the range is `[0,1/2)` for real `x`, and `[0,1)`
for complex x. If Fs is omitted entirely, the range is [0, \pi) for real x, and [0,2\pi) for complex x. half is the default range.

- whole, to compute the PSD over the range [0,Fs) for all x. If Fs is left blank, [], the range is [0,1) for all x. If Fs is omitted entirely, the range is [0,2\pi) for all x.

pburg(...) plots the power spectral density in the current figure window. The frequency range on the plot is the same as the range of output freq for a given set of parameters.

pburg(...,'squared') plots the magnitude of Pxx directly, rather than converting the values to dB.

Example

Because the method estimates the spectrum by fitting an AR model to the signal, first define the AR system (all-pole filter) that generates the input. Check the magnitude response of the process with freqz.

```matlab
a = [1 -2.2137 2.9403 -2.1697 0.9606]; % AR system coefficients
freqz(1,a,'phase','no') % AR system magnitude response
title('AR System Magnitude Response')
```

![AR System Magnitude Response](image)
Now generate the input signal \( x \) by filtering white noise through the AR system. Estimate the PSD of \( x \) based on a fourth-order AR model (since we know that the original AR system, \( a \), has order 4).

\[
x = \text{filter}(1,a,\text{randn}(150,1)); \quad \% \text{AR system output}
pburg(x,4) \quad \% \text{4th order estimate}
\]

**Diagnostics**

The first input argument must be a full vector; otherwise \textit{pburg} generates the following error message:

\begin{quote}
Input signal cannot be sparse.
\end{quote}

If you specify an empty matrix for the second argument, \textit{pburg} generates the following error message:

\begin{quote}
Model order must be an integer.
\end{quote}
pburg

See Also

arburg
Compute an estimate of AR model parameters using the Burg method.

lpc
Linear prediction coefficients.

pcov
Power spectrum estimate using the covariance method.

pmcov
Power spectrum estimate using the modified covariance method.

pmtm
Power spectrum estimate using the multitaper method (MTM).

pmusic
Power spectrum estimate using MUSIC eigenvector method.

prony
Prony's method for time domain IIR filter design.

pwelch
Estimate the power spectral density (PSD) of a signal using Welch's method.

pyulear
Power spectrum estimate using Yule-Walker AR method.

References

Purpose

Power spectrum estimate using the covariance method.

Syntax

Pxx = pcov(x,p,nfft)
[Pxx,freq] = pcov(x,p,nfft)
[Pxx,freq] = pcov(x,p,nfft,Fs)
[Pxx,freq] = pcov(x,p,nfft,Fs,'range')
pcov(...)
pcov(...,'squared')

Description

pcov estimates the power spectral density (PSD) of the signal vector \(x[n]\) using the covariance method. This method fits an autoregressive (AR) model to the signal by minimizing the forward prediction error in the least-squares sense. The spectral estimate returned by pcov is the magnitude squared frequency response of this AR model. The correct choice of the model order \(p\) is important.

\(Pxx = pcov(x,p,nfft)\) returns \(Pxx\), the power spectrum estimate. \(x\) is the input signal, \(p\) is the model order for the all-pole filter, and \(nfft\) is the FFT length (defaults to 256 if not specified). \(Pxx\) has length \((nfft/2+1)\) for \(nfft\) even, \((nfft+1)/2\) for \(nfft\) odd, and \(nfft\) if \(x\) is complex.

\([Pxx,freq] = pcov(x,p,nfft)\) returns \(Pxx\), the power spectrum estimate, and \(freq\), a vector of frequencies at which the PSD was estimated. If the input signal is real-valued, the range of \(freq\) is \([0,\pi)\). If the input signal is complex, the range of \(freq\) is \([0,2\pi)\).

\([Pxx,freq] = pcov(x,p,nfft,Fs)\) uses the signal’s sampling frequency, \(Fs\), to scale both the PSD vector \((Pxx)\) and the frequency vector \((freq)\). \(Pxx\) is scaled by \(1/Fs\). If the input signal is real-valued, the range of \(freq\) is \([0,Fs/2)\). If the input signal is complex, the range of \(freq\) is \([0,Fs)\). \(Fs\) defaults to 1 if left empty, \([]\).

\([Pxx,freq] = pcov(x,p,nfft,Fs,'range')\) specifies the range of frequency values to include in \(freq\). range can be:

- half, to compute the PSD over the range \([0,Fs/2)\) for real \(x\), and \([0,Fs)\) for complex \(x\). If \(Fs\) is left blank, \([]\), the range is \([0,1/2)\) for real \(x\), and \([0,1)\)
for complex x. If Fs is omitted entirely, the range is \([0, \pi]\) for real x, and 
\([0, 2\pi]\) for complex x. half is the default range.

- whole, to compute the PSD over the range \([0, Fs]\) for all x. If Fs is left 
  blank, [], the range is \([0, 1]\) for all x. If Fs is omitted entirely, the range is 
  \([0, 2\pi]\) for all x.

`pcov(...)` plots the power spectral density in the first available figure window. 
The frequency range on the plot is the same as the range of output freq for a 
given set of parameters.

`pcov(...,'squared')` plots the magnitude of Pxx directly, rather than 
converting the values to dB.

**Example**

Because the method estimates the spectrum by fitting an AR model to the 
signal, first define the AR system (all-pole filter) that generates the input. 
Check the magnitude response of the process with `freqz`.

```matlab
a = [1 -2.2137 2.9403 -2.1697 0.9606]; % AR system coefficients
freqz(1,a,'phase','no') % AR system magnitude response
title('AR System Magnitude Response')
```

![AR System Magnitude Response](image)
Now generate the input signal $x$ by filtering white noise through the AR system. Estimate the PSD of $x$ based on a fourth-order AR model (since we know that the original AR system, $a$, has order 4).

```matlab
x = filter(1,a(randn(150,1))); % AR system output
pcov(x,4) % 4th order estimate
```

**Diagnostics**

The first input argument must be a full vector; otherwise `pcov` generates the following error message:

Input signal cannot be sparse.

If you specify an empty matrix for the second argument, `pcov` generates the following error message:

Model order must be an integer.
See Also

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References

pmcov

Purpose

Power spectrum estimate using the modified covariance method.

Syntax

Pxx = pmcov(x,p,nfft)
[Pxx,freq] = pmcov(x,p,nfft)
[Pxx,freq] = pmcov(x,p,nfft,Fs)
[Pxx,freq] = pmcov(x,p,nfft,Fs,'range')
pmcov(...)  
pmcov(...,'squared')

Description

pmcov estimates the power spectral density (PSD) of the signal vector \( x[n] \) using the modified covariance method. This method fits an autoregressive (AR) model to the signal by minimizing the forward and backward prediction errors in the least-squares sense. The spectral estimate returned by pmcov is the magnitude squared frequency response of this AR model. The correct choice of the model order \( p \) is important.

\[
Pxx = pmcov(x,p,nfft)
\]
returns \( Pxx \), the power spectrum estimate. \( x \) is the input signal, \( p \) is the model order for the all-pole filter, and \( nfft \) is the FFT length (defaults to 256 if not specified). \( Pxx \) has length \( (nfft/2+1) \) for \( nfft \) even, \( (nfft+1)/2 \) for \( nfft \) odd, and \( nfft \) if \( x \) is complex.

\[
[Pxx,freq] = pmcov(x,p,nfft)
\]
returns \( Pxx \), the power spectrum estimate, and \( freq \), a vector of frequencies at which the PSD was estimated. If the input signal is real-valued, the range of \( freq \) is \([0,\pi)\). If the input signal is complex, the range of \( freq \) is \([0,2\pi)\).

\[
[Pxx,freq] = pmcov(x,p,nfft,Fs)
\]
uses the signal’s sampling frequency, \( Fs \), to scale both the PSD vector (\( Pxx \)) and the frequency vector (\( freq \)). \( Pxx \) is scaled by \( 1/Fs \). If the input signal is real-valued, the range of \( freq \) is \([0,Fs/2)\). If the input signal is complex, the range of \( freq \) is \([0,Fs)\). \( Fs \) defaults to 1 if left empty, \( \)\.

\[
[Pxx,freq] = pmcov(x,p,nfft,Fs,'range')
\]
specifies the range of frequency values to include in \( freq \). \( range \) can be:

- half, to compute the PSD over the range \([0,Fs/2)\) for real \( x \), and \([0,Fs)\) for complex \( x \). If \( Fs \) is left blank, \( \), the range is \([0,1/2)\) for real \( x \), and \([0,1)\)
pmcov

for complex $x$. If $F_s$ is omitted entirely, the range is $[0, \pi)$ for real $x$, and $[0, 2\pi)$ for complex $x$. `half` is the default range.

- `whole`, to compute the PSD over the range $[0, F_s)$ for all $x$. If $F_s$ is left blank, [], the range is $[0, 1)$ for all $x$. If $F_s$ is omitted entirely, the range is $[0, 2\pi)$ for all $x$.

`pmcov(...)` plots the power spectral density in the first available figure window. The frequency range on the plot is the same as the range of output `freq` for a given set of parameters.

`pmcov(...,'squared')` plots the magnitude of $P_{xx}$ directly, rather than converting the values to dB.

**Example**

Because the method estimates the spectrum by fitting an AR model to the signal, first create the AR system (all-pole filter) that generates the input. Check the magnitude response of the process with `freqz`.

```plaintext
a = [1 -2.2137 2.9403 -2.1697 0.9606]; % AR system coefficients
freqz(1,a,'phase','no') % AR system magnitude response
title('AR System Magnitude Response')
```

![AR System Magnitude Response](image_url)
Now generate the input signal \( x \) by filtering white noise through the AR system. Estimate the PSD of \( x \) based on a fourth-order AR model (since we know that the original AR system, \( a \), has order 4).

\[
x = \text{filter}(1,a,\text{randn(150,1)}); \quad \% \text{AR system output}
\]

\[
\text{pmcov}(x,4) \quad \% \text{4th order estimate}
\]

**Diagnostics**

The first input argument must be a full vector; otherwise \texttt{pmcov} generates the following error message:

Input signal cannot be sparse.

If you specify an empty matrix for the second argument, \texttt{pmcov} generates the following error message:

Model order must be an integer.
pmcov

See Also

armcov  Compute an estimate of AR model parameters using the modified covariance method.
lpc  Linear prediction coefficients.
pburg  Power spectrum estimate using the Burg method.
pcov  Power spectrum estimate using the covariance method.
pmtm  Power spectrum estimate using the multitaper method (MTM).
pmusic  Power spectrum estimate using MUSIC eigenvector method.
prony  Prony’s method for time domain IIR filter design.
pwelch  Estimate the power spectral density (PSD) of a signal using Welch’s method.
pyulear  Power spectrum estimate using Yule-Walker AR method.

References

pmtm

Purpose

Power spectrum estimate using the multitaper method (MTM).

Syntax

\[
Pxx = \text{pmtm}(x,nw) \\
Pxx = \text{pmtm}(x,nw,nfft) \\
[Pxx,f] = \text{pmtm}(x,nw,nfft,Fs) \\
[Pxx,f] = \text{pmtm}(x,nw,nfft,'method') \\
[Pxx,Pxxc,f] = \text{pmtm}(x,nw,nfft,Fs,'method') \\
[Pxx,Pxxc,f] = \text{pmtm}(x,nw,nfft,Fs,'method',p) \\
[Pxx,Pxxc,f] = \text{pmtm}(x,e,v,nfft,Fs,'method',p) \\
[Pxx,Pxxc,f] = \text{pmtm}(x,dpss_params,nfft,Fs,'method',p)
\]

Description

\text{pmtm} estimates the power spectral density (PSD) of the real time series \(x\) using the multitaper method (MTM), described in [1].

\[Pxx = \text{pmtm}(x,nw)\] estimates the PSD using \(nw\) as the time-bandwidth product for the discrete prolate spheroidal sequences (Slepian sequences) that are used as data windows. The default for \(nw\) is 4; other typical choices are 2, 5/2, 3, 7/2. The number of sequences used to form \(Pxx\) is \(2\times nw - 1\).

\[Pxx = \text{pmtm}(x,nw,nfft)\] defines the frequency grid as length \(nfft\). When \(x\) is real, \(Pxx\) is length \((nfft/2+1)\) for \(nfft\) even and \((nfft+1)/2\) for \(nfft\) odd; when \(x\) is complex, \(Pxx\) is length \(nfft\). The default for \(nfft\) is 256 or the next power of 2 greater than the length of \(x\), whichever is larger.

\[Pxx = \text{pmtm}(x,nw,nfft,Fs)\] returns \(f\), the vector of frequencies at which the PSD is estimated, for the sampling frequency \(Fs\). The default for \(Fs\) is 1 Hz.

\[Pxx = \text{pmtm}(x,nw,nfft,Fs,'method')\] specifies the algorithm used for combining the individual spectral estimates, where method is:

- adapt, to specify Thomson’s adaptive nonlinear combination (default)
- unity, to specify a linear combination with unity weights
- eigen, to specify a linear combination with eigenvalue weights

\[Pxx = \text{pmtm}(x,nw,nfft,Fs,'method',p)\] returns \(Pxxc\), the 95% confidence interval for \(Pxx\), and

\[Pxx = \text{pmtm}(x,nw,nfft,Fs,'method',p)\] returns \(Pxxc\), the \(p\times100\%\) confidence interval for \(Pxx\), where \(p\) is a scalar between 0 and 1. Confidence
intervals are computed using a chi-squared approach, where $P_{xxc(:,1)}$ is the lower bound and $P_{xxc(:,2)}$ is the upper bound of the confidence interval.

$[P_{xx}, P_{xxc}, f] = \text{pmtm}(x, e, v, nfft, Fs, 'method', p)$ returns the PSD estimate $P_{xx}$, the confidence interval $P_{xxc}$, and the frequency vector $f$ from the data tapers in $e$ and their concentrations $v$.

$[P_{xx}, P_{xxc}, f] = \text{pmtm}(x, \text{dpss}_\text{params}, nfft, Fs, 'method', p)$ returns the PSD estimate $P_{xx}$, the confidence interval $P_{xxc}$, and the frequency vector $f$ from the data tapers computed using $\text{dpss}$ with parameters from the cell array $\text{dpss}_\text{params}$, whose first element is the second input to $\text{dpss}$. The first $\text{dpss}$ parameter ($n$) is determined by the length of $x$. For example, $\text{pmtm}(x, \{3.5, 'trace'\}, 512, Fs)$ calculates the Slepian sequences for $nw=3.5$, and displays the method that $\text{dpss}$ uses. See $\text{dpss}$ for other options.

**Remarks**

$\text{pmtm}$ with no output arguments plots the PSD in the current or next available figure, with confidence intervals.

To use default parameters for any argument in an expression, insert an empty matrix $[]$. For example, $\text{pmtm}(x,[],[],1000)$ uses defaults for the second and third elements, in this case, $nw$ and $nfft$. 
Example

This example analyzes a sinusoid in white noise:

```matlab
Fs = 1000; t = 0:1/Fs:0.3;
x = cos(2*pi*t*200) + randn(size(t));
[Pxx,Pxxc,f] = pmtm(x,3.5,512,Fs,[],0.99);
plot(f,10*log10([Pxx Pxxc]))
```

See Also

- **dpss**: Discrete prolate spheroidal sequences (Slepian sequences).
- **pburg**: Power spectrum estimate using the Burg method.
- **pcov**: Power spectrum estimate using the covariance method.
- **pmcov**: Power spectrum estimate using the modified covariance method.
- **pmusic**: Power spectrum estimate using MUSIC eigenvector method.
- **pwelch**: Estimate the power spectral density (PSD) of a signal using Welch's method.
- **pyulear**: Power spectrum estimate using Yule-Walker AR method.
References


pmusic

Purpose

Power spectrum estimate using MUSIC eigenvector method.

Syntax

\[
\begin{align*}
[Pxx,f] &= \text{pmusic}(x,p) \\
[Pxx,f] &= \text{pmusic}(x,[p \text{ thresh}]) \\
[Pxx,f] &= \text{pmusic}(x,[p \text{ thresh}],\text{nfft},\text{Fs},\text{window},\text{noverlap}) \\
[Pxx,f] &= \text{pmusic}(x,...,'\text{corr}') \\
[Pxx,f] &= \text{pmusic}(x,...,'\text{ev}') \\
[Pxx,f,evects,svals] &= \text{pmusic}(x,...)
\end{align*}
\]

Description

pmusic estimates the power spectral density (PSD) of a signal or correlation matrix using Schmidt’s eigen-analysis method [1]. The name MUSIC is an acronym for MUltiple SIgnal Classification. The eigenvector method, which uses eigenvalue weighting, is also supported [2]. The calling syntax is similar to that of pwelch, which also performs spectrum estimation. pwelch uses the classical FFT-based approach while pmusic performs eigen-analysis of the signal’s correlation matrix.

\[
[Pxx,f] = \text{pmusic}(x,p) \text{ and }
\]

\[
[Pxx,f] = \text{pmusic}(x,[p \text{ thresh}]) \text{ return } Pxx, \text{ the power spectrum estimate,}
\]

and \( f \), a vector of frequencies at which the PSD is estimated. \( x \) is the input signal, where:

- A row or column vector represents one observation of the process output (for example, one “signal”)
- A rectangular (possibly square) array assumes that each column of \( x \) is a separate observation of the process output (for example, each column is one output of an array of sensors, as in array processing)
- A square matrix, given the trailing argument ‘corr’, represents a correlation matrix

The second argument is a one- or two-element vector, either \( p \) or \( [p \text{ thresh}] \). If only \( p \) is specified, the signal subspace dimension is \( p \). If \( [p \text{ thresh}] \) is specified, \( \text{thresh} \) is multiplied by \( \lambda_{\text{min}} \), the smallest eigenvalue; eigenvalues below the threshold \( \lambda_{\text{min}} \times \text{thresh} \) are assigned to the noise subspace. In this case, \( p \) is the maximum dimension of the signal subspace.
pmusic

**NOTE** pmusic must assign eigenvectors to the noise and signal subspaces, but this is very difficult to do in practice. The two parameters p and thresh are provided for flexibility and control.

\[ [Pxx,f] = \text{pmusic}(x,[p \text{ thresh}],nfft,Fs,\text{window},\text{nooverlap}) \]

specifies the FFT length nfft (default is 256) and the sampling frequency for the signal Fs (default is 1). If Fs is specified, the output frequency vector f is scaled by this value. If the input signal is real-valued, the frequency range is 0 to Fs/2; for the complex case, it is 0 to Fs. window is a scalar specifying the rectangular window length, or a vector giving the actual window coefficients. nooverlap, used in conjunction with window, is a scalar that gives the number of points by which to overlap successive windows.

\[ [Pxx,f] = \text{pmusic}(x,...,\text{'}corr\text{'}) \]

forces x to be taken as a correlation matrix. In this case, the arguments window and nooverlap are ignored.

\[ [Pxx,f] = \text{pmusic}(x,...,\text{'}ev\text{'}) \]

selects the eigenvector variant of the MUSIC estimator. See the “Algorithm” section below for an explanation of how this is different from the MUSIC method.

\[ [Pxx,f,\text{evects},\text{svals}] = \text{pmusic}(x,...) \]

returns two additional arguments. evects is a matrix of eigenvectors spanning the noise subspace (one per column). svals is either a vector of singular values (squared) from svd or a vector of eigenvalues of the correlation matrix when the ‘corr’ option is present.

**Remarks**

The input x can be a vector or a matrix. x can be interpreted as signal data or as a correlation matrix, in one of three ways:

- x is a vector of signal values (row or column). In this case, the dimension of the eigenvectors must be given. This is done either by taking the default value of 2*p or by specifying a window length using window.
- x is a rectangular (m-by-n, possibly square) matrix. In this case, each column of x is a separate observation signal that enters into the SVD analysis, n is
the number of observations, and the dimension of the eigenvectors is equal to \( m \), the length of a column.

• \( x \) is a square matrix and the trailing 'corr' is present. \( x \) is treated as a correlation matrix. In this case, the matrix must have only real, nonnegative eigenvalues.

The inputs \( p \) and \( \text{thresh} \) can determine the number of noise eigenvectors in one of four ways:

• If \( \text{thresh} < 1 \), or if it is unspecified, the number of eigenvectors spanning the signal subspace will be equal to \( p \). \( p \) must be an integer satisfying \( 0 \leq p < n \), where \( n \) is the dimension of the eigenvectors. This dimension \( n \) is the column length in the data matrix case, the matrix size in the correlation matrix case, or the window length for signal vectors. The value of \( \text{thresh} \) is unused.

• If \( p \geq n \), \( \text{thresh} \) must be at least 1. \( \text{thresh} \) is used as the multiplier to determine an absolute threshold for splitting the eigenvalues between the signal and noise subspaces:

\[
\lambda_k \leq (\text{thresh}) \min(\lambda_k) \quad \Rightarrow \{\lambda_k, v_k\} \text{ belong to noise subspace}
\]

• If \( \text{thresh} < 1 \), there will be no noise eigenvectors. This case is not allowed and gives the following error message:

Note subspace dimension cannot be zero.

• When \( p < n \) and \( \text{thresh} \geq 1 \), \( p \) specifies the maximum number of signal eigenvectors. However, the threshold test specified by \( \text{thresh} \) can also take eigenvectors from the signal subspace and assign them to the noise subspace.

**Examples**

This example analyzes a signal vector \( xx \), assuming that two real signals are present in the signal subspace. In this case, the dimension of the signal subspace is 4 because each real sinusoid is the sum of two complex exponentials:

\[
\begin{align*}
nn &= 0:199; \\
xx &= \cos(0.257*\text{pi}*nn) + \sin(0.2*\text{pi}*nn) + 0.01*\text{randn}(\text{size}(nn)); \\
[PP,ff] &= \text{pmusic}(xx,4);
\end{align*}
\]

This example analyzes the same signal vector \( xx \) with an eigenvalue cutoff of 10% above the minimum. Setting \( p = \text{Inf} \) forces the signal/noise subspace
decision to be based on thresh. Use eigenvectors of dimension 7 and a sampling frequency Fs of 8 kHz:

\[ [PP, ff] = \text{pmusic}(xx, [\text{Inf}, 1.1], [\text{Inf}], 8000, 7); \] % window length = 7

With the third and fourth outputs, by plotting the zeros of the noise-eigenvector polynomials, it is possible to create a “Root-MUSIC” algorithm, as the following zplane plot illustrates:

\[ [PP, ff, v\_noise] = \text{pmusic}(xx, 4); \]
\[ \text{for} \ kk = 1:\text{size}(v\_noise, 2) \]
\[ \quad \text{rr}(\cdot, kk) = \text{roots}(v\_noise(\cdot, kk)); \]
\[ \text{end} \]
\[ \text{zplane}(\text{rr}) \]

Assume that RR is a square correlation matrix (for example, 7-by-7):

\[ RR = \text{toeplitz}(\cos(0.1\pi*[0:6])) + 0.1*\text{eye}(7); \]
\[ [PP, ff] = \text{pmusic}(RR, 4, 'corr'); \]

Make an observation matrix xx that is rectangular (100-by-7):

\[ xx = \text{reshape}(\cos(0.257\pi*(0:699)), 7, 100) + 0.1*\text{randn}(7, 100); \]
\[ [PP, ff] = \text{pmusic}(xx, 4); \]

Use the same signal, but let pmusic form the 100-by-7 data matrix using its window and overlap inputs. In addition, use a longer FFT:

\[ yy = xx(:); \]
\[ [PP, ff] = \text{pmusic}(yy, 4, 512, [\text{Inf}], 7, 0); \]

If we set \( p = 0 \), all the eigenvectors are assigned to the noise subspace. ‘ev’ specifies the eigenvector weighting. This turns out to be equivalent to MVDL (Capon’s MLM):

\[ PP, ff] = \text{pmusic}(RR, 0, 'ev', 'corr'); \]

**Algorithm**

The MUSIC estimate is given by the formula

\[
P_{\text{music}}(f) = \frac{1}{N} \left( \sum_{k=p+1}^{N} v_k v_k^H \right) e(f) = \frac{1}{N} \sum_{k=p+1}^{N} |v_k^H e(f)|^2
\]
where $N$ is the dimension of the eigenvectors and $v_k$ is the $k$-th eigenvector of the correlation matrix of the input signal. The integer $p$ is the dimension of the signal subspace, so the eigenvectors $v_k$ used in the sum correspond to the smallest eigenvalues and also span the noise subspace. The vector $e(f)$ consists of complex exponentials, so the inner product

$$v_k^H e(f)$$

amounts to a Fourier transform. The second form is preferred for computation because the FFT is computed for each $v_k$ and then the squared magnitudes are summed.

In the eigenvector method, the summation is weighted by the eigenvalues $\lambda_k$ of the correlation matrix:

$$P_{ev}(f) = \frac{1}{N} \sum_{k=p+1}^{N} \frac{|v_k^H e(f)|^2}{\lambda_k}$$

The function relies on the svd matrix decomposition in the signal case, and it uses the eig function for analyzing the correlation matrix. If SVD is used, the correlation matrix is never explicitly computed, but the singular values are the $\lambda_k$.

**Diagnostics**

There must be at least one output argument and at least two inputs; otherwise, pmusic stops and gives one of the following error messages:

Must have at least 1 output argument.
Must have at least 2 input arguments.

The first argument must be a full matrix, otherwise pmusic gives the following error message:

Input signal or correlation cannot be sparse.
If the second argument was entered as an empty matrix, or if it has more than
two elements, or if it has negative or non-integer elements, pmusic gives one of
the following error messages:

P cannot be empty.
Second input must have only 1 or 2 elements.
P must be an integer.
Second input must contain non-negative entries.

If the value of p is too large with respect to the eigenvector dimension, and
thresh is less than 1, no eigenvectors can be assigned to the noise subspace and
the algorithm fails. In this case, pmusic gives the following error message:

Noise subspace dimension cannot be zero.

If the 'corr' parameter is used, then the first input must be a square
correlation matrix. If it is not, pmusic gives the following error message:

Correlation matrix (R) is not square.

The correlation matrix is then checked for validity; if it fails, pmusic gives the
following error message:

Correlation matrix (R) has negative or complex eigenvalue.

See Also

lpc Linear prediction coefficients.
pburg Power spectrum estimate using the Burg method.
pcov Power spectrum estimate using the covariance method.
pmcov Power spectrum estimate using the modified covariance method.
pmtm Power spectrum estimate using the multitaper method (MTM).
prony Prony's method for time domain IIR filter design.
pwelch Estimate the power spectral density (PSD) of a signal using Welch's method.
pyulear Power spectrum estimate using Yule-Walker AR method.
References


**poly2ac**

**Purpose**
Conversion of prediction polynomial to autocorrelation sequence.

**Syntax**
```matlab
r = poly2ac(a,efinal)
```

**Description**
`r = poly2ac(a,efinal)` finds the autocorrelation sequence corresponding to prediction polynomial `a` and final prediction error `efinal`. If `a(1)` is not equal to 1, `poly2ac` normalizes the prediction polynomial by `a(1)`. `a(1)` cannot be 0.

**See Also**
- `ac2poly` Conversion of autocorrelation sequence to prediction polynomial.
- `poly2rc` Conversion of prediction polynomial to reflection coefficients.
- `rc2ac` Conversion of reflection coefficients to autocorrelation sequence.

**References**
**Purpose** Conversion of prediction polynomial to reflection coefficients.

**Syntax**

- `k = poly2rc(a)`
- `[k,r0] = poly2rc(a,efinal)`

**Description**

- `k = poly2rc(a)` finds the reflection coefficients of the AR lattice structure corresponding to prediction polynomial `a`. `a` must be real, and `a(1)` cannot be 0. If `a(1)` is not equal to 1, `poly2rc` normalizes the prediction polynomial by `a(1)`. 
  - `k` is a row vector of size `length(a)–1`.

- `[k,r0] = poly2rc(a,efinal)` returns the zero-lag autocorrelation, `r0`, based on the final prediction error, `efinal`.

A simple, fast way to check if `a` has all of its roots inside the unit circle is to check if each of the elements of `k` has magnitude less than 1:

- `stable = all(abs(poly2rc(a))<1)`

**Example**

Consider an IIR filter given by

- `a = [1.0000 0.6149 0.9899 0.0000 0.0031 -0.0082];`

Its reflection coefficient representation is

- `k = poly2rc(a)`

```matlab
k =
  0.3090
  0.9801
  0.0031
  0.0081
-0.0082
```

**Limitations**

- If `abs(k(i)) == 1` for any `i`, finding the reflection coefficients is an ill-conditioned problem. `poly2rc` will return some NaNs and provide a warning message in this case.

**Algorithm**

- `poly2rc` implements the recursive relationship:
This relationship is based on Levinson's recursion [1]. To implement it, poly2rc loops through a in reverse order after discarding its first element. For each loop iteration i, the function

1. sets k(i) equal to a(i)
2. applies the second relationship above to elements 1 through i of the vector a:

\[
a = (a-k(i) * \text{fliplr}(a))/(1-k(i)^2);
\]

See Also

- ac2rc Conversion of autocorrelation sequence to reflection coefficients.
- latc2tf Lattice filter to transfer function conversion.
- latcfilt Lattice and lattice-ladder filter implementation.
- poly2ac Conversion of prediction polynomial to autocorrelation sequence.
- rc2poly Conversion of reflection coefficients to prediction polynomial.
- tf2latc Conversion of transfer function to lattice filter.

References

**Purpose**
Stabilize polynomial.

**Syntax**
```
b = polystab(a)
```

**Description**
polystab stabilizes a polynomial with respect to the unit circle; it reflects roots with magnitudes greater than 1 inside the unit circle.

```
b = polystab(a)
```
where `a` is a vector of polynomial coefficients normally in the `z`-domain:

\[ a(z) = a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na} \]

returns a row vector containing the stabilized polynomial.

**Example**
polystab can convert a linear-phase filter into a minimum-phase filter with the same magnitude response:

```
h = fir1(25,0.4);
hmin = polystab(h)*norm(h)/norm(polystab(h));
```

**Algorithm**
polystab finds the roots of the polynomial and maps those roots found outside the unit circle to the inside of the unit circle:

```
v = roots(a);
vs = 0.5*(sign(abs(v)-1)+1);
v = (1-vs).*v + vs./conj(v);
```

**See Also**
roots\\nPolynomial roots (see the online MATLAB Function Reference).
**Purpose**

Prony's method for time domain IIR filter design.

**Syntax**

\[
b, a = \text{prony}(h, nb, na)\]

**Description**

Prony's method is an algorithm for finding an IIR filter with a prescribed time domain impulse response. It has applications in filter design, exponential signal modeling, and system identification (parametric modeling).

\[
b, a = \text{prony}(h, nb, na)\]

finds a filter with numerator order \(nb\), denominator order \(na\), and the time domain impulse response in \(h\). **prony** returns the filter coefficients in row vectors \(b\) and \(a\), of length \(nb + 1\) and \(na + 1\), respectively. The filter coefficients are in descending powers of \(z\):

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na + 1)z^{-na}}
\]

**Example**

Recover the coefficients of a Butterworth filter from its impulse response:

\[
[b, a] = \text{butter}(4, 0.2)
\]

\[
b = \\
0.0048 \quad 0.0193 \quad 0.0289 \quad 0.0193 \quad 0.0048
\]

\[
a = \\
1.0000 \quad -2.3695 \quad 2.3140 \quad -1.0547 \quad 0.1874
\]

\[h = \text{filter}(b, a, [1 \ zeros(1, 25)]);
\]

\[[bb, aa] = \text{prony}(h, 4, 4)\]

\[
bb = \\
0.0048 \quad 0.0193 \quad 0.0289 \quad 0.0193 \quad 0.0048
\]

\[
ab = \\
1.0000 \quad -2.3695 \quad 2.3140 \quad -1.0547 \quad 0.1874
\]

**Algorithm**

**prony** implements the method described in reference [1]. This method uses a variation of the covariance method of AR modeling to find the denominator coefficients \(a\) and then finds the numerator coefficients \(b\) for which the impulse response of the output filter matches exactly the first \(nb + 1\) samples of \(x\). The filter is not necessarily stable, but potentially can recover the coefficients.
exactly if the data sequence is truly an autoregressive moving average (ARMA) process of the correct order.

See Also

- butter: Butterworth analog and digital filter design.
- cheby1: Chebyshev type I filter design (passband ripple).
- cheby2: Chebyshev type II filter design (stopband ripple).
- ellip: Elliptic (Cauer) filter design.
- invfreqz: Discrete-time filter identification from frequency data.
- levinson: Levinson-Durbin recursion.
- lpc: Linear prediction coefficients.
- stmcb: Linear model using Steiglitz-McBride iteration.

References

Superseded

The `psd` function for spectral estimation by Welch's method has been superseded by `pwelch`.

The `pwelch` function improves upon `psd` in three ways:

- The magnitude of the spectrum is scaled by $1/F_s$
- The default value of $F_s$ is 1
- No detrending is performed

The `psd` function will continue to be included in the Signal Processing Toolbox for backwards compatibility.
Purpose

Pulse train generator.

Syntax

\[
y = \text{pulstran}(t, d, 'func')
\]

\[
y = \text{pulstran}(t, d, 'func', p1, p2, \ldots)
\]

\[
y = \text{pulstran}(t, d, p, Fs)
\]

\[
y = \text{pulstran}(t, d, p)
\]

Description

\text{pulstran} generates pulse trains from continuous functions or sampled prototype pulses.

\[
y = \text{pulstran}(t, d, 'func')
\] generates a pulse train based on samples of a continuous function, 'func', where \text{func} is:

- \text{gauspuls}, for Gaussian-modulated sinusoidal pulse generator
- \text{rectpuls}, for sampled aperiodic rectangle generator
- \text{tripuls}, for sampled aperiodic triangle generator

\text{pulstran} is evaluated \text{length}(d) times and returns the sum of the evaluations

\[
y = \text{func}(t-d(1)) + \text{func}(t-d(2)) + \ldots
\]

The function is evaluated over the range of argument values specified in array \(t\), after removing a scalar argument offset taken from the vector \(d\). Note that \text{func} must be a vectorized function that can take an array \(t\) as an argument.

An optional gain factor may be applied to each delayed evaluation by specifying \(d\) as a two-column matrix, with the offset defined in column 1 and associated gain in column 2 of \(d\). Note that a row vector will be interpreted as specifying delays only.

\text{pulstran}(t, d, 'func', p1, p2, \ldots) allows additional parameters to be passed to 'func' as necessary. For example,

\[
\text{func}(t-d(1), p1, p2, \ldots) + \text{func}(t-d(2), p1, p2, \ldots) + \ldots
\]

\text{pulstran}(t, d, p, Fs) generates a pulse train that is the sum of multiple delayed interpolations of the prototype pulse in vector \(p\), sampled at the rate \(Fs\), where \(p\) spans the time interval \([0, (\text{length}(p)-1)/Fs]\), and its samples are identically 0 outside this interval. By default, linear interpolation is used for generating delays.
pulstran

pulstran(t,d,p) assumes that the sampling rate Fs is equal to 1 Hz.

pulstran(...,'func') specifies alternative interpolation methods. See interp1 for a list of available methods.

Examples

This example generates an asymmetric sawtooth waveform with a repetition frequency of 3 Hz and a sawtooth width of 0.1 sec. It has a signal length of 1 sec and a 1 kHz sample rate:

```matlab
t = 0 : 1/1e3 : 1;       % 1 kHz sample freq for 1 sec
d = 0 : 1/3 : 1;         % 3 Hz repetition freq
y = pulstran(t,d,'tripuls',0.1,-1);
plot(t,y)
```

This example generates a periodic Gaussian pulse signal at 10 kHz, with 50% bandwidth. The pulse repetition frequency is 1 kHz, sample rate is 50 kHz, and
pulse train length is 10 msec. The repetition amplitude should attenuate by 0.8 each time:

```matlab
t = 0 : 1/50E3 : 10e–3;
d = [0 : 1/1E3 : 10e–3 ; 0.8.^(0:10)];
y = pulstran(t,d,'gauspuls',10e3,0.5);
plot(t,y)
```
This example generates a train of 10 Hamming windows:

```matlab
p = hamming(32);
t = 0:320; d = (0:9)'*32;
y = pulstran(t,d,p);
plot(t,y)
```

See Also

- `chirp`: Swept-frequency cosine generator.
- `cos`: Cosine of vector/matrix elements (see the online MATLAB Function Reference).
- `diric`: Dirichlet or periodic sinc function.
- `gauspuls`: Gaussian-modulated sinusoidal pulse generator.
- `rectpuls`: Sampled aperiodic rectangle generator.
- `sawtooth`: Sawtooth or triangle wave generator.
- `sin`: Sine of vector/matrix elements (see the online MATLAB Function Reference).
- `sinc`: Sinc or sin(\pi t)/\pi t function.
- `square`: Square wave generator.
- `tripuls`: Sampled aperiodic triangle generator.
Purpose

Estimate the power spectral density (PSD) of a signal using Welch's method.

Syntax

Pxx = pwelch(x)
Pxx = pwelch(x,nfft)
[Pxx,w] = pwelch(x,nfft)
[Pxx,f] = pwelch(x,nfft,Fs)
[Pxx,f] = pwelch(x,nfft,Fs,window)
[Pxx,f] = pwelch(x,nfft,Fs,window,noverlap)
[Pxx,Pxxc,f] = pwelch(x,nfft,Fs,window,noverlap,p)
[pwelch(...)
pwelch(...,...,'magunits')

Description

Pxx = pwelch(x) estimates the power spectrum of the sequence x using the Welch method of spectral estimation. If x is real, pwelch estimates the spectrum at positive frequencies only; in this case, output Pxx is a column vector of length nfft/2+1 for nfft even and (nfft+1)/2 for nfft odd. If x is complex, pwelch estimates the spectrum at both positive and negative frequencies and Pxx has length nfft.

Pxx = pwelch(x,nfft) uses the specified FFT length nfft in estimating the power spectrum for x. This value determines the number of different frequencies at which the power spectrum is estimated. Specify nfft as a power of 2 for fastest execution. Specify an empty matrix for nfft, [], to use the default value of min(256,length(x)).

[Pxx,w] = pwelch(x,nfft) returns a vector w of normalized angular frequencies (in rads/sample) at which the function evaluates the PSD. The range of w is [0,π) for real x and [0,2π) for complex x. Since the frequency vector w is the same size as Pxx, plot(w,Pxx) plots the power spectrum versus the normalized angular frequency.

[Pxx,f] = pwelch(x,nfft,Fs) returns a vector f of linear frequencies (in Hz) at which the function evaluates the PSD. Fs is a scalar that specifies the sampling frequency of x, and Pxx is scaled by 1/Fs. The range of f is [0,Fs/2) for real x and [0,Fs) for complex x. Since the frequency vector f is the same size as Pxx, plot(f,Pxx) plots the power spectrum versus the linear frequency. Specify an empty matrix for Fs, [], to use the default value of 1 Hz.
[Pxx,f] = pwelch(x,nfft,Fs,window) specifies a windowing function and the number of samples per windowed section of the x vector (e.g., kaiser(64)). The length of the window must be less than or equal to nfft; pwelch zero pads the sections if the length of the window is less than nfft. Specify a scalar for window to use a Hanning window of that length; specify an empty matrix, [], to use the default value of hanning(nfft).

[Pxx,f] = pwelch(x,nfft,Fs,window,noverlap) overlaps the windowed sections of x by nooverlap samples. Specify an empty matrix for nooverlap, [], to use the default value of 0.

[Pxx,Pxxc,f] = pwelch(x,nfft,Fs,window,noverlap,p) where p is a positive scalar between 0 and 1 returns a vector Pxxc that contains an estimate of the p*100 percent confidence interval for Pxx. Pxxc is a two-column matrix that is the same length as Pxx. The interval [Pxxc(:,1),Pxxc(:,2)] covers the true PSD with probability p. plot(f,[Pxx Pxxc]) plots the power spectrum inside the p*100 percent confidence interval. Specify an empty matrix for p, [], to use the default value of 0.95.

[Pxx,Pxxc,f] = pwelch(...,'range') specifies the range of frequency values to include in f. range can be:

- half, to compute the PSD over the range [0,π) for real or complex x. If Fs is specified, the range is [0,Fs/2) for real or complex x. If Fs is specified by an empty matrix, [], the default value of 1 is used, and the range is [0,1/2) for real or complex x.
- whole, to compute the PSD over the range [0,2π) for real or complex x. If Fs is specified, the range is [0,Fs) for real or complex x. If Fs is specified by an empty matrix, [], the default value of 1 is used, and the range is [0,1) for real or complex x.

pwelch(x,...) with no output arguments plots the PSD against frequency in the current figure window. If p is specified, the plot includes the confidence interval.

pwelch(...,'magunits') specifies the units in which to express to the magnitude axis of the PSD plot. The options are:
• 'db', to express the PSD data in decibels (dB). This is the default.
• 'squared', to express the PSD in linear units (magnitude squared).

**Example**
Generate a colored noise signal and plot its PSD with a confidence interval of 95%. Specify a length 1024 FFT, a 512-point Kaiser window with no overlap, and a sampling frequency of 10 kHz:

```matlab
h = fir1(30,0.2,boxcar(31)); % design a lowpass filter
r = randn(16384,1); % white noise
x = filter(h,1,r); % color the noise
pwelch(x,1024,10000,kaiser(512,5),0,0.95)
```

**Algorithm**
`pwelch` calculates the power spectral density using Welch’s method (see references [1] and [2]):

1. It applies the window specified by the `window` vector to each successive section of input `x`.
2. It transforms each section with an `nfft`-point FFT.
3. It forms the periodogram of each section by scaling the magnitude squared of each transformed section.
It averages the periodograms of the overlapping sections to form $P_{xx}(f)$.

It scales $P_{xx}(f)$ by $1/F_s$ to form $P_{xx}(f)/F_s$, the power spectrum of $x$.

The number of sections that `pwelch` averages is

$$k = \text{fix}\left(\frac{\text{length}(x)-\text{noverlap}}{\text{length}(\text{window})-\text{noverlap}}\right)$$

### Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments to `pwelch` are used:

- Requires window's length to be no greater than FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires confidence parameter to be a scalar between 0 and 1.
- Requires vector input.

### See Also

- `cohere` Estimate magnitude squared coherence function between two signals.
- `csd` Estimate the cross spectral density (CSD) of two signals.
- `pburg` Power spectrum estimate using the Burg method.
- `pcov` Power spectrum estimate using the covariance method.
- `pmcov` Power spectrum estimate using the modified covariance method.
- `pmtm` Power spectrum estimate using the multitaper method (MTM).
- `pmusic` Power spectrum estimate using MUSIC eigenvector method.
- `pyulear` Power spectrum estimate using Yule-Walker AR method.
- `specgram` Time-dependent frequency analysis (spectrogram).
- `tfe` Transfer function estimate from input and output.
References


**Purpose**

Power spectrum estimate using Yule-Walker AR method.

**Syntax**

```
Pxx = pyulear(x,p,nfft)
[Pxx,freq] = pyulear(x,p,nfft)
[Pxx,freq] = pyulear(x,p,nfft,Fs)
[Pxx,freq] = pyulear(x,p,nfft,Fs,'range')
pyulear(...)
pyulear(...,'squared')
```

**Description**

`pyulear` estimates the power spectral density (PSD) of the signal vector `x[n]` using the Yule-Walker AR method. This method, also called the autocorrelation method, fits an autoregressive (AR) model to the signal by minimizing the forward prediction error in the least-squares sense. This formulation leads to the Yule-Walker equations, which are solved by the Levinson-Durbin recursion. The spectral estimate returned by `pyulear` is the magnitude squared frequency response of this AR model. The correct choice of the model order `p` is important.

`Pxx = pyulear(x,p,nfft)` returns `Pxx`, the power spectrum estimate. `x` is the input signal, `p` is the model order for the all-pole filter, and `nfft` is the FFT length (defaults to 256 if not specified). `Pxx` has length `(nfft/2+1)` for `nfft` even, `(nfft+1)/2` for `nfft` odd, and `nfft` if `x` is complex.

`[Pxx,freq] = pyulear(x,p,nfft)` returns `Pxx`, the power spectrum estimate, and `freq`, a vector of frequencies at which the PSD was estimated. If the input signal is real-valued, the range of `freq` is `[0,π)`. If the input signal is complex, the range of `freq` is `[0,2π)`. `Fs` defaults to 1 if left empty, `[]`.

`[Pxx,freq] = pyulear(x,p,nfft,Fs)` uses the signal's sampling frequency, `Fs`, to scale both the PSD vector (`Pxx`) and the frequency vector (`freq`). `Pxx` is scaled by `1/Fs`. If the input signal is real-valued, the range of `freq` is `[0,Fs/2)`. If the input signal is complex, the range of `freq` is `[0,Fs)`. `Fs` defaults to 1 if left empty, `[]`.

`[Pxx,freq] = pyulear(x,p,nfft,Fs,'range')` specifies the range of frequency values to include in `freq`. `range` can be:

- `half`, to compute the PSD over the range `[0,Fs/2)` for real `x`, and `[0,Fs)` for complex `x`. If `Fs` is left blank, `[]`, the range is `[0,1/2)` for real `x`, and `[0,1)`
pyulear for complex \( x \). If \( Fs \) is omitted entirely, the range is \([0, \pi)\) for real \( x \), and \([0, 2\pi)\) for complex \( x \). half is the default range.

- whole, to compute the PSD over the range \([0, Fs)\) for all \( x \). If \( Fs \) is left blank, [], the range is \([0, 1)\) for all \( x \). If \( Fs \) is omitted entirely, the range is \([0, 2\pi)\) for all \( x \).

pyulear(...) plots the power spectral density in the first available figure window. The frequency range on the plot is the same as the range of output \( freq \) for a given set of parameters.

pyulear(...,'squared') plots the magnitude of \( Pxx \) directly, rather than converting the values to dB.

**Example**

Since the method estimates the spectrum by fitting an AR model to the signal, first define the AR system (all-pole filter) that generates the input. Check the magnitude response of the process with \( freqz \).

\[
a = [1 -2.2137 2.9403 -2.1697 0.9606]; \quad \text{% AR system coefficients}
\]

\[
freqz(a,'phase','no') \quad \text{% AR system magnitude response}
\]

title('AR System Magnitude Response')

![AR System Magnitude Response](image)
Now generate the input signal $x$ by filtering white noise through the AR system. Estimate the PSD of $x$ based on a fourth-order AR model (since we know that the original AR system, $a$, has order 4).

```matlab
x = filter(1,a,randn(150,1));  % AR system output
pyulear(x,4) % 4th order estimate
```

**Diagnostics**

The first input argument must be a full vector; otherwise `pyulear` generates the following error message:

Input signal cannot be sparse.

If you specify an empty matrix for the second argument, `pyulear` generates the following error message:

Model order must be an integer.
See Also

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References

**rc2ac**

**Purpose**  
Conversion of reflection coefficients to autocorrelation sequence.

**Syntax**  
\[ r = rc2ac(k,r0) \]

**Description**  
\[ r = rc2ac(k,r0) \] finds the autocorrelation coefficients, \( r \), of the discrete filter with reflection coefficients \( k \) and zero-lag autocorrelation \( r0 \).

**See Also**  
- `ac2rc`  
  Conversion of autocorrelation sequence to reflection coefficients.
- `poly2ac`  
  Conversion of prediction polynomial to autocorrelation sequence.
- `rc2poly`  
  Conversion of reflection coefficients to prediction polynomial.

**References**  
Purpose  
Conversion of reflection coefficients to prediction polynomial.

Syntax  
a = rc2poly(k)  
[a,efinal] = rc2poly(k,r0)

Description  
a = rc2poly(k) finds the prediction polynomial a, with a(1) = 1, from the reflection coefficients k of the corresponding lattice structure. Output a is row vector of length k + 1.

[a,efinal] = rc2poly(k,r0) returns the final prediction error, efinal, based on the zero-lag autocorrelation, r0.

Example  
Consider an IIR filter given by reflection coefficients k:

\[
\begin{bmatrix}
0.3090 & 0.9800 & 0.0031 & 0.0082 & -0.0082
\end{bmatrix}
\]

Its polynomial representation is

\[
a = rc2poly(k)
\]

\[
a =
\begin{bmatrix}
1.0000 & 0.6148 & 0.9899 & 0.0000 & 0.0032 & -0.0082
\end{bmatrix}
\]

Algorithm  
rc2poly computes output a using Levinson’s recursion [1]. The function:

1. Sets the output vector a to the first element of k
2. Loops through the remaining elements of k
   For each loop iteration i, a = [a + a(i-1:-1:1)*k(i) k(i)].
3. Implements a = [1 a]
**rc2poly**

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**References**

Purpose
Real cepstrum and minimum phase reconstruction.

Syntax
\[ y = \text{rceps}(x) \]
\[ [y, ym] = \text{rceps}(x) \]

Description
The real cepstrum is the inverse Fourier transform of the real logarithm of the magnitude of the Fourier transform of a sequence.

\( \text{rceps}(x) \) returns the real cepstrum of the real sequence \( x \). The real cepstrum is a real-valued function.

\( [y, ym] = \text{rceps}(x) \) returns both the real cepstrum \( y \) and a minimum phase reconstructed version \( ym \) of the input sequence.

Algorithm
\( \text{rceps} \) is an M-file implementation of algorithm 7.2 in [2], that is:
\[ y = \text{real}(\text{ifft}(\log(\text{abs}(\text{fft}(x))))); \]

Appropriate windowing in the cepstral domain forms the reconstructed minimum phase signal:
\[ w = [1; 2*\text{ones}(n/2-1,1); \text{ones}(1 - \text{rem}(n,2),1); \text{zeros}(n/2-1,1)]; \]
\[ ym = \text{real}(\text{ifft}(\exp(\text{fft}(w.*y)))); \]

See Also
- \text{cceps} Complex cepstral analysis.
- \text{fft} One-dimensional fast Fourier transform.
- \text{hilbert} Hilbert transform.
- \text{icceps} Inverse complex cepstrum.
- \text{unwrap} Unwrap phase angles.

References

rectpuls

Purpose
Sampled aperiodic rectangle generator.

Syntax

\[ y = \text{rectpuls}(t) \]

\[ y = \text{rectpuls}(t,w) \]

Description

\[ y = \text{rectpuls}(t) \] returns a continuous, aperiodic, unity-height rectangular pulse at the sample times indicated in array \( t \), centered about \( t = 0 \) and with a default width of 1. Note that the interval of non-zero amplitude is defined to be open on the right, that is, \( \text{rectpuls}(-0.5) = 1 \) while \( \text{rectpuls}(0.5) = 0 \).

\[ y = \text{rectpuls}(t,w) \] generates a rectangle of width \( w \).

\text{rectpuls} is typically used in conjunction with the pulse train generating function, \text{pulstran}.

See Also

- chirp: Swept-frequency cosine generator.
- cos: Cosine of vector/matrix elements (see the online MATLAB Function Reference).
- diric: Dirichlet or periodic sinc function.
- gauspuls: Gaussian-modulated sinusoidal pulse generator.
- pulstran: Pulse train generator.
- sawtooth: Sawtooth or triangle wave generator.
- sin: Sine of vector/matrix elements (see the online MATLAB Function Reference).
- sinc: Sinc or \( \sin(\pi t)/\pi t \) function.
- square: Square wave generator.
- tripuls: Sampled aperiodic triangle generator.
Purpose  

Parks-McClellan optimal FIR filter design.

Syntax

\[
b = \text{remez}(n,f,a) \\
b = \text{remez}(n,f,a,w) \\
b = \text{remez}(n,f,a,'ftype') \\
b = \text{remez}(n,f,a,w,'ftype') \\
b = \text{remez}(\ldots,[lgrid]) \\
b = \text{remez}(n,f,'fresp',w) \\
b = \text{remez}(n,f,'fresp',w,'ftype') \\
b = \text{remez}(n,f,\{'fresp',p1,p2,\ldots\}',w) \\
b = \text{remez}(n,f,\{'fresp',p1,p2,\ldots\}',w,'ftype') \\
[b,delta] = \text{remez}(\ldots) \\
[b,delta,opt] = \text{remez}(\ldots)
\]

Description

\text{remez} designs a linear-phase FIR filter using the Parks-McClellan algorithm [1]. The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with an optimal fit between the desired and actual frequency responses. The filters are optimal in the sense that the maximum error between the desired frequency response and the actual frequency response is minimized. Filters designed this way exhibit an equiripple behavior in their frequency responses and hence are sometimes called equiripple filters.

\[b = \text{remez}(n,f,a)\] returns row vector \(b\) containing the \(n+1\) coefficients of the order \(n\) FIR filter whose frequency-amplitude characteristics match those given by vectors \(f\) and \(a\).

The output filter coefficients (taps) in \(b\) obey the symmetry relation

\[b(k) = b(n+2-k), \quad k = 1, \ldots, n+1\]

Vectors \(f\) and \(a\) specify the frequency-magnitude characteristics of the filter:
remez

- f is a vector of pairs of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order.
- a is a vector containing the desired amplitudes at the points specified in f.
  The desired amplitude at frequencies between pairs of points (f(k), f(k+1)) for k odd is the line segment connecting the points (f(k), a(k)) and (f(k+1), a(k+1)).
  The desired amplitude at frequencies between pairs of points (f(k), f(k+1)) for k even is unspecified. The areas between such points are transition or "don't care" regions.
- f and a must be the same length. The length must be an even number.

The relationship between the f and a vectors in defining a desired frequency response is shown below:

\[
\begin{align*}
  f &= [0 \ .3 \ .4 \ .6 \ .7 \ .9] \\
  a &= [0 \ 1 \ 0 \ .5 \ .5]
\end{align*}
\]

\[\text{Desired amplitude response (a)}\]

\[\text{Normalized frequency (f)}\]

remez(n,f,a,w) uses the weights in vector w to weight the fit in each frequency band. The length of w is half the length of f and a, so there is exactly one weight per band.

\[b = \text{remez}(n,f,a,'ftype')\] and

\[b = \text{remez}(n,f,a,w,'ftype')\] specify a filter type, where ftype is
hilbert, for linear-phase filters with odd symmetry (type III and type IV)
The output coefficients in $b$ obey the relation $b(k) = -b(n+2-k)$, $k = 1, \ldots, n+1$.
This class of filters includes the Hilbert transformer, which has a desired amplitude of 1 across the entire band.
For example,
\[ h = \text{remez}(30,[0.1 \ 0.9],[1 \ 1],\text{'}hilbert\text{'}); \]
designs an approximate FIR Hilbert transformer of length 31.
differentiator, for type III and IV filters, using a special weighting technique
For nonzero amplitude bands, it weights the error by a factor of $1/f$ so that the error at low frequencies is much smaller than at high frequencies. For FIR differentiators, which have an amplitude characteristic proportional to frequency, these filters minimize the maximum relative error (the maximum of the ratio of the error to the desired amplitude).
\[ b = \text{remez}(\ldots,\{lgrid\}) \]
uses the integer $lgrid$ to control the density of the frequency grid, which has roughly $(lgrid*n)/(2*bw)$ frequency points, where $bw$ is the fraction of the total frequency band interval $[0,1]$ covered by $f$. Increasing $lgrid$ often results in filters that are more exactly equiripple, but which take longer to compute. The default value of 16 is the minimum value that should be specified for $lgrid$. Note that the $\{lgrid\}$ argument must be a 1-by-1 cell array.
\[ b = \text{remez}(n,f,\text{'}fresp\text{',}w) \]
returns row vector $b$ containing the $n+1$ coefficients of the order $n$ FIR filter whose frequency-amplitude characteristics best approximate the response specified by function $fresp$. The function is called from within remez with the following syntax:
\[ [dh,dw] = fresp(n,f,gf,w) \]
The arguments are similar to those for remez:
- $n$ is the filter order.
- $f$ is the vector of frequency band edges that appear monotonically between 0 and 1, where 1 is the Nyquist frequency.
- $gf$ is a vector of grid points that have been linearly interpolated over each specified frequency band by remez. $gf$ determines the frequency grid at
which the response function must be evaluated, and contains the same data
returned by \texttt{remez} in the \texttt{fgrid} field of the \texttt{opt} structure.

- \( w \) is a vector of real, positive weights, one per band, used during optimization.
  \( w \) is optional in the call to \texttt{remez}; if not specified, it is set to unity weighting
  before being passed to \textit{fresp}.

- \( dh \) and \( dw \) are the desired complex frequency response and band weight
  vectors, respectively, evaluated at each frequency in grid \textit{gf}.

The predefined frequency response function (\textit{fresp}) that \texttt{remez} calls is
\texttt{remezfrf} in the \texttt{signal/private} directory.

\begin{verbatim}
b = remez(n,f,\{'fresp',p1,p2,...\},w)
b = remez(n,f,\{'fresp',w,\'ftype\}')
b = remez(n,f,\{'fresp',p1,p2,...\},w,\'ftype\}')
\end{verbatim}

allows you to specify additional
parameters (\( p1, p2, \text{etc.} \)) to pass to \textit{fresp}. Note that \( b = \text{remez}(n,f,a,w) \) is a
synonym for \( b = \text{remez}(n,f,\{'\text{remezfrf}',a\},w) \), where \( a \) is a vector
containing the desired amplitudes at the points specified in \( f \).

\begin{verbatim}
b = remez(n,f,\{'fresp',w,\'ftype\}') and
b = remez(n,f,\{'fresp',p1,p2,...\},w,\'ftype\}') design antisymmetric
(odd) rather than symmetric (even) filters, where \textit{ftype} is either \texttt{d}
for a differentiator or \texttt{h} for a Hilbert transformer.
\end{verbatim}

In the absence of a specification for \textit{ftype}, a preliminary call is made to \textit{fresp}
to determine the default symmetry property \textit{sym}. This call is made using the
syntax:

\begin{verbatim}
sym = fresp(\'defaults\',{n,f,\[],w,p1,p2,...})
\end{verbatim}

The arguments \( n, f, w, \text{etc.} \), may be used as necessary in determining an
appropriate value for \textit{sym}, which \texttt{remez} expects to be either \texttt{even} or \texttt{odd}.
If the \textit{fresp} function does not support this calling syntax, \texttt{remez} defaults to even
symmetry.

\( [b,\text{delta}] = \text{remez}(\ldots) \) returns the maximum ripple height in \textit{delta}. 

[b,delta,opt] = remez(...) returns a structure, opt, of optional results with the following fields.

<table>
<thead>
<tr>
<th>opt.fgrid</th>
<th>Frequency grid vector used for the filter design optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>opt.des</td>
<td>Desired frequency response for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.wt</td>
<td>Weighting for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.H</td>
<td>Actual frequency response for each point in opt.fgrid</td>
</tr>
<tr>
<td>opt.error</td>
<td>Error at each point in opt.fgrid (opt.des–opt.H)</td>
</tr>
<tr>
<td>opt.iextr</td>
<td>Vector of indices into opt.fgrid for extremal frequencies</td>
</tr>
<tr>
<td>opt.fextr</td>
<td>Vector of extremal frequencies</td>
</tr>
</tbody>
</table>

**Example**

Graph the desired and actual frequency responses of a 17th-order Parks-McClellan bandpass filter:

```matlab
f = [0 0.3 0.4 0.6 0.7 1]; a = [0 1 1 0 0];
b = remez(17,f,a);
[h,w] = freqz(b,1,512);
plot(f,a,w/pi,abs(h))
```

![Graph of frequency responses](image.png)
**Algorithm**

`remez` is a MEX-file version of the original Fortran code from [1], altered to design arbitrarily long filters with arbitrarily many linear bands.

`remez` designs type I, II, III, and IV linear-phase filters. Type I and Type II are the defaults for \( n \) even and \( n \) odd, respectively, while Type III (\( n \) even) and Type IV (\( n \) odd) are obtained with the 'hilbert' and 'differentiator' flags. The different types of filters have different symmetries and certain constraints on their frequency responses (see [5] for more details).

<table>
<thead>
<tr>
<th>Linear Phase Filter Type</th>
<th>Filter Order ( n )</th>
<th>Symmetry of Coefficients</th>
<th>Response ( H(f) ), ( f = 0 )</th>
<th>Response ( H(f) ), ( f = 1 ) (Nyquist)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>Even</td>
<td>even: ( b(k) = b(n + 2 - k), \ k = 1, \ldots, n + 1 )</td>
<td>No restriction</td>
<td>No restriction</td>
</tr>
<tr>
<td>Type II</td>
<td>Odd</td>
<td>( b(k) = b(n + 2 - k), \ k = 1, \ldots, n + 1 )</td>
<td>No restriction</td>
<td>( H(1) = 0 )</td>
</tr>
<tr>
<td>Type III</td>
<td>Even</td>
<td>odd: ( b(k) = -b(n + 2 - k), \ k = 1, \ldots, n + 1 )</td>
<td>( H(0) = 0 )</td>
<td>( H(1) = 0 )</td>
</tr>
<tr>
<td>Type IV</td>
<td>Odd</td>
<td>( b(k) = -b(n + 2 - k), \ k = 1, \ldots, n + 1 )</td>
<td>( H(0) = 0 )</td>
<td>No restriction</td>
</tr>
</tbody>
</table>

**Diagnostics**

An appropriate diagnostic message is displayed if incorrect arguments are used:

- Filter order must be 3 or more.
- There should be one weight per band.
- Frequency and amplitude vectors must be the same length.
- The number of frequency points must be even.
- Frequencies must lie between 0 and 1.
- Frequencies must be specified in bands.
- Frequencies must be nondecreasing.
- Adjacent bands not allowed.

A more serious warning message is

--- Failure to Converge ---

Probable cause is machine rounding error.

In the rare event that you see this message, it is possible that the filter design may still be correct. Verify the design by checking its frequency response.
See Also

butter  Butterworth analog and digital filter design.
cheby1  Chebyshev type I filter design (passband ripple).
cheby2  Chebyshev type II filter design (stopband ripple).
cremez  Complex and nonlinear-phase equiripple FIR filter design.
ellip   Elliptic (Cauer) filter design.
fir1    Window-based finite impulse response filter design - standard response.
fir2    Window-based finite impulse response filter design - arbitrary response.
fircls  Constrained least square FIR filter design for multiband filters.
fircls1 Constrained least square filter design for lowpass and highpass linear phase FIR filters.
firls   Least square linear-phase FIR filter design.
firrcos  Raised cosine FIR filter design.
remezord Parks-McClellan optimal FIR filter order estimation.
yulewalk Recursive digital filter design.

References


remezord

Purpose

Parks-McClellan optimal FIR filter order estimation.

Syntax

\[ [n, fo, ao, w] = \text{remezord}(f, a, dev) \]
\[ [n, fo, ao, w] = \text{remezord}(f, a, dev, Fs) \]
\[ c = \text{remezord}(f, a, dev, Fs, 'cell') \]

Description

\[ [n, fo, ao, w] = \text{remezord}(f, a, dev) \] finds the approximate order, normalized frequency band edges, frequency band amplitudes, and weights that meet input specifications \( f \), \( a \), and \( dev \), to use with the \text{remez} command.

- \( f \) is a vector of frequency band edges (between 0 and \( Fs/2 \), where \( Fs \) is the sampling frequency), and \( a \) is a vector specifying the desired amplitude on the bands defined by \( f \). The length of \( f \) is twice the length of \( a \), minus 2. The desired function is piecewise constant.

- \( dev \) is a vector the same size as \( a \) that specifies the maximum allowable deviation or ripples between the frequency response and the desired amplitude of the output filter, for each band.

Use \text{remez} with the resulting order \( n \), frequency vector \( fo \), amplitude response vector \( ao \), and weights \( w \) to design the filter \( b \) which approximately meets the specifications given by \text{remezord} input parameters \( f \), \( a \), and \( dev \):

\[ b = \text{remez}(n, fo, ao, w) \]

\[ [n, fo, ao, w] = \text{remezord}(f, a, dev, Fs) \] specifies a sampling frequency \( Fs \). \( Fs \) defaults to 2 Hz, implying a Nyquist frequency of 1 Hz. You can therefore specify band edges scaled to a particular application's sampling frequency.

In some cases \text{remezord} underestimates the order \( n \). If the filter does not meet the specifications, try a higher order such as \( n+1 \) or \( n+2 \).

\[ c = \text{remezord}(f, a, dev, Fs, 'cell') \] specifies a cell-array whose elements are the parameters to \text{remez}.

Examples

Design a minimum-order lowpass filter with a 500 Hz passband cutoff frequency and 600 Hz stopband cutoff frequency, with a sampling frequency of
2000 Hz), at least 40 dB attenuation in the stopband, and less than 3 dB of ripple in the passband:

```matlab
rp = 3;          % passband ripple
rs = 40;         % stopband ripple
Fs = 2000;       % sampling frequency
f = [500 600];   % cutoff frequencies
a = [1 0];       % desired amplitudes

% compute deviations
dev = [(10^(rp/20)-1)/(10^(rp/20)+1)  10^(-rs/20)];

[n,fo,ao,w] = remezord(f,a,dev,Fs);
b = remez(n,fo,ao,w);
[h,f] = freqz(b,1,1024,Fs);
plot(f,20*log10(abs(h)));
title('Lowpass Filter Designed to Specifications');
xlabel('Frequency (Hz)'); ylabel('Magnitude (dB)'); grid;
```

Note that the filter falls slightly short of meeting the stopband attenuation and passband ripple specifications. Using \( n+1 \) in the call to \texttt{remez} instead of \( n \) achieves the desired amplitude characteristics.
Design a lowpass filter with a 1500 Hz passband cutoff frequency and 2000 Hz stopband cutoff frequency, with a sampling frequency of 8000 Hz, a maximum stopband amplitude of 0.1, and a maximum passband error (ripple) of 0.01:

\[
[n,fo,ao,w] = \text{remezord}([1500 \ 2000], [1 \ 0], [0.01 \ 0.1], 8000);
b = \text{remez}(n,fo,ao,w);
\]

This is equivalent to

\[
c = \text{remezord}([1500 \ 2000], [1 \ 0], [0.01 \ 0.1], 8000, \text{'cell'});
b = \text{remez}(c{:});
\]

**NOTE** In some cases, `remezord` underestimates or overestimates the order \(n\). If the filter does not meet the specifications, try a higher order such as \(n+1\) or \(n+2\).

Results are inaccurate if the cutoff frequencies are near 0 or the Nyquist frequency.

**Algorithm** `remezord` uses the algorithm suggested in [1]. This method is inaccurate for band edges close to either 0 or the Nyquist frequency (\(Fs/2\)).

**Diagnostics** If the input parameter lengths are not consistent, `remezord` gives the following error messages:

Requires \(M\) and \(DEV\) to be the same length.
Length of \(F\) must be \(\text{length}(M)-2\).

**See Also**
- `buttord` Butterworth filter order selection.
- `cheb1ord` Chebyshev type I filter order selection.
- `cheb2ord` Chebyshev type II filter order selection.
- `ellipord` Elliptic filter order selection.
- `kaiserord` Estimate parameters for an FIR filter design with Kaiser window.
- `remez` Parks-McClellan optimal FIR filter design.
References


resample

Purpose

Change sampling rate by any rational factor.

Syntax

\[
\begin{align*}
    y &= \text{resample}(x,p,q) \\
    y &= \text{resample}(x,p,q,n) \\
    y &= \text{resample}(x,p,q,n,beta) \\
    y &= \text{resample}(x,p,q,b) \\
    [y,b] &= \text{resample}(x,p,q)
\end{align*}
\]

Description

\( y = \text{resample}(x,p,q) \) resamples the sequence in vector \( x \) at \( p/q \) times the original sampling rate, using a polyphase filter implementation. The length of \( y \) is equal to ceil\( (\text{length}(x) \times p/q) \). \( p \) and \( q \) must be positive integers. If \( x \) is a matrix, \text{resample} works down the columns of \( x \).

\( \text{resample} \) applies an anti-aliasing (lowpass) FIR filter to \( x \) during the resampling process. It designs the filter using \text{firls} with a Kaiser window.

\( y = \text{resample}(x,p,q,n) \) uses \( n \) terms on either side of the current sample, \( x(k) \), to perform the resampling. The length of the FIR filter \text{resample} uses is proportional to \( n \); larger values of \( n \) provide better accuracy at the expense of more computation time. The default for \( n \) is 10. If you let \( n = 0 \), \text{resample} performs a nearest-neighbor interpolation:

\[
y(k) = x(\text{round}(k \times q/p) + 1)
\]

where \( y(k) = 0 \) if the index to \( x \) is greater than \( \text{length}(x) \).

\( y = \text{resample}(x,p,q,n,beta) \) uses \( \beta \) as the design parameter for the Kaiser window that \text{resample} employs in designing the lowpass filter. The default for \( \beta \) is 5.

\( y = \text{resample}(x,p,q,b) \) filters \( x \) with \( b \), a vector of filter coefficients.

\( [y,b] = \text{resample}(x,p,q) \) returns the vector \( b \), which contains the coefficients of the filter applied to \( x \) during the resampling process.
Examples

Resample a simple linear sequence at 3/2 the original rate:

\[
\begin{align*}
Fs1 &= 10; \quad \% \text{original sampling frequency in Hz} \\
t1 &= 0:1/Fs1:1; \quad \% \text{time vector} \\
x &= t1; \quad \% \text{define a linear sequence} \\
y &= \text{resample}(x,3,2); \quad \% \text{now resample it} \\
t2 &= (0:(\text{length}(y)-1))*2/(3*Fs1); \quad \% \text{new time vector} \\
\text{plot}(t1,x,'*',t2,y,'o',-0.5:0.01:1.5,-0.5:0.01:1.5,':') \\
\text{legend}('\text{original}','\text{resampled}'); \text{xlabel('Time')} 
\end{align*}
\]

Notice that the last few points of the output \( y \) are inaccurate. In its filtering process, \text{resample} assumes the samples at times before and after the given samples in \( x \) are equal to zero. Thus large deviations from zero at the end
points of the sequence x can cause inaccuracies in y at its end points. The following two plots illustrate this side effect of resample:

```matlab
x = [1:10 9:-1:1]; y = resample(x,3,2);
subplot(2,1,1);
plot(1:19,x,'*',(0:28)*2/3 + 1,y,'o');
title('Edge Effects Not Noticeable');
legend('original','resampled');

x = [10:-1:1 2:10]; y = resample(x,3,2);
subplot(2,1,2);
plot(1:19,x,'*',(0:28)*2/3 + 1,y,'o')
title('Edge Effects Very Noticeable');
legend('original','resampled');
```

**Diagnostics**

If p or q are not positive integers, resample gives the appropriate error message:

- P must be a positive integer.
- Q must be a positive integer.

If x is not a vector, resample gives the following error message:

- Input X must be a vector.
See Also

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decimate</td>
<td>Decrease the sampling rate for a sequence (decimation).</td>
</tr>
<tr>
<td>firls</td>
<td>Least square linear-phase FIR filter design.</td>
</tr>
<tr>
<td>interp</td>
<td>Increase sampling rate by an integer factor (interpolation).</td>
</tr>
<tr>
<td>interp1</td>
<td>One-dimensional data interpolation (table lookup) (see the online MATLAB Function Reference).</td>
</tr>
<tr>
<td>intfilt</td>
<td>Interpolation FIR filter design.</td>
</tr>
<tr>
<td>kaiser</td>
<td>Kaiser window.</td>
</tr>
<tr>
<td>spline</td>
<td>Cubic spline interpolation (see the online MATLAB Function Reference).</td>
</tr>
<tr>
<td>upfirdn</td>
<td>Upsample, apply an FIR filter, and downsample.</td>
</tr>
</tbody>
</table>
residuez

Purpose
z-transform partial-fraction expansion.

Syntax
[r, p, k] = residuez(b, a)
[b, a] = residuez(r, p, k)

Description
residuez converts a discrete time system, expressed as the ratio of two polynomials, to partial fraction expansion, or residue, form. It also converts the partial fraction expansion back to the original polynomial coefficients.

[r, p, k] = residuez(b, a) finds the residues, poles, and direct terms of a partial fraction expansion of the ratio of two polynomials, b(z) and a(z). Vectors b and a specify the coefficients of the polynomials of the discrete-time system b(z)/a(z) in descending powers of z:

\[ b(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \]
\[ a(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n} \]

If there are no multiple roots and \( a > n-1 \),

\[ \frac{b(z)}{a(z)} = \frac{r(1)}{1 - p(1) z^{-1}} + \cdots + \frac{r(n)}{1 - p(n) z^{-1}} + k(1) + k(2) z^{-1} + \cdots + k(m - n + 1) z^{-(m-n)} \]

The returned column vector \( r \) contains the residues, column vector \( p \) contains the pole locations, and row vector \( k \) contains the direct terms. The number of poles is

\[ n = \text{length}(a) - 1 = \text{length}(r) = \text{length}(p) \]

The direct term coefficient vector \( k \) is empty if \( \text{length}(b) < \text{length}(a) \); otherwise

\[ \text{length}(k) = \text{length}(b) - \text{length}(a) + 1 \]

If \( p(j) = \ldots = p(j+s-1) \) is a pole of multiplicity \( s \), then the expansion includes terms of the form

\[ \frac{r(j)}{1 - p(j) z^{-1}} + \frac{r(j+1)}{(1 - p(j) z^{-1})^2} + \cdots + \frac{r(j+s-1)}{(1 - p(j) z^{-1})^s} \]
residuez

[b,a] = residuez(r,p,k) with three input arguments and two output arguments, converts the partial fraction expansion back to polynomials with coefficients in row vectors b and a.

The residue function in the standard MATLAB language is very similar to residuez. It computes the partial fraction expansion of continuous-time systems in the Laplace domain (see reference [1]), rather than discrete-time systems in the z-domain as does residuez.

Algorithm

residuez applies standard MATLAB functions and partial fraction techniques to find r, p, and k from b and a. It finds:

1. The direct terms a using deconv (polynomial long division) when length(b) > length(a) - 1.
2. The poles using p = roots(a). mpoles finds repeated poles and reorders the poles according to their multiplicities.
3. The residue for each nonrepeating pole p_i by multiplying b(z)/a(z) by 1/(1-p_i z^-1) and evaluating the resulting rational function at z = p_i.
4. The residues for the repeated poles by solving

   \[ S2 \cdot r2 = h - S1 \cdot r1 \]

   for r2 using \( h \) is the impulse response of the reduced b(z)/a(z), S1 is a matrix whose columns are impulse responses of the first-order systems made up of the nonrepeating roots, and r1 is a column containing the residues for the nonrepeating roots. Each column of matrix S2 is an impulse response. For each root p_j of multiplicity s_j, S2 contains s_j columns representing the impulse responses of each of the following systems:

   \[ \frac{1}{1 - p_j z^{-1}} \frac{1}{1 - p_j z^{-1}}^2 \ldots \frac{1}{1 - p_j z^{-1}}^s \]

   The vector h and matrices S1 and S2 have n + xtra rows, where n is the total number of roots and the internal parameter xtra, set to 1 by default, determines the degree of overdetermination of the system of equations.

Diagnostics

If a(1) == 0 while computing the partial fraction decomposition using [r,p,k] = residuez(b,a), residuez gives the following error message:

   First coefficient in A vector must be nonzero.
If the number of residues \( r \) and poles \( p \) is not the same, \texttt{residuez} gives the following error message:

\[ \text{R and P vectors must be the same size.} \]

\section*{See Also}

- \texttt{convmtx}  
  Convolution matrix.
- \texttt{deconv}  
  Deconvolution and polynomial division (see the online MATLAB Function Reference).
- \texttt{poly}  
  Polynomial with specified roots (see the online MATLAB Function Reference).
- \texttt{prony}  
  Prony's method for time domain IIR filter design.
- \texttt{residue}  
  Partial fraction expansion (see the online MATLAB Function Reference).
- \texttt{roots}  
  Polynomial roots (see the online MATLAB Function Reference).
- \texttt{ss2tf}  
  Conversion of state-space to zero-pole-gain.
- \texttt{tf2ss}  
  Conversion of transfer function to state-space.
- \texttt{tf2zp}  
  Conversion of transfer function to zero-pole-gain.
- \texttt{zp2ss}  
  Conversion of zero-pole-gain to state-space.

\section*{References}

Purpose
Reverse Levinson-Durbin recursion.

Syntax
- \( r = rlevinson(a, efinal) \)
- \([r,u] = rlevinson(a, efinal)\)
- \([r,u,k] = rlevinson(a, efinal)\)
- \([r,u,k,e] = rlevinson(a, efinal)\)

Description
The reverse Levinson-Durbin recursion implements the step-down algorithm for solving the following symmetric Toeplitz system of linear equations for \( r \), where \( r = [r(1) \ldots r(p+1)] \) and \( r(i)* \) denotes the complex conjugate of \( r(i) \).

\[
\begin{bmatrix}
  r(1) & r(2)* & \cdots & r(p)* \\
r(2) & r(1) & \cdots & r(p-1)* \\
  \vdots & \ddots & \ddots & \vdots \\
r(p) & \cdots & r(2) & r(1)
\end{bmatrix}
\begin{bmatrix}
a(2) \\
a(3) \\
\vdots \\
a(p+1)
\end{bmatrix}
= 
\begin{bmatrix}
-r(2) \\
-r(3) \\
\vdots \\
-r(p+1)
\end{bmatrix}
\]

\( r = rlevinson(a, efinal) \) solves the above system of equations for \( r \) given vector \( a \), where \( a = [1 \ a(2) \ldots \ a(p+1)] \). In linear prediction applications, \( r \) represents the autocorrelation sequence of the input to the prediction error filter, where \( r(1) \) is the zero-lag element. The figure below shows the typical filter of this type, where \( H(z) \) is the optimal linear predictor, \( x(n) \) is the input signal, \( \hat{x}(n) \) is the predicted signal, and \( e(n) \) is the prediction error.

Prediction Error Filter

\[ H(z) = -a(2)z^{-1} - a(3)z^{-2} - \cdots - a(n+1)z^{-p} \]

Input vector \( a \) represents the polynomial coefficients of this prediction error filter in descending powers of \( z \)

\[ A(z) = 1 + a(2)z^{-1} + \cdots + a(n+1)z^{-p} \]
The filter must be minimum phase to generate a valid autocorrelation sequence. e\text{final} is the scalar prediction error power, which is equal to the variance of the prediction error signal, \sigma^2(e).

\[ [r, u] = \text{rlevinson}(a, e\text{final}) \]
returns upper triangular matrix \( U \) from the \( \text{UDU}^* \) decomposition

\[ R^{-1} = UE^{-1}U^* \]

where

\[ R = \begin{bmatrix} r(1) & r(2)^* & \cdots & r(p)^* \\ r(2) & r(1) & \cdots & r(p-1)^* \\ \vdots & \vdots & \ddots & \vdots \\ r(p) & \cdots & r(2) & r(1) \end{bmatrix} \]

and \( E \) is a diagonal matrix with elements returned in output \( e \) (see below). This decomposition permits the efficient evaluation of the inverse of the autocorrelation matrix, \( R^{-1} \).

Output matrix \( u \) contains the prediction polynomial, \( a \), from each iteration of the reverse Levinson-Durbin recursion

\[ U = \begin{bmatrix} a_1(1)^* & a_2(2)^* & \cdots & a_{p+1}(p+1)^* \\ 0 & a_2(1)^* & \cdots & a_{p+1}(p)^* \\ 0 & 0 & \cdots & a_{p+1}(p-1)^* \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{p+1}(1)^* \end{bmatrix} \]

where \( a_i(j) \) is the \( j \)th coefficient of the \( i \)th order prediction polynomial (i.e., step \( i \) in the recursion). For example, the 5th order prediction polynomial is

\[ a_5 = u(5:-1:1, 5)' \]

Note that \( u(p+1:-1:1, p+1)' \) is the input polynomial coefficient vector \( a \).
[r,u,k] = rlevinson(a,efinal) returns a vector k of length (p+1) containing the reflection coefficients. The reflection coefficients are the conjugates of the values in the first row of u

k = conj(u(1,2:end))

[r,u,k,e] = rlevinson(a,efinal) returns a vector of length (p+1) containing the prediction errors from each iteration of the reverse Levinson-Durbin recursion: e(1) is the prediction error from the first-order model, e(2) is the prediction error from the second-order model, and so on.

These prediction error values form the diagonal of the matrix E in the UDU* decomposition of $R^{-1}$,

$$R^{-1} = U E^{-1} U^*.$$ 

See Also
levinson
Levinson-Durbin recursion.
lpc
Linear prediction coefficients.
prony
Prony's method for time domain IIR filter design.
stmcb
Linear model using Steiglitz-McBride iteration.

References
sawtooth

Purpose
Sawtooth or triangle wave generator.

Syntax
x = sawtooth(t)
x = sawtooth(t,width)

Description
sawtooth(t) generates a sawtooth wave with period 2\pi for the elements of time vector t. sawtooth(t) is similar to sin(t), but it creates a sawtooth wave with peaks of -1 and 1 instead of a sine wave. The sawtooth wave is defined to be -1 at multiples of 2\pi and to increase linearly with time with a slope of 1/\pi at all other times.

sawtooth(t,width) generates a modified triangle wave where width, a scalar parameter between 0 and 1, determines the fraction between 0 and 2\pi at which the maximum occurs. The function increases from -1 to 1 on the interval 0 to 2\pi*width, then decreases linearly from 1 to -1 on the interval 2\pi*width to 2\pi. Thus a parameter of 0.5 specifies a standard triangle wave, symmetric about time instant \pi with peak-to-peak amplitude of 1. sawtooth(t,1) is equivalent to sawtooth(t).

Diagnostics
If the width parameter is not a scalar, sawtooth gives the following error message:

Requires WIDTH parameter to be a scalar.

See Also
chirp Swept-frequency cosine generator.
cos Cosine of vector/matrix elements (see the online MATLAB Function Reference).
diric Dirichlet or periodic sinc function.
gauspuls Gaussian-modulated sinusoidal pulse generator.
pulstran Pulse train generator.
rectpuls Sampled aperiodic rectangle generator.
sin Sine of vector/matrix elements (see the online MATLAB Function Reference).
sinc Sinc or sin(\pi t)/\pi t function.
square Square wave generator.
tripuls Sampled aperiodic triangle generator.
Savitzky-Golay filter design.

Syntax

\[ b = \text{sgolay}(k,f) \]
\[ b = \text{sgolay}(k,f,w) \]

Description

\[ b = \text{sgolay}(k,f) \] designs a Savitzky-Golay FIR smoothing filter \( b \). The polynomial order \( k \) must be less than the frame size, \( f \), which must be odd. If \( k = f-1 \), the designed filter produces no smoothing. The output, \( b \), is an \( f \)-by-\( f \) matrix whose rows represent the time-varying FIR filter coefficients. In a smoothing filter implementation (for example, \text{sgolayfilt})\), the last \((f-1)/2\) rows (each an FIR filter) are applied to the signal during the startup transient, and the first \((f-1)/2\) rows are applied to the signal during the terminal transient. The center row is applied to the signal in the steady state.

\[ b = \text{sgolay}(k,f,w) \] specifies a weighting vector \( w \) with length \( f \), which contains the real, positive-valued weights to be used during the least-squares minimization.

Remarks

Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least squares smoothing filters) are typically used to “smooth out” a noisy signal whose frequency span (without noise) is large. In this type of application, Savitzky-Golay smoothing filters perform much better than standard averaging FIR filters, which tend to filter out a significant portion of the signal’s high frequency content along with the noise. Although Savitzky-Golay filters are more effective at preserving the pertinent high frequency components of the signal, they are less successful than standard averaging FIR filters at rejecting noise.

Savitzky-Golay filters are optimal in the sense that they minimize the least-squares error in fitting a polynomial to each frame of noisy data.

See Also

- \text{fir1} Window-based finite impulse response filter design – standard response.
- \text{firls} Least square linear-phase FIR filter design.
- \text{filter} Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- \text{sgolayfilt} Savitzky-Golay filtering.
References

Purpose
Savitzky-Golay filtering.

Syntax
\begin{align*}
y &= \text{sgolayfilt}(x,k,f) \\
y &= \text{sgolayfilt}(x,k,f,w)
\end{align*}

Description
\( y = \text{sgolayfilt}(x,k,f) \) applies a Savitzky-Golay FIR smoothing filter to the data in vector \( x \). If \( x \) is a matrix, \text{sgolayfilt} operates on each column. The polynomial order \( k \) must be less than the frame size, \( f \), which must be odd. If \( k = f-1 \), the filter produces no smoothing.

\( y = \text{sgolayfilt}(x,k,f,w) \) specifies a weighting vector \( w \) with length \( f \), which contains the real, positive-valued weights to be used during the least-squares minimization.

Remarks
Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least-squares smoothing filters) are typically used to “smooth out” a noisy signal whose frequency span (without noise) is large. In this type of application, Savitzky-Golay smoothing filters perform much better than standard averaging FIR filters, which tend to filter out a significant portion of the signal’s high frequency content along with the noise. Although Savitzky-Golay filters are more effective at preserving the pertinent high frequency components of the signal, they are less successful than standard averaging FIR filters at rejecting noise.

Savitzky-Golay filters are optimal in the sense that they minimize the least-squares error in fitting a polynomial to frames of noisy data.
Example

Smooth the mtlb signal by applying a cubic Savitzky-Golay filter to data frames of length 41.

```matlab
load mtlb
smtlb = sgolayfilt(mtlb,3,41);
```

```matlab
subplot(2,1,1)
plot([1:2000],mtlb(1:2000)); axis([0 2000 −4 4]);
title('mtlb'); grid;
```

```matlab
subplot(2,1,2)
plot([1:2000],smtlb(1:2000)); axis([0 2000 −4 4]);
title('smtlb'); grid;
```

See Also

medfilt1 One-dimensional median filtering.
filter Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
sgolay Savitzky-Golay filter design.
sosfilt Second-order (biquadratic) IIR filtering.

References

**Purpose**
Sinc function.

**Syntax**
y = sinc(x)

**Description**
sinc computes the sinc function of an input vector or array, where the sinc function is

\[
sinc(t) = \begin{cases} 
1, & t = 0 \\
\frac{\sin(\pi t)}{\pi t}, & t \neq 0 
\end{cases}
\]

This function is the continuous inverse Fourier transform of the rectangular pulse of width \(2\pi\) and height 1:

\[
sinc(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{\text{i}\omega t} d\omega
\]

\(y = \text{sinc}(x)\) returns an array \(y\) the same size as \(x\), whose elements are the sinc function of the elements of \(x\).

The space of functions bandlimited in the frequency band \(\omega \in [-\pi, \pi]\) is spanned by the infinite (yet countable) set of sinc functions shifted by integers. Thus any such bandlimited function \(g(t)\) can be reconstructed from its samples at integer spacings:

\[
g(t) = \sum_{n = -\infty}^{\infty} g(n) sinc(t - n)
\]
**sinc**

**Example**

Perform ideal bandlimited interpolation by assuming that the signal to be interpolated is 0 outside of the given time interval and that it has been sampled at exactly the Nyquist frequency:

```matlab
t = (1:10)'; % a column vector of time samples
randn('seed',0);
x = randn(size(t)); % a column vector of data
% ts is times at which to interpolate data
ts = linspace(-5,15,600)';
y = sinc(ts(:,ones(size(t)))) - t(:,ones(size(ts))))' * x;
plot(t,x,'o',ts,y)
```

![Graph showing sinc function interpolation](image)
### See Also

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<td>tripuls</td>
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sos2ss

Purpose
Conversion of second-order sections to state-space.

Syntax
\[ [A,B,C,D] = \text{sos2ss}(sos) \]
\[ [A,B,C,D] = \text{sos2ss}(sos,g) \]

Description
sos2ss converts a second-order section representation of a given system to an equivalent state-space representation.

\[ [A,B,C,D] = \text{sos2ss}(sos) \]
converts the system \( sos \) in second-order section form, to a single-input, single-output state-space representation:

\[
\begin{align*}
    x[n+1] &= Ax[n] + Bu[n] \\
    y[n] &= Cx[n] + Du[n]
\end{align*}
\]

The discrete transfer function in second-order section form is given by

\[
H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

\( sos \) is a \( L \)-by-6 matrix organized as

\[
\text{sos} = \begin{bmatrix}
    b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
    b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L}
\end{bmatrix}
\]

The entries of \( sos \) must be real for proper conversion to state space. The returned matrix \( A \) is size \( N \)-by-\( N \), where \( N = 2L - 1 \), \( B \) is a length \( N - 1 \) column vector, \( C \) is a length \( N - 1 \) row vector, and \( D \) is a scalar.

\[ [A,B,C,D] = \text{sos2ss}(sos,g) \]
converts the system \( sos \) in second-order section form with gain \( g \):

\[
H(z) = g \prod_{k=1}^{L} H_k(z)
\]
Example

Compute the state-space representation of a simple second-order section system with a gain of 2:

\[
\text{sos} = \begin{bmatrix}
1 & 1 & 1 & 0 & -1 \\
-2 & 3 & 1 & 1 & 10 & 1
\end{bmatrix};
\]

\[
[A,B,C,D] = \text{sos2ss}(\text{sos})
\]

\[
A = \\
\begin{bmatrix}
-10 & 0 & 10 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
B = \\
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
C = \\
\begin{bmatrix}
21 & 2 & -16 & -1
\end{bmatrix}
\]

\[
D = \\
\begin{bmatrix}
-2
\end{bmatrix}
\]

Algorithm

\text{sos2ss} first converts from second-order sections to transfer function using \text{sos2tf}, and then from transfer function to state-space using \text{tf2ss}.

See Also

\text{sos2tf} Conversion of second-order sections to transfer function.

\text{sos2zp} Conversion of second-order sections to zero-pole-gain.

\text{ss2sos} Conversion of state-space to second-order sections.

\text{tf2ss} Conversion of transfer function to state-space.

\text{zp2ss} Conversion of zero-pole-gain to state-space.
sos2tf

Purpose
Conversion of second-order sections to transfer function.

Syntax
\[
[b,a] = \text{sos2tf}(sos)
\]
\[
[b,a] = \text{sos2tf}(sos,g)
\]

Description
sos2tf converts a second-order section representation of a given system to an equivalent transfer function representation.

\[ [b,a] = \text{sos2tf}(sos) \]
returns the numerator coefficients \( b \) and denominator coefficients \( a \) of the transfer function that describes a discrete-time system given by \( sos \) in second-order section form. The second-order section format of \( H(z) \) is given by

\[
H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

\( sos \) is an \( L \)-by-6 matrix that contains the coefficients of each second-order section stored in its rows:

\[
sos = \begin{bmatrix}
 b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
 b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L}
\end{bmatrix}
\]

Row vectors \( b \) and \( a \) contain the numerator and denominator coefficients of \( H(z) \) stored in descending powers of \( z \):

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b_1 + b_2z^{-1} + \cdots + b_nz^{-n}}{a_1 + a_2z^{-1} + \cdots + a_mz^{-m}}
\]

\[ [b,a] = \text{sos2tf}(sos,g) \]
returns the transfer function that describes a discrete-time system given by \( sos \) in second-order section form with gain \( g \):

\[
H(z) = g \prod_{k=1}^{L} H_k(z)
\]
Algorithm  
sos2tf uses the conv function to multiply all of the numerator and denominator second-order polynomials together.

Example  
Compute the transfer function representation of a simple second-order section system:

```matlab
sos = [1 1 1 0 -1; -2 3 1 10 1];
b,a] = sos2tf(sos)
```

```
b = 
   -2     1     2     4     1
a = 
    1    10     0   -10     -1
```

See Also  
lattc2tf  Lattice filter to transfer function conversion.
sos2ss  Conversion of second-order sections to state-space.
sos2zp  Conversion of second-order sections to zero-pole-gain.
ss2tf  Conversion of state-space to transfer function.
tf2sos  Conversion of transfer function to second-order sections.
zp2tf  Conversion of zero-pole-gain to transfer function.
sos2zp

Purpose
Conversion of second-order sections to zero-pole-gain.

Syntax

\[ [z, p, k] = \text{sos2zp}(sos) \]
\[ [z, p, k] = \text{sos2zp}(sos, g) \]

Description
sos2zp converts a second-order section representation of a given system to an equivalent zero-pole-gain representation.

\[ [z, p, k] = \text{sos2zp}(sos) \]
returns the zeros \(z\), poles \(p\), and gain \(k\) of the system given by sos in second-order section form. The second-order section format of \(H(z)\) is given by

\[
H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

\(sos\) is an \(L\)-by-6 matrix that contains the coefficients of each second-order section stored in its rows:

\[
sos = \begin{bmatrix} b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L} \end{bmatrix}
\]

Column vectors \(z\) and \(p\) contain the zeros and poles of the transfer function \(H(z)\):

\[
H(z) = k \frac{(z-z_1)(z-z_2) \cdots (z-z_n)}{(p-p_1)(p-p_2) \cdots (p-p_m)}
\]

where the orders \(n\) and \(m\) are determined by the matrix sos.

\[ [z, p, k] = \text{sos2zp}(sos, g) \]
returns the zeros \(z\), poles \(p\), and gain \(k\) of the system given by sos in second-order section form with gain \(g\):

\[
H(z) = g \prod_{k=1}^{L} H_k(z)
\]

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Example
Compute the poles, zeros, and gain of a simple system in second-order section form:

```matlab
sos = [1 1 1 0 -1; -2 3 1 10 1];
[z,p,k] = sos2zp(sos)
```

```
z =
   -0.5000 + 0.8660i
   -0.5000 - 0.8660i
     1.7808
   -0.2808

p =
   -1.0000
     1.0000
   -9.8990
   -0.1010

k
   -2
```

Algorithm
`sos2zp` finds the poles and zeros of each second-order section by repeatedly calling `tf2zp`.

See Also
- `sos2ss` Conversion of second-order sections to state-space.
- `sos2tf` Conversion of second-order sections to transfer function.
- `ss2zp` Conversion of state-space to zero-pole-gain.
- `tf2zp` Conversion of transfer function to zero-pole-gain.
- `zp2sos` Conversion of zero-pole-gain to second-order sections.
sosfilt

Purpose
Second-order (biquadratic) IIR filtering.

Syntax
y = sosfilt(sos,x)

Description
y = sosfilt(sos,x) applies the second-order section filter sos to vector x. The output, y, is the same length as x.
sos represents the second-order section filter \( H(z) \)

\[
H(z) = \prod_{k=1}^{L} H_k(z) = \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

by an L-by-6 matrix containing the coefficients of each second-order section in its rows

\[
sos = \begin{bmatrix}
    b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
    b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
        : & : & : & : & : & : \\
    b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L}
\end{bmatrix}
\]

If x is a matrix, sosfilt applies the filter to each column of x independently. Output y is a matrix of the same size, containing the filtered data corresponding to each column of x.

See Also
filter Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
medfilt1 One-dimensional median filtering.
sgolayfilt Savitzky-Golay filtering.

References
specgram

Purpose
Time-dependent frequency analysis (spectrogram).

Syntax
B = specgram(a)
B = specgram(a,nfft)
[B,f] = specgram(a,nfft,Fs)
[B,f,t] = specgram(a,nfft,Fs)
B = specgram(a,nfft,Fs,window)
B = specgram(a,nfft,Fs,window,nnoverlap)
specgram(a)
B = specgram(a,f,Fs,window,nnoverlap)

Description
specgram computes the windowed discrete-time Fourier transform of a signal using a sliding window. The spectrogram is the magnitude of this function.

B = specgram(a) calculates the spectrogram for the signal in vector a. This syntax uses the default values:
• nfft = min(256,length(a))
• Fs = 2
• window = hanning(nfft)
• noverlap = length(window)/2

nfft specifies the FFT length that specgram uses. This value determines the frequencies at which the discrete-time Fourier transform is computed. Fs is a scalar that specifies the sampling frequency. window specifies a windowing function and the number of samples specgram uses in its sectioning of vector a. noverlap is the number of samples by which the sections overlap. Any arguments that you omit from the end of the input parameter list use the default values shown above.

If a is real, specgram computes the discrete-time Fourier transform at positive frequencies only. If n is even, specgram returns nfft/2+1 rows (including the zero and Nyquist frequency terms). If n is odd, specgram returns nfft/2 rows. The number of columns in B is

\[ k = \frac{\text{fix}\left((n-noverlap)/(\text{length(window)}-noverlap)\right)}{2} \]

If a is complex, specgram computes the discrete-time Fourier transform at both positive and negative frequencies. In this case, B is a complex matrix with nfft
rows. Time increases linearly across the columns of B, starting with sample 1 in column 1. Frequency increases linearly down the rows, starting at 0.

\[ B = \text{specgram}(a,\text{nfft}) \] uses the specified FFT length \( nfft \) in its calculations. Specify \( nfft \) as a power of 2 for fastest execution.

\[ [B,f] = \text{specgram}(a,\text{nfft,Fs}) \] returns a vector \( f \) of frequencies at which the function computes the discrete-time Fourier transform. \( Fs \) has no effect on the output \( B \); it is a frequency scaling multiplier.

\[ [B,f,t] = \text{specgram}(a,\text{nfft,Fs}) \] returns frequency and time vectors \( f \) and \( t \) respectively. \( t \) is a column vector of scaled times, with length equal to the number of columns of \( B \). \( t(j) \) is the earliest time at which the \( j \)-th window intersects \( a \). \( t(1) \) is always equal to 0.

\[ B = \text{specgram}(a,\text{nfft,Fs,window}) \] specifies a windowing function and the number of samples per section of the \( x \) vector. If you supply a scalar for \( \text{window} \), \text{specgram} uses a Hanning window of that length. The length of the window must be less than or equal to \( nfft \); \text{specgram} zero pads the sections if the length of the window exceeds \( nfft \).

\[ B = \text{specgram}(a,\text{nfft,Fs,window,overlap}) \] overlaps the sections of \( x \) by \( \text{overlap} \) samples.

You can use the empty matrix [ ] to specify the default value for any input argument. For example,

\[ B = \text{specgram}(x,[],10000) \]

is equivalent to

\[ B = \text{specgram}(x) \]

but with a sampling frequency of 10,000 Hz instead of the default 2 Hz.

\text{specgram} with no output arguments displays the scaled logarithm of the spectrogram in the current figure window using

\[ \text{imagesc}(t,f,20\times\log10(\text{abs}(b))),\text{axis xy,colormap(jet)} \]
The axis xy mode displays the low-frequency content of the first portion of the signal in the lower-left corner of the axes. *specgram* uses Fs to label the axes according to true time and frequency.

\[ B = \text{specgram}(a,f,Fs,\text{window}, \text{noverlap}) \]

computes the spectrogram at the frequencies specified in \( f \), using either the chirp z-transform (for more than 20 evenly spaced frequencies) or a polyphase decimation filter bank. \( f \) is a vector of frequencies in Hertz; it must have at least two elements.

**Algorithm**

*specgram* calculates the spectrogram for a given signal as follows:

1. It splits the signal into overlapping sections and applies the window specified by the \text{window} parameter to each section.

2. It computes the discrete-time Fourier transform of each section with a length \( nfft \) FFT to produce an estimate of the short-term frequency content of the signal; these transforms make up the columns of \( B \). *specgram* zero pads the windowed sections if \( nfft > \text{length(window)} \), so the quantity \((\text{length(window)} - \text{noverlap})\) specifies by how many samples *specgram* shifts the window.

3. For real input, *specgram* truncates the spectrogram to the first \( nfft/2 + 1 \) points for \( nfft \) even and \((nfft + 1)/2 \) for \( nfft \) odd.
specgram

Example

Plot the spectrogram of a digitized speech signal:

```matlab
load mtlb
specgram(mtlb,512,Fs,kaiser(500,5),475)
title('Spectrogram')
```

Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector input.

See Also

- `cohere` Estimate magnitude squared coherence function between two signals.
- `csd` Estimate the cross spectral density (CSD) of two signals.
- `pwelch` Estimate the power spectral density (PSD) of a signal using Welch's method.
- `tfe` Transfer function estimate from input and output.
References


Purpose
Interactive digital signal processing tool (SPTool).

Syntax
sptool

Description
The `sptool` command invokes a suite of graphical user interface (GUI) tools that provides access to many of the signal, filter, and spectral analysis functions in the toolbox in a powerful, easy-to-use interactive signal display and exploration environment.

Using SPTool, you can import, export, and manage signals, filters, and spectra. From SPTool, you can activate its four integrated signal processing tools:

- The Signal Browser, for viewing, measuring, and analyzing the time-domain information of one or more signals.
• The Filter Designer, for designing and editing FIR and IIR filters of various lengths and types, with standard (lowpass, highpass, bandpass, bandstop, and multiband) configurations.
The Filter Viewer, for viewing the characteristics of a designed or imported filter, including its magnitude response, phase response, group delay, pole-zero plot, impulse response, and step response.
The Spectrum Viewer, for graphical analysis of frequency-domain data using a variety of methods of spectral density estimation, including the Burg method, the FFT method, the multitaper method (MTM), the MUSIC eigenvector method, Welch's method, and the Yule-Walker AR method.

See Chapter 5, “Interactive Tools” for a full discussion of how to use SPTool.
Purpose
Square wave generator.

Syntax
\[ x = \text{square}(t) \]
\[ x = \text{square}(t, \text{duty}) \]

Description
\[ x = \text{square}(t) \] generates a square wave with period $2\pi$ for the elements of time vector \( t \). \( \text{square}(t) \) is similar to \( \sin(t) \), but it creates a square wave with peaks of $\pm 1$ instead of a sine wave.

\[ x = \text{square}(t, \text{duty}) \] generates a square wave with specified duty cycle, \( \text{duty} \). The duty cycle is the percent of the period in which the signal is positive.

See Also
chirp
Swept-frequency cosine generator.

cos
Cosine of vector/matrix elements (see the online MATLAB Function Reference).
diric
Dirichlet or periodic sinc function.
gauspuls
Gaussian-modulated sinusoidal pulse generator.
pulstran
Pulse train generator.
rectpuls
Sampled aperiodic rectangle generator.
sawtooth
Sawtooth or triangle wave generator.
sin
Sine of vector/matrix elements (see the online MATLAB Function Reference).
sinc
Sinc or $\sin(\pi t)/\pi t$ function.
tripuls
Sampled aperiodic triangle generator.
Purpose

Conversion of state-space to second-order sections.

Syntax

\[
[sos, g] = \text{ss2sos}(A, B, C, D)
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, iu)
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, 'order')
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, iu, 'order')
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, iu, 'order', 'scale')
\]

sos = ss2sos(...)

Description

ss2sos converts a state-space representation of a given system to an equivalent second-order section representation.

\[
[sos, g] = \text{ss2sos}(A, B, C, D)
\]

determines a matrix sos in second-order section form with gain g that is equivalent to the state-space system represented by input arguments A, B, C and D. The input system must be single output and real. sos is an L-by-6 matrix:

\[
sos = \begin{bmatrix}
    b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
    b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L}
\end{bmatrix}
\]

whose rows contain the numerator and denominator coefficients \( b_{ik} \) and \( a_{ik} \) of the second-order sections of \( H(z) \):

\[
H(z) = g \prod_{k=1}^{L} H_k(z) = g \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, iu)
\]

determines a scalar iu that determines which input of the state-space system A, B, C, D is used in the conversion. The default for iu is 1.

\[
[sos, g] = \text{ss2sos}(A, B, C, D, 'order')
\]

\[
[sos, g] = \text{ss2sos}(A, B, C, D, iu, 'order', 'scale')
\]

specify the order of the rows in sos, where order is:
ss2sos

- down, to order the sections so the first row of sos contains the poles closest to the unit circle
- up, to order the sections so the first row of sos contains the poles farthest from the unit circle (default)

The zeros are always paired with the poles closest to them.

\[ [sos, g] = \text{ss2sos}(A, B, C, D, iu, \text{'order'}, \text{'scale'}) \] specifies the desired scaling of the gain and the numerator coefficients of all second-order sections, where scale is:

- none, to apply no scaling (default)
- inf, to apply infinity-norm scaling
- two, to apply 2-norm scaling

Using infinity-norm scaling in conjunction with up-ordering minimizes the probability of overflow in the realization. Using 2-norm scaling in conjunction with down-ordering minimizes the peak round-off noise.

\[ sos = \text{ss2sos}(\ldots) \] embeds the overall system gain, \( g \), in the first section, \( H_1(z) \), so that:

\[ H(z) = \prod_{k=1}^{L} H_k(z) \]

Example

Find a second-order section form of a Butterworth lowpass filter:

\[
[A, B, C, D] = \text{butter}(5, 0.2);
\]
\[
sos = \text{ss2sos}(A, B, C, D)
\]

\[
sos =
\begin{bmatrix}
0.0013 & 0.0013 & 0 & 1.0000 & -0.5095 & 0 \\
1.0000 & 2.0008 & 1.0008 & 1.0000 & -1.0966 & 0.3554 \\
1.0000 & 1.9979 & 0.9979 & 1.0000 & -1.3693 & 0.6926
\end{bmatrix}
\]

Algorithm

ss2sos uses a four-step algorithm to determine the second-order section representation for an input state-space system:
It finds the poles and zeros of the system given by A, B, C and D.

It uses the function `zp2sos`, which first groups the zeros and poles into complex conjugate pairs using the `cplxpair` function. `zp2sos` then forms the second-order sections by matching the pole and zero pairs according to the following rules:

a. Match the poles closest to the unit circle with the zeros closest to those poles.
b. Match the poles next closest to the unit circle with the zeros closest to those poles.
c. Continue until all of the poles and zeros are matched.

`ss2sos` groups real poles into sections with the real poles closest to them in absolute value. The same rule holds for real zeros.

It orders the sections according to the proximity of the pole pairs to the unit circle. `ss2sos` normally orders the sections with poles closest to the unit circle last in the cascade. You can tell `ss2sos` to order the sections in the reverse order by specifying the `'down'` flag.

`ss2sos` scales the sections by the norm specified in the `'scale'` argument. For arbitrary $H(\omega)$, the scaling is defined by:

$$
\|H\|_p = \left[ \frac{1}{2\pi} \int_0^{2\pi} |H(\omega)|^p d\omega \right]^{1/p}
$$

where $p$ can be either $\infty$ or 2. See the references for details on the scaling. This scaling is an attempt to minimize overflow or peak round-off noise in fixed point filter implementations.

**Diagnostics**

If there is more than one input to the system, `ss2sos` gives the following error message:

State-space system must have only one input.
ss2sos

See Also

cplxpair  Group complex numbers into complex conjugate pairs.
sos2ss   Conversion of second-order sections to state-space.
ss2tf    Conversion of state-space to transfer function.
ss2zp    Conversion of state-space to zero-pole-gain.
tf2sos   Conversion of transfer function to second-order sections.
zp2sos   Conversion of zero-pole-gain to second-order sections.

References


Purpose
Conversion of state-space to transfer function.

Syntax
[b, a] = ss2tf(A, B, C, D, iu)

Description
ss2tf converts a state-space representation of a given system to an equivalent transfer function representation.

[b, a] = ss2tf(A, B, C, D, iu) returns the transfer function

\[ H(s) = \frac{B(s)}{A(s)} = C(sI - A)^{-1}B + D \]

of the system

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

from the \(iu\)-th input. Vector \(a\) contains the coefficients of the denominator in descending powers of \(s\). The numerator coefficients are returned in array \(b\) with as many rows as there are outputs \(y\). ss2tf also works with systems in discrete time, in which case it returns the z-transform representation.

The ss2tf function is part of the standard MATLAB language.

Algorithm
The ss2tf function uses poly to find the characteristic polynomial \(\det(sI - A)\) and the equality

\[
H(s) = C(sI - A)^{-1}B = \frac{\det(sI - A + BC) - \det(sI - A)}{\det(sI - A)}
\]

See Also
latc2tf  Lattice filter to transfer function conversion.
sos2tf  Conversion of second-order sections to transfer function.
ss2sos  Conversion of state-space to second-order sections.
ss2zp  Conversion of state-space to zero-pole-gain.
tf2ss  Conversion of transfer function to state-space.
zp2tf  Conversion of zero-pole-gain to transfer function.
**Purpose**

Conversion of state-space to zero-pole-gain.

**Syntax**

\[ [z,p,k] = \text{ss2zp}(A,B,C,D,iu) \]

**Description**

`ss2zp` converts a state-space representation of a given system to an equivalent zero-pole-gain representation. The zeros, poles, and gains of state-space systems represent the transfer function in factored form.

\[ [z,p,k] = \text{ss2zp}(A,B,C,D,iu) \]

 calculates the transfer function in factored form

\[
H(s) = \frac{Z(s)}{P(s)} = k \frac{(s-Z_1)(s-Z_2)\cdots(s-Z_n)}{(s-p_1)(s-p_2)\cdots(s-p_n)}
\]

of the system

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]

from the \(iu\)-th input. Returned column vector \(p\) contains the pole locations of the denominator coefficients of the transfer function. Matrix \(z\) contains the numerator zeros in its columns, with as many columns as there are outputs \(y\). Column vector \(k\) contains the gains for each numerator transfer function.

`ss2zp` also works with systems in discrete time, in which case it returns the \(z\)-transform representation. The input state-space system must be real.

The `ss2zp` function is part of the standard MATLAB language.
Example

Here are two ways of finding the zeros, poles, and gains of a system:

\[ b = [2 \ 3]; \]
\[ a = [1 \ 0.4 \ 1]; \]
\[ [z,p,k] = \text{tf2zp}(b,a) \]

\[ z = \]
\[ -1.5000 \]

\[ p = \]
\[ -0.2000 + 0.9798i \]
\[ -0.2000 - 0.9798i \]

\[ k = \]
\[ 2 \]

\[ [A,B,C,D] = \text{tf2ss}(b,a); \]
\[ [z,p,k] = \text{ss2zp}(A,B,C,D,1) \]

\[ z = \]
\[ -1.5000 \]

\[ p = \]
\[ -0.2000 + 0.9798i \]
\[ -0.2000 - 0.9798i \]

\[ k = \]
\[ 2 \]

Algorithm

\text{ss2zp} \text{ finds the poles from the eigenvalues of the } A \text{ array. The zeros are the finite solutions to a generalized eigenvalue problem:}

\[ z = \text{eig}([A \ B; C \ D], \text{diag}([\text{ones}(1,n) \ 0])); \]

In many situations this algorithm produces spurious large, but finite, zeros. \text{ss2zp} \text{ interprets these large zeros as infinite.}

\text{ss2zp} \text{ finds the gains by solving for the first nonzero Markov parameters.}
ss2zp

See Also

<table>
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<tbody>
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<td>pzmap</td>
<td>Pole-zero map of LTI system (Control System Toolbox User’s Guide).</td>
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<tr>
<td>sos2zp</td>
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References

Purpose

Linear model using Steiglitz-McBride iteration.

Syntax

\[ [b,a] = stmcb(x,nb,na) \]
\[ [b,a] = stmcb(x,u,nb,na) \]
\[ [b,a] = stmcb(x,nb,na,niter) \]
\[ [b,a] = stmcb(x,u,nb,na,niter) \]
\[ [b,a] = stmcb(x,nb,na,niter,ai) \]
\[ [b,a] = stmcb(x,u,nb,na,niter,ai) \]

Description

Steiglitz-McBride iteration is an algorithm for finding an IIR filter with a prescribed time domain impulse response. It has applications in both filter design and system identification (parametric modeling).

\[ [b,a] = stmcb(x,nb,na) \]
finds the coefficients \( b \) and \( a \) of the system \( \frac{b(z)}{a(z)} \) with approximate impulse response \( x \), exactly \( nb \) zeros, and exactly \( na \) poles.

\[ [b,a] = stmcb(x,u,nb,na) \]
finds the system coefficients \( b \) and \( a \) of the system that, given \( u \) as input, has \( x \) as output. \( x \) and \( u \) must be the same length.

\[ [b,a] = stmcb(x,nb,na,niter) \]
and
\[ [b,a] = stmcb(x,u,nb,na,niter) \]
use \( niter \) iterations. The default for \( niter \) is 5.

\[ [b,a] = stmcb(x,nb,na,niter,ai) \]
and
\[ [b,a] = stmcb(x,u,nb,na,niter,ai) \]
use the vector \( ai \) as the initial estimate of the denominator coefficients. If \( ai \) is not specified, \( stmcb \) uses the output argument from \( [b,ai] = prony(x,0,na) \) as the vector \( ai \).

\( stmcb \) returns the IIR filter coefficients in length \( nb + 1 \) and \( na + 1 \) row vectors \( b \) and \( a \). The filter coefficients are ordered in descending powers of \( z \):

\[
H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(nb + 1)z^{-nb}}{a(1) + a(2)z^{-1} + \ldots + a(na + 1)z^{-na}}
\]
Example

Approximate the impulse response of a Butterworth filter with a system of lower order:

\[
\begin{align*}
[b, a] &= \text{butter}(6, 0.2); \\
h &= \text{filter}(b, a, [1 \text{ zeros}(1, 100)]); \\
\text{freqz}(b, a, 128)
\end{align*}
\]

\[
\begin{align*}
[bb, aa] &= \text{stmcb}(h, 4, 4); \\
\text{freqz}(bb, aa, 128)
\end{align*}
\]
stmcb attempts to minimize the squared error between the impulse response $x'$ of $b(z)/a(z)$ and the input signal $x$:

$$\min_{a,b} \sum_{i=0}^{\infty} |x(i) - x'(i)|^2$$

stmcb iterates using two steps:

1. Itprefilters $x$ and $u$ using $1/a(z)$.
2. It solves a system of linear equations for $b$ and $a$ using $\backslash$.

stmcb repeats this process $niter$ times. No checking is done to see if the $b$ and $a$ coefficients have converged in fewer than $niter$ iterations.

Diagnostics

If $x$ and $u$ have different lengths, stmcb gives the following error message:

$X$ and $U$ must have same length.

See Also

levinson  
Levinson-Durbin recursion.

lpc  
Linear prediction coefficients.

aryule  
Compute an estimate of AR model parameters using the Yule-Walker method.

prony  
Prony's method for time domain IIR filter design.

References


Purpose
Strip plot.

Syntax
strips(x)
strips(x,n)
strips(x,sd,Fs)
strips(x,sd,Fs,scale)

Description
strips(x) plots vector x in horizontal strips of length 250. If x is a matrix, strips(x) plots each column of x. The left-most column (column 1) is the top horizontal strip.

strips(x,n) plots vector x in strips that are each n samples long.

strips(x,sd,Fs) plots vector x in strips of duration sd seconds, given a sampling frequency of Fs samples per second.

strips(x,sd,Fs,scale) scales the vertical axes.

If x is a matrix, strips(x,n), strips(x,sd,Fs), and strips(x,sd,Fs,scale) plot the different columns of x on the same strip plot.

strips ignores the imaginary part of x if it is complex.
Example

Plot two seconds of a frequency modulated sinusoid in 0.25 second strips:

```matlab
Fs = 1000; % sampling frequency
t = 0:1/Fs:2; % time vector
x = vco(sin(2*pi*t),[10 490],Fs); % FM waveform
strips(x,0.25,Fs)
```

See Also

- `plot` Linear two-dimensional plot (see the online MATLAB Function Reference).
- `stem` Plot discrete sequence data (see the online MATLAB Function Reference).
tf2latc

Purpose
Conversion of transfer function to lattice filter.

Syntax
[k,v] = tf2latc(b,a)
k = tf2latc(1,a)
[k,v] = tf2latc(1,a)
k = tf2latc(b)

Description
[k,v] = tf2latc(b,a) finds the lattice parameters k and the ladder parameters v for an IIR (ARMA) lattice-ladder filter, normalized by a(1). Note that an error is generated if one or more of the lattice parameters are exactly equal to 1.

k = tf2latc(1,a) finds the lattice parameters k for an IIR all-pole (AR) lattice filter.

[k,v] = tf2latc(1,a) returns the scalar ladder coefficient at the correct position in vector v. All other elements of v are zero.

k = tf2latc(b) finds the lattice parameters k for an FIR (MA) lattice filter, normalized by b(1).

See Also
latc2tf Lattice filter to transfer function conversion.
latcfilt Lattice and lattice-ladder filter implementation.
tf2sos Conversion of transfer function to second-order sections.
tf2ss Conversion of transfer function to state-space.
tf2zp Conversion of transfer function to zero-pole-gain.
Purpose

Conversion of transfer function to second-order sections.

Syntax

\[ [\text{sos},g] = \text{tf2sos}(b,a) \]
\[ [\text{sos},g] = \text{tf2sos}(b,a,'order') \]
\[ [\text{sos},g] = \text{tf2sos}(b,a,'order','scale') \]
\[ \text{sos} = \text{tf2sos}(...) \]

Description

\text{tf2sos} converts a transfer function representation of a given system to an equivalent second-order section representation.

\[ [\text{sos},g] = \text{tf2sos}(b,a) \] finds a matrix \text{sos} in second-order section form with gain \( g \) that is equivalent to the system represented by transfer function coefficient vectors \( a \) and \( b \).

\[ H(z) = \frac{B(z)}{A(z)} = \frac{b_1 + b_2 z^{-1} + \cdots + b_{nb+1} z^{-nb}}{a_1 + a_2 z^{-1} + \cdots + a_{na+1} z^{-na}} \]

\text{sos} is an \( L \)-by-6 matrix

\[
\text{sos} = 
\begin{bmatrix}
    b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
    b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L} \\
\end{bmatrix}
\]

whose rows contain the numerator and denominator coefficients \( b_k \) and \( a_k \) of the second-order sections of \( H(z) \):

\[ H(z) = g \prod_{k=1}^{L} H_k(z) = g \prod_{k=1}^{L} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}} \]

\[ [\text{sos},g] = \text{tf2sos}(b,a,'order') \] specifies the order of the rows in \text{sos}, where \textit{order} is:
tf2sos

- **down**, to order the sections so the first row of sos contains the poles closest to the unit circle
- **up**, to order the sections so the first row of sos contains the poles farthest from the unit circle (default)

\[ [\text{sos}, g] = \text{tf2sos}(b, a, 'order', 'scale') \] specifies the desired scaling of the gain and numerator coefficients of all second-order sections, where \text{scale} is:

- **none**, to apply no scaling (default)
- **inf**, to apply infinity-norm scaling
- **two**, to apply 2-norm scaling

Using infinity-norm scaling in conjunction with up-ordering minimizes the probability of overflow in the realization. Using 2-norm scaling in conjunction with down-ordering minimizes the peak round-off noise.

\[ \text{sos} = \text{tf2sos}(...) \] embeds the overall system gain, \( g \), in the first section, \( H_1(z) \), so that

\[
H(z) = \prod_{k=1}^{L} H_k(z)
\]

**See Also**

- \text{cplxpair} Group complex numbers into complex conjugate pairs.
- \text{sos2tf} Conversion of second-order sections to transfer function.
- \text{ss2sos} Conversion of state-space to second-order sections.
- \text{tf2ss} Conversion of transfer function to state-space.
- \text{tf2zp} Conversion of transfer function to zero-pole-gain.
- \text{zp2sos} Conversion of zero-pole-gain to second-order sections.
References


tf2ss

**Purpose**
Conversion of transfer function to state-space.

**Syntax**

\[ [A, B, C, D] = \text{tf2ss}(b, a) \]

**Description**

The `tf2ss` function converts a transfer function representation of a given system to an equivalent state-space representation.

\[ [A, B, C, D] = \text{tf2ss}(b, a) \]

finds a state-space representation:

\[
\begin{align*}
    x &= Ax + Bu \\
    y &= Cx + Du
\end{align*}
\]

given a system in transfer function form:

\[
H(s) = \frac{\text{B}(s)}{\text{A}(s)} = C(sI - A)^{-1}B + D
\]

from a single input. Input vector `a` contains the denominator coefficients in descending powers of `s`. Matrix `b` contains the numerator coefficients with as many rows as there are outputs `y`. `tf2ss` returns the `A`, `B`, `C`, and `D` matrices in controller canonical form.

`tf2ss` also works for discrete systems, but you must pad the numerator with trailing zeros to make it the same length as the denominator.

The `tf2ss` function is part of the standard MATLAB language.

**Example**

Consider the system

\[
H(s) = \frac{2s + 3}{s^2 + 2s + 1} - \frac{1}{s^2 + 0.4s + 1}
\]
To convert this system to state-space:

```matlab
b = [0 2 3; 1 2 1];
a = [1 0.4 1];
[A,B,C,D] = tf2ss(b,a)
```

\[
A = \begin{bmatrix}
-0.4000 & -1.0000 \\
1.0000 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 2.0000 & 3.0000 \\
1.6000 & 0 \end{bmatrix}
\]

\[
D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

There is disagreement in the literature on naming conventions for the canonical forms. It is easy, however, to generate similarity transformations that convert to other forms. For example:

```matlab
T = fliplr(eye(n));
A = T\A*T;
```

**Algorithm**

`tf2ss` writes the output in controller canonical form by inspection.

**See Also**

- `sos2ss`: Conversion of second-order sections to state-space.
- `ss2tf`: Conversion of state-space to transfer function.
- `tf2sos`: Conversion of transfer function to second-order sections.
- `tf2zp`: Conversion of transfer function to zero-pole-gain.
- `zp2ss`: Conversion of zero-pole-gain to state-space.
tf2zp

**Purpose**  Conversion of transfer function to zero-pole-gain.

**Syntax**  
\[ [z, p, k] = \text{tf2zp}(b, a) \]

**Description**  
\text{tf2zp} finds the zeros, poles, and gains of a system in polynomial transfer function form.

\[ [z, p, k] = \text{tf2zp}(b, a) \]
finds the single-input, multi-output (SIMO) factored transfer function form:

\[
H(s) = \frac{Z(s)}{P(s)} = k \frac{(s - Z_1)(s - Z_2) \cdots (s - Z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}
\]

given a SIMO system in polynomial transfer function form:

\[
\frac{B(s)}{A(s)} = \frac{b_1 s^{(n_b - 1)} + \cdots + b_{(n_b - 1)} s + b_{n_b}}{a_1 s^{(n_a - 1)} + \cdots + a_{(n_a - 1)} s + a_{n_a}}
\]

Vector \(a\) specifies the coefficients of the denominator in descending powers of \(s\). Matrix \(b\) indicates the numerator coefficients with as many rows as there are outputs. The zero locations are returned in the columns of matrix \(z\), with as many columns as there are rows in \(b\). The pole locations are returned in column vector \(p\) and the gains for each numerator transfer function in vector \(k\).

The \text{tf2zp} function also works for discrete systems, and is part of the standard MATLAB language.
Example

Find the zeros, poles, and gains of the system

\[ H(s) = \frac{2s + 3}{s^2 + 0.4s + 1} \]

\[ b = [2 \ 3]; \]
\[ a = [1 \ 0.4 \ 1]; \]
\[ [z,p,k] = \text{tf2zp}(b,a) \]

\[ z = -1.5000 \]

\[ p = \begin{array}{c}
-0.2000 + 0.9798i \\
-0.2000 - 0.9798i 
\end{array} \]

\[ k = 2 \]

Algorithm

The system is converted to state-space using \text{tf2ss} and then to zeros, poles, and gains using \text{ss2zp}.

See Also

- \text{sos2zp} Conversion of second-order sections to zero-pole-gain.
- \text{ss2zp} Conversion of state-space to zero-pole-gain.
- \text{tf2sos} Conversion of transfer function to second-order sections.
- \text{tf2ss} Conversion of transfer function to state-space.
- \text{zp2tf} Conversion of zero-pole-gain to transfer function.
**Purpose**

Transfer function estimate from input and output.

**Syntax**

\[ T_{xy} = \text{tfe}(x,y) \]

\[ T_{xy} = \text{tfe}(x,y,nfft) \]

\[ [T_{xy},f] = \text{tfe}(x,y,nfft,Fs) \]

\[ T_{xy} = \text{tfe}(x,y,nfft,Fs,\text{window}) \]

\[ T_{xy} = \text{tfe}(x,y,nfft,Fs,\text{window},\text{noverlap}) \]

\[ T_{xy} = \text{tfe}(x,y,...,'dflag') \]

\[ \text{tfe}(x,y) \]

**Description**

\[ T_{xy} = \text{tfe}(x,y) \] finds a transfer function estimate \( T_{xy} \) given input signal vector \( x \) and output signal vector \( y \). The transfer function is the quotient of the cross spectrum of \( x \) and \( y \) and the power spectrum of \( x \):

\[ T_{xy}(f) = \frac{P_{xy}(f)}{P_{xx}(f)} \]

The relationship between the input \( x \) and output \( y \) is modeled by the linear, time-invariant transfer function \( T_{xy} \).

Vectors \( x \) and \( y \) must be the same length. \( T_{xy} = \text{tfe}(x,y) \) uses the following default values:

- \( nfft = \min(256, \text{length}(x)) \)
- \( Fs = 2 \)
- \( \text{window} = \text{hanning}(nfft) \)
- \( \text{noverlap} = 0 \)

\( nfft \) specifies the FFT length that \text{tfe} uses. This value determines the frequencies at which the power spectrum is estimated. \( Fs \) is a scalar that specifies the sampling frequency. \( \text{window} \) specifies a windowing function and the number of samples \text{tfe} uses in its sectioning of the \( x \) and \( y \) vectors. \( \text{noverlap} \) is the number of samples by which the sections overlap. Any arguments that are omitted from the end of the parameter list use the default values shown above.

If \( x \) is real, \text{tfe} estimates the transfer function at positive frequencies only; in this case, the output \( T_{xy} \) is a column vector of length \( nfft/2+1 \) for \( nfft \) even and \( (nfft+1)/2 \) for \( nfft \) odd. If \( x \) or \( y \) is complex, \text{tfe} estimates the transfer function for both positive and negative frequencies and \( T_{xy} \) has length \( nfft \).
Txy = tfe(x,y,nfft) uses the specified FFT length nfft in estimating the transfer function. Specify nfft as a power of 2 for fastest execution.

[Txy,f] = tfe(x,y,nfft,Fs) returns a vector f of frequencies at which tfe estimates the transfer function. Fs is the sampling frequency. f is the same size as Txy, so plot(f,Txy) plots the transfer function estimate versus properly scaled frequency. Fs has no effect on the output Txy; it is a frequency scaling multiplier.

Txy = tfe(x,y,nfft,Fs,window) specifies a windowing function and the number of samples per section of the x vector. If you supply a scalar for window, Txy uses a Hanning window of that length. The length of the window must be less than or equal to nfft; tfe zero pads the sections if the length of the window exceeds nfft.

Txy = tfe(x,y,nfft,Fs,window,noverlap) overlaps the sections of x by noverlap samples.

You can use the empty matrix [] to specify the default value for any input argument except x or y. For example,

Txy = tfe(x,y,[],[],kaiser(128,5))

uses 256 as the value for nfft and 2 as the value for Fs.

Txy = tfe(x,y,...,'dflag') specifies a detrend option, where dflag is:

• linear, to remove the best straight-line fit from the prewindowed sections of x and y
• mean, to remove the mean from the prewindowed sections of x and y
• none, for no detrending (default)

The dflag parameter must appear last in the list of input arguments. tfe recognizes a dflag string no matter how many intermediate arguments are omitted.

tfe(...) with no output arguments plots the magnitude of the transfer function estimate in decibels versus frequency in the current figure window.
Example

Compute and plot the transfer function estimate between two colored noise sequences \( x \) and \( y \):

\[
h = \text{fir1}(30,0.2,\text{boxcar}(31));
\]
\[
x = \text{randn}(16384,1);
\]
\[
y = \text{filter}(h,1,x);
\]
\[
\text{tfe}(x,y,1024,[],[],512)
\]
\[
\text{title('Transfer Function Estimate')}
\]

Algorithm

tfe uses a four-step algorithm:

1. It multiplies the detrended sections by \( \text{window} \).
2. It takes the length \( \text{nfft} \) FFT of each section.
3. It averages the squares of the spectra of the \( x \) sections to form \( \text{Pxx} \) and averages the products of the spectra of the \( x \) and \( y \) sections to form \( \text{Pxy} \).
4. It calculates \( \text{Txy} \):
   \[
   \text{Txy} = \frac{\text{Pxy}}{\text{Pxx}}
   \]
Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector (either row or column) input.
- Requires inputs X and Y to have the same length.

See Also

- **etfe**
  Compute empirical transfer function estimate and periodogram (see System Identification Toolbox User’s Guide).

- **cohere**
  Estimate magnitude squared coherence function between two signals.

- **csd**
  Estimate the cross spectral density (CSD) of two signals.

- **pwelch**
  Estimate the power spectral density (PSD) of a signal using Welch’s method.

- **spa**
  Perform spectral analysis for input-output data (see System Identification Toolbox User’s Guide).
triang

**Purpose**
Triangular window.

**Syntax**
\[ w = \text{triang}(n) \]

**Description**
\( \text{triang}(n) \) returns an \( n \)-point triangular window in the column vector \( w \). The coefficients of a triangular window are

For \( n \) odd:
\[
  w[k] = \begin{cases} 
    \frac{2k}{n+1}, & 1 \leq k \leq \frac{n+1}{2} \\
    \frac{2(n-k+1)}{n+1}, & \frac{n+1}{2} \leq k \leq n 
  \end{cases}
\]

For \( n \) even:
\[
  w[k] = \begin{cases} 
    \frac{2k-1}{n}, & 1 \leq k \leq \frac{n}{2} \\
    \frac{2(n-k+1)}{n}, & \frac{n}{2} + 1 \leq k \leq n 
  \end{cases}
\]

The triangular window is very similar to a Bartlett window. The Bartlett window always ends with zeros at samples 1 and \( n \), while the triangular window is nonzero at those points. For \( n \) odd, the center \( n-2 \) points of \( \text{triang}(n-2) \) are equivalent to \( \text{bartlett}(n) \).

**See Also**
- bartlett
- blackman
- boxcar
- chebwin
- hamming
- hanning
- kaiser

**References**
### Purpose
Sampled aperiodic triangle generator.

### Syntax
- `y = tripuls(T)`
- `y = tripuls(T,w)`
- `y = tripuls(T,w,s)`

### Description
- `y = tripuls(T)` returns a continuous, aperiodic, symmetric, unity-height triangular pulse at the times indicated in array `T`, centered about `T=0` and with a default width of 1.
- `y = tripuls(T,w)` generates a triangular pulse of width `w`.
- `y = tripuls(T,w,s)` generates a triangular pulse with skew `s`, where `-1<s<1`. When `s` is 0, a symmetric triangular pulse is generated.

### See Also
- chirp: Swept-frequency cosine generator.
- cos: Cosine of vector/matrix elements (see the online MATLAB Function Reference).
- diric: Dirichlet or periodic sinc function.
- gauspuls: Gaussian-modulated sinusoidal pulse generator.
- pulstran: Pulse train generator.
- rectpuls: Sampled aperiodic rectangle generator.
- sawtooth: Sawtooth or triangle wave generator.
- sin: Sine of vector/matrix elements (see the online MATLAB Function Reference).
- sinc: Sinc or sin(αt)/αt function.
- square: Square wave generator.
unwrap

**Purpose**
Unwrap phase angles.

**Syntax**
```
p = unwrap(p)
```

**Description**
```
p = unwrap(p) corrects the phase angles in vector p by adding multiples of 
±2π, where needed, to smooth the transitions across branch cuts. When p is a 
matrix, unwrap corrects the phase angles down each column. The phase must 
be in radians.
```

The `unwrap` function is part of the standard MATLAB language.

**Limitations**
`unwrap` tries to detect branch cut crossings, but it can be fooled by sparse, 
rapidly changing phase values.

**See Also**
- `abs` Absolute value (magnitude).
- `angle` Phase angle.
Purpose
Upsample, apply an FIR filter, and downsample.

Syntax
yout = upfirdn(xin,h)
yout = upfirdn(xin,h,p)
yout = upfirdn(xin,h,p,q)

Description
upfirdn performs a cascade of three operations:

1. Upsampling by \( p \) (zero inserting)
2. FIR filtering with the impulse response given in \( h \)
3. Downsampling by \( q \) (throwing away samples)

upfirdn has been implemented as a MEX-file for maximum speed, so only the outputs actually needed are computed. The FIR filter is usually a lowpass filter, which you must design using another function such as remez or fir1.

NOTE
The function resample performs an FIR design using firls, followed by rate changing implemented with upfirdn.

\[
yout = upfirdn(xin,h) \text{ returns the output signal } yout. \text{ If } yout \text{ is a row or column vector, then it represents one signal; if } yout \text{ is an array, then each column is a separate output. } xin \text{ is the input signal. If } xin \text{ is a row or column vector, then it represents one signal; if } xin \text{ is an array, then each column is filtered. } h \text{ is the impulse response of the FIR filter. If } h \text{ is a row or column vector, then it represents one filter; if } h \text{ is an array, then each column is a separate impulse response.}
\]

\[
yout = upfirdn(xin,h,p) \text{ specifies the upsampling factor } p. p \text{ is an integer with a default of 1.}
\]

\[
yout = upfirdn(xin,h,p,q) \text{ specifies the downsampling factor } q. q \text{ is an integer with a default of 1.}
\]
**NOTE** Since `upfirdn` performs convolution and rate changing, the `yout` signals have a different length than `xin`. The length of `y[n]` is approximately \( \frac{p}{q} \) times the length of `x[n]`.

**Remarks**

Usually the inputs `xin` and the filter `h` are vectors, in which case only one output signal is produced. However, when these arguments are arrays, each column is treated as a separate signal or filter. Valid combinations are:

1. `xin` is a vector and `h` is a vector.
   - There is one filter and one signal, so the function convolves `xin` with `h`. The output signal `yout` is a row vector if `xin` is a row; otherwise, it is a column vector.

2. `xin` is an array and `h` is a vector.
   - There is one filter and many signals, so the function convolves `h` with each column of `xin`. The resulting `yout` will be an array with the same number of columns as `xin`.

3. `xin` is a vector and `h` is an array.
   - There are many filters and one signal, so the function convolves each column of `h` with `xin`. The resulting `yout` will be an array with the same number of columns as `h`.

4. `xin` is an array and `h` is an array, both with the same number of columns.
   - There are many filters and many signals, so the function convolves corresponding columns of `xin` and `h`. The resulting `yout` is an array with the same number of columns as `xin` and `h`.

**Examples**

If both `p` and `q` are equal to 1 (that is, there is no rate changing), the result is ordinary convolution of two signals (equivalent to `conv`):

```matlab
yy = upfirdn(xx,hh);
```

This example implements a seven-channel filter bank by convolving seven different filters with one input signal, then downsamples by five:

```matlab
% Assume that hh is an L-by-7 array of filters.
yy = upfirdn(xx,hh,1,5);
```
Implement a rate change from 44.1 kHz (CD sampling rate) to 48 kHz (DAT rate), a ratio of 160/147. This requires a lowpass filter with cutoff frequency at $\omega_c = 2\pi/160$:

% Design lowpass filter with cutoff at 1/160th of Fs.
hh = fir1(300,2/160); % need a very long lowpass filter
yy = upfirdn(xx,hh,160,147);

In this example, the filter design and resampling are separate steps. Note that resample would do both steps as one.

**Algorithm**

`upfirdn` uses a polyphase interpolation structure. The number of multiply-add operations in the polyphase structure is approximately $(L_hL_x-pL_x)/q$ where $L_h$ and $L_x$ are the lengths of $h[n]$ and $x[n]$, respectively.

A more accurate flops count is computed in the program, but the actual count is still approximate. For long signals $x[n]$, the formula is quite often exact.

**Diagnostics**

There must be one output argument and at least two input arguments. If either of these conditions are violated, `upfirdn` gives the appropriate error message:

- **UPFIRDN needs at least two input arguments.**
- **UPFIRDN should have exactly one output argument.**

If the arrays are sparse, `upfirdn` gives the error message

- **H must be full numeric matrix.**

When the input signals are in the columns of a matrix and there are multiple filters also in the columns of a matrix, the number of signals and filters must be the same. If they are not, `upfirdn` gives the error message

- **X and H must have the same number of columns, if more than one.**

The arguments $p$ and $q$ must be integers. If they are not, `upfirdn` gives the error message

- **P and/or Q must be greater than zero**

If the arguments $p$ and $q$ are not relatively prime, `upfirdn` gives the warning message

- **WARNING (upfirdn) p & q have common factor**
### See Also

- **conv**: Convolution and polynomial multiplication.
- **decimate**: Decrease the sampling rate for a sequence (decimation).
- **filter**: Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
- **interp**: Increase sampling rate by an integer factor (interpolation).
- **intfilt**: Interpolation FIR filter design.
- **resample**: Change sampling rate by any rational factor.

### References


Purpose
Voltage controlled oscillator.

Syntax
\[ y = \text{vco}(x,F_c,F_s) \]
\[ y = \text{vco}(x,[F_{\text{min}} F_{\text{max}}],F_s) \]

Description
\[ y = \text{vco}(x,F_c,F_s) \] creates a signal that oscillates at a frequency determined by the real input vector or array \( x \) with sampling frequency \( F_s \). \( F_c \) is the carrier or reference frequency; when \( x \) is 0, \( y \) is an \( F_c \) Hz cosine with amplitude 1 sampled at \( F_s \) Hz. \( x \) ranges from -1 to 1, where -1 corresponds to a 0 frequency output, 0 to \( F_c \), and 1 to 2*\( F_c \). \( y \) is the same size as \( x \).

\[ y = \text{vco}(x,[F_{\text{min}} F_{\text{max}}],F_s) \] scales the frequency modulation range so that -1 and 1 values of \( x \) yield oscillations of \( F_{\text{min}} \) Hz and \( F_{\text{max}} \) Hz respectively. For best results, \( F_{\text{min}} \) and \( F_{\text{max}} \) should be in the range 0 to \( F_s/2 \).

By default, \( F_s \) is 1 and \( F_c \) is \( F_s/4 \).

If \( x \) is a matrix, \text{vco} produces a matrix whose columns oscillate according to the columns of \( x \).

Example
Generate two seconds of a signal sampled at 10,000 samples/second whose instantaneous frequency is a triangle function of time:

\[
\begin{align*}
F_s &= 10000; \\
t &= 0:1/F_s:2; \\
x &= \text{vco(sawtooth(2*pi*t,0.75),[.1 0.4]*F_s,F_s)};
\end{align*}
\]
**vco**

Plot the spectrogram of the generated signal.

```
specgram(x,512,Fs,kaiser(256,5),220)
```

**Algorithm**  
vco performs FM modulation using the `modulate` function.

**Diagnostics**  
If any values of \( x \) lie outside \([-1,1]\), vco gives the following error message:

\[ X \text{ outside of range } [-1,1]. \]

**See Also**  
demod  
Modulation for communications simulation.  
modulate  
Demodulation for communications simulation.
Purpose

Cross-correlation function estimate.

Syntax

c = xcorr(x,y)
c = xcorr(x)
c = xcorr(x,y,'option')
c = xcorr(x,'option')
c = xcorr(x,y,maxlags)
c = xcorr(x,maxlags)
c = xcorr(x,y,maxlags,'option')
c = xcorr(x,maxlags,'option')
[c,lags] = xcorr(...)

Description

xcorr estimates the cross-correlation sequence of a random process. Autocorrelation is handled as a special case.

The true cross-correlation sequence is

\[ \gamma_{xy}(m) = \mathbb{E}\{x_n y_{n+m}\} \]

where \( x_n \) and \( y_n \) are stationary random processes, \( -\infty < n < \infty \), and \( \mathbb{E}\{\} \) is the expected value operator. xcorr must estimate the sequence because, in practice, only a finite segment of the infinite-length random process is available.

\( c = xcorr(x,y) \) returns the cross-correlation sequence in a length 2N-1 vector, where \( x \) and \( y \) are length N vectors (N>1). If \( x \) and \( y \) are not the same length, the shorter vector is zero-padded to the length of the longer vector.

By default, xcorr computes raw correlations with no normalization:

\[
c_{xy}(m) = \begin{cases} 
\sum_{n=0}^{N-|m|-1} x_n y_{n+m} & m \geq 0 \\
 c_{yx}^*(-m) & m < 0 
\end{cases}
\]

The output vector \( c \) has elements given by \( c(m) = c_{xy}(m-N) \), \( m=1, \ldots, 2N-1 \).

In general, the correlation function requires normalization to produce an accurate estimate (see below).
\texttt{c = xcorr(x)} is the autocorrelation sequence for the vector \( x \). If \( x \) is an \( N \)-by-\( P \) matrix, \( c \) is a matrix with \( 2N-1 \) rows whose \( P^2 \) columns contain the cross-correlation sequences for all combinations of the columns of \( x \).

\( c = xcorr(x,y,\text{'option'}) \) specifies a normalization option for the cross-correlation, where \text{'option'} is:

- \text{biased}, for biased estimates of the cross-correlation function
  \[
  c_{xy,\text{biased}}(m) = \frac{1}{N} c_{xy}(m)
  \]
- \text{unbiased}, for unbiased estimates of the cross-correlation function
  \[
  c_{xy,\text{unbiased}}(m) = \frac{1}{N - |m|} c_{xy}(m)
  \]
- \text{coeff}, to normalize the sequence so the autocorrelations at zero lag are identically 1.0
- \text{none}, to use the raw, unscaled cross-correlations (default)

See reference [1] for more information on the properties of biased and unbiased correlation estimates.

\( c = xcorr(x,\text{'option'}) \) specifies one of the above normalization options for the autocorrelation.

\( c = xcorr(x,y,\text{maxlags}) \) returns the cross-correlation sequence over the lag range \([-\text{maxlags} : \text{maxlags}]\). Output \( c \) has length \( 2 \times \text{maxlags} + 1 \).

\( c = xcorr(x,\text{maxlags}) \) returns the autocorrelation sequence over the lag range \([-\text{maxlags} : \text{maxlags}]\). Output \( c \) has length \( 2 \times \text{maxlags} + 1 \). If \( x \) is an \( N \)-by-\( P \) matrix, \( c \) is a matrix with \( 2 \times \text{maxlags} + 1 \) rows whose \( P^2 \) columns contain the autocorrelation sequences for all combinations of the columns of \( x \).

\( c = xcorr(x,y,\text{maxlags},\text{'option'}) \) specifies both a maximum number of lags and a scaling option for the cross-correlation.

\( c = xcorr(x,\text{maxlags},\text{'option'}) \) specifies both a maximum number of lags and a scaling option for the autocorrelation.
[c, lags] = xcorr(...) returns a vector of the lag indices at which c was estimated, with the range \([-\text{maxlags}:\text{maxlags}]\). When maxlags is not specified, the range of lags is \([-N+1:N-1]\).

In all cases, the cross-correlation or autocorrelation computed by xcorr has the 0-th lag in the middle of the sequence, at element or row maxlags+1 (element or row N if maxlags is not specified).

**Examples**

The second output, lags, is useful for plotting the cross-correlation or autocorrelation. For example, the estimated autocorrelation of zero-mean Gaussian white noise \(c_{ww}(m)\) can be displayed for \(-10 \leq m \leq 10\) using

\[
\begin{align*}
\text{ww} &= \text{randn}(1000,1); \\
[c_{ww}, \text{lags}] &= \text{xcorr}(\text{ww}, 10, \text{'coeff'}); \\
\text{stem}(\text{lags}, c_{ww})
\end{align*}
\]

Swapping the \(x\) and \(y\) input arguments reverses (and conjugates) the output correlation sequence. For row vectors, the resulting sequences are reversed left to right; for column vectors, up and down. The following example illustrates this property (mat2str is used for a compact display of complex numbers).

\[
\begin{align*}
x &= [1, 2i, 3]; \\
y &= [4, 5, 6]; \\
[c1, \text{lags}] &= \text{xcorr}(x, y); \\
c1 &= \text{mat2str}(c1, 2), \text{ lags}
\end{align*}
\]

\[
c1 =
\begin{bmatrix}
12-i*8.9e-016 & 15-i*8 & 22-i*10 & 5-i*12 & 6+i*8.9e-016
\end{bmatrix}
\]

\[
\text{lags} =
\begin{bmatrix}
-2 & -1 & 0 & 1 & 2
\end{bmatrix}
\]

\[
c2 = \text{conj(} \text{flip}(\text{xcorr}(y, x)))
\]

\[
c2 =
\begin{bmatrix}
12-i*8.9e-016 & 15-i*8 & 22-i*10 & 5-i*12 & 6+i*8.9e-016
\end{bmatrix}
\]

For the case where input argument \(x\) is a matrix, the output columns are arranged so that extracting a row and rearranging it into a square array
produces the cross-correlation matrix corresponding to the lag of the chosen row. For example, the cross-correlation at zero lag can be retrieved by

```matlab
randn('seed',0)
X = randn(2,2);
[M,P] = size(X);
c = xcorr(X);
c0 = zeros(P); c0(:) = c(M,:)  % Extract zero-lag row
```

```
c0 =
    1.7500    0.3079
    0.3079    0.1293
```

You can calculate the matrix of correlation coefficients that the MATLAB function `corrcoef` generates by substituting

```matlab
c = xcov(X,'coef')
```

in the last example. The function `xcov` subtracts the mean and then calls `xcorr`.

Use `fftshift` to move the second half of the sequence starting at the zeroth lag to the front of the sequence. `fftshift` swaps the first and second halves of a sequence.

**Algorithm**

For more information on estimating covariance and correlation functions, see [1] and [2].

**Diagnostics**

There must be at least one vector input argument; otherwise, `xcorr` gives the following error message:

```
1st arg must be a vector or matrix.
```

The string 'option' must be the last argument; otherwise, `xcorr` gives the following error message:

```
Argument list not in correct order.
```

If the second argument was entered as a scalar, it is taken to be `maxlags` and no succeeding input can be a scalar. When the second argument is a vector, the first must also be a signal vector. The third argument, when present, must be
a scalar or a string. If they are not, \texttt{xcorr} gives the appropriate error message(s):

3rd arg is \texttt{maxlag}, 2nd arg cannot be scalar.
When \texttt{b} is a vector, \texttt{a} must be a vector.
Maxlag must be a scalar.

Normally the lengths of the vector inputs should be the same; if they are not, then the only allowable scaling option is 'none'. If it is not, \texttt{xcorr} gives the following error message:

OPTION must be 'none' for different length vectors \texttt{A} and \texttt{B}.

\textbf{See Also}

- \texttt{conv} Convolution and polynomial multiplication.
- \texttt{corrcoef} Correlation coefficient matrix.
- \texttt{cov} Covariance matrix.
- \texttt{xcorr2} Two-dimensional cross-correlation.
- \texttt{xcov} Cross-covariance function estimate (equal to mean-removed cross-correlation).

\textbf{References}


xcorr2

**Purpose**
Two-dimensional cross-correlation.

**Syntax**

```
C = xcorr2(A)
C = xcorr2(A,B)
```

**Description**

`C = xcorr2(A,B)` returns the cross-correlation of matrices `A` and `B` with no scaling. `xcorr2` is the two-dimensional version of `xcorr`. It has its maximum value when the two matrices are aligned so that they are shaped as similarly as possible.

`xcorr2(A)` is the autocorrelation matrix of input matrix `A`. It is identical to `xcorr2(A,A)`.

**See Also**

- `conv2` Two-dimensional convolution.
- `filter2` Two-dimensional digital filtering.
- `xcorr` Cross-correlation function estimate.
**Purpose**
Cross-covariance function estimate (equal to mean-removed cross-correlation).

**Syntax**

\[ v = \text{xcov}(x, y) \]
\[ v = \text{xcov}(x) \]
\[ v = \text{xcov}(x, 'option') \]
\[ [c, lags] = \text{xcov}(x, y, \text{maxlags}) \]
\[ [c, lags] = \text{xcov}(x, \text{maxlags}) \]
\[ [c, lags] = \text{xcov}(x, y, \text{maxlags}, 'option') \]

**Description**
\text{xcov} estimates the cross-covariance sequence of random processes. Autocovariance is handled as a special case.

The true cross-covariance sequence is the mean-removed cross-correlation sequence
\[ \phi_{xy}(m) = E \{ (x_n - m_x)(y_{n+m} - m_y)^* \} \]
where \( m_x \) and \( m_y \) are the mean values of the two stationary random processes, and \( E \{ \} \) is the expected value operator. \text{xcov} estimates the sequence because, in practice, access is available to only a finite segment of the infinite-length random process.

\( v = \text{xcov}(x, y) \) returns the cross-covariance sequence in a length 2N-1 vector, where \( x \) and \( y \) are length N vectors.

\( v = \text{xcov}(x) \) is the autocovariance sequence for the vector \( x \). Where \( x \) is an N-by-P array, \( v = \text{xcov}(X) \) returns a matrix with 2N-1 rows whose \( P^2 \) columns contain the cross-covariance sequences for all combinations of the columns of \( X \).

By default, \text{xcov} computes raw covariances with no normalization. For a length N vector:

\[ c_{xy}(m) = \begin{cases} \sum_{n=0}^{N-|m|-1} \left( x(n) - \frac{1}{N} \sum_{i=0}^{N-1} x_i \right) \left( y_{n+m} - \frac{1}{N} \sum_{j=0}^{N-1} y_j \right) & m \geq 0 \\ c_{yx}^*(-m) & m < 0 \end{cases} \]

The output vector \( c \) has elements given by \( c(m) = c_{xy}(m-N), m=1,...,2N-1. \)
The covariance function requires normalization to estimate the function properly.

\[ v = \text{xcov}(x, 'option') \] specifies a scaling option, where \( option \) is:

- **biased**, for biased estimates of the cross-covariance function
- **unbiased**, for unbiased estimates of the cross-covariance function
- **coeff**, to normalize the sequence so the auto-covariances at zero lag are identically 1.0
- **none**, to use the raw, unscaled cross-covariances (default)

See [1] for more information on the properties of biased and unbiased correlation and covariance estimates.

\[ [c, \text{lags}] = \text{xcov}(x, y, \text{maxlags}) \] where \( x \) and \( y \) are length \( m \) vectors, returns the cross-covariance sequence in a length \( 2 \times \text{maxlags} + 1 \) vector \( c \). \( \text{lags} \) is a vector of the lag indices where \( c \) was estimated, that is, \([-\text{maxlags} : \text{maxlags}]\).

\[ [c, \text{lags}] = \text{xcov}(x, \text{maxlags}) \] is the autocovariance sequence over the range of lags \([-\text{maxlags} : \text{maxlags}]\).

\[ [c, \text{lags}] = \text{xcov}(x, \text{maxlags}) \] where \( x \) is an \( m \)-by-\( p \) array, returns array \( c \) with \( 2 \times \text{maxlags} + 1 \) rows whose \( P \) columns contain the cross-covariance sequences for all combinations of the columns of \( x \).

\[ [c, \text{lags}] = \text{xcov}(x, y, \text{maxlags}, 'option') \] specifies a scaling option, where \( option \) is the last input argument.

In all cases, \text{xcov} gives an output such that the zeroth lag of the covariance vector is in the middle of the sequence, at element or row \( \text{maxlag}+1 \) or at \( m \).

**Examples**

The second output \( \text{lags} \) is useful when plotting. For example, the estimated autocovariance of uniform white noise \( c_{ww}(m) \) can be displayed for \(-10 \leq m \leq 10\) using

\[ w = \text{randn}(1000,1); \% \text{generate uniform noise with mean} = 1/2 \]
\[ [\text{cov}_w, \text{lags}] = \text{xcov}(w, 10, 'coeff'); \]
\[ \text{stem(lags, cov}_w) \]
Algorithms

**xcov** computes the mean of its inputs, subtracts the mean, and then calls **xcorr**. For more information on estimating covariance and correlation functions, see [1] and [2].

**Diagnostics**

**xcov** does not check for any errors other than the correct number of input arguments. Instead, it relies on the error checking in **xcorr**, which it calls.

**See Also**

- **conv**: Convolution and polynomial multiplication.
- **corrcoef**: Correlation coefficient matrix.
- **cov**: Covariance matrix.
- **xcorr**: Cross-correlation function estimate.
- **xcorr2**: Two-dimensional cross-correlation.

**References**


Purpose
Recursive digital filter design.

Syntax
[b,a] = yulewalk(n,f,m)

Description
yulewalk designs recursive IIR digital filters using a least-squares fit to a specified frequency response.

[b,a] = yulewalk(n,f,m) returns row vectors b and a containing the n+1 coefficients of the order n IIR filter whose frequency-magnitude characteristics approximately match those given in vectors f and m:

- f is a vector of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sample frequency (the Nyquist frequency). The first point of f must be 0 and the last point 1, with all intermediate points in increasing order. Duplicate frequency points are allowed, corresponding to steps in the frequency response.
- m is a vector containing the desired magnitude response at the points specified in f.
- f and m must be the same length.
- plot(f,m) displays the filter shape.

The output filter coefficients are ordered in descending powers of z,

\[ \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \ldots + b(n+1)z^{-n}}{a(1) + a(2)z^{-1} + \ldots + a(n+1)z^{-n}} \]

When specifying the frequency response, avoid excessively sharp transitions from passband to stopband. You may need to experiment with the slope of the transition region to get the best filter design.

Example
Design an 8th-order lowpass filter and overplot the desired frequency response with the actual frequency response:

\[
\begin{align*}
    f & = [0 \ 0.6 \ 0.6 \ 1]; \\
    m & = [1 \ 1 \ 0 \ 0];
\end{align*}
\]
Algorithm

*yulewalk* performs a least-squares fit in the time domain. It computes the denominator coefficients using modified Yule-Walker equations, with correlation coefficients computed by inverse Fourier transformation of the specified frequency response. To compute the numerator, *yulewalk* takes the following steps:

1. Computes a numerator polynomial corresponding to an additive decomposition of the power frequency response.
2. Evaluates the complete frequency response corresponding to the numerator and denominator polynomials.
3. Uses a spectral factorization technique to obtain the impulse response of the filter.
4. Obtains the numerator polynomial by a least-squares fit to this impulse response.
See Also

- butter: Butterworth analog and digital filter design.
- cheby1: Chebyshev type I filter design (passband ripple).
- cheby2: Chebyshev type II filter design (stopband ripple).
- ellip: Elliptic (Cauer) filter design.
- fir2: Window-based finite impulse response filter design - arbitrary response.
- firls: Least square linear-phase FIR filter design.
- maxflat: Generalized digital Butterworth filter design.
- remez: Parks-McClellan optimal FIR filter design.

References

**Purpose**  
Conversion of zero-pole-gain to second-order sections.

**Syntax**  

\[
[sos, g] = \text{zp2sos}(z, p, k) \\
[sos, g] = \text{zp2sos}(z, p, k, 'order') \\
[sos, g] = \text{zp2sos}(z, p, k, 'order', 'scale') \\
sos = \text{zp2sos}(...) 
\]

**Description**  
\text{zp2sos} converts a zero-pole-gain representation of a given system to an equivalent second-order section representation.

\[
[sos, g] = \text{zp2sos}(z, p, k) \text{ finds a matrix } sos \text{ in second-order section form with gain } g \text{ equivalent to the zero-pole-gain system represented by input arguments } z, p, \text{ and } k. \text{ Vectors } z \text{ and } p \text{ contain the zeros and poles of the system } H(z), \text{ not necessarily in any order:}
\]

\[
H(z) = k \frac{(z - z_1)(z - z_2)\cdots(z - z_n)}{(z - p_1)(z - p_2)\cdots(z - p_m)}
\]

where \(n\) and \(m\) are the lengths of \(z\) and \(p\), respectively, and \(k\) is a scalar gain. The zeros and poles must be real or complex conjugate pairs. \(sos\) is an \(L\)-by-6 matrix:

\[
sos = \begin{bmatrix}
b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\
b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
b_{0L} & b_{1L} & b_{2L} & 1 & a_{1L} & a_{2L} 
\end{bmatrix}
\]

whose rows contain the numerator and denominator coefficients \(b_{ik}\) and \(a_{ik}\) of the second-order sections of \(H(z)\):

\[
H(z) = g \prod_{k=1}^{L} H_k(z) = g \prod_{k=1}^{L} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}
\]

The number of rows \(L\) of matrix \(sos\) is the maximum of the ceiling of \(n/2\) and the ceiling of \(m/2\).
[sos,g] = zp2sos(z,p,k,'order') specifies the order of the rows in sos, where order is:

- **down**, to order the sections so the first row of sos contains the poles closest to the unit circle
- **up**, to order the sections so the first row of sos contains the poles farthest from the unit circle (default)

[sos,g] = zp2sos(z,p,k,'order','scale') specifies the desired scaling of the gain and the numerator coefficients of all second-order sections, where scale is:

- **none**, to apply no scaling (default)
- **inf**, to apply infinity-norm scaling
- **two**, to apply 2-norm scaling

Using infinity-norm scaling in conjunction with up-ordering minimizes the probability of overflow in the realization. Using 2-norm scaling in conjunction with down-ordering minimizes the peak round-off noise.

sos = zp2sos(...) embeds the overall system gain, g, in the first section, \( H_1(z) \), so that

\[
H(z) = \prod_{k=1}^{L} H_k(z)
\]

**Example**

Find a second-order section form of a Butterworth lowpass filter:

\[
[z,p,k] = butter(5,0.2);
sos = zp2sos(z,p,k);
\]

**Algorithm**

zp2sos uses a four-step algorithm to determine the second-order section representation for an input zero-pole-gain system:

1. It groups the zeros and poles into complex conjugate pairs using the cplxpair function.
2. It forms the second-order section by matching the pole and zero pairs according to the following rules:
Match the poles closest to the unit circle with the zeros closest to those poles.

b Match the poles next closest to the unit circle with the zeros closest to those poles.

c Continue until all of the poles and zeros are matched.

zp2sos groups real poles into sections with the real poles closest to them in absolute value. The same rule holds for real zeros.

3 It orders the sections according to the proximity of the pole pairs to the unit circle. zp2sos normally orders the sections with poles closest to the unit circle last in the cascade. You can tell zp2sos to order the sections in the reverse order by specifying the down flag.

4 zp2sos scales the sections by the norm specified in the 'scale' argument. For arbitrary $H(\omega)$, the scaling is defined by:

$$
\|H\|_p = \left[ \frac{1}{2\pi} \int_{0}^{2\pi} |H(\omega)|^p d\omega \right]^{\frac{1}{p}}
$$

where $p$ can be either $\infty$ or $2$. See the references for details on the scaling. This scaling is an attempt to minimize overflow or peak round-off noise in fixed point filter implementations.

**See Also**

cplxpair Group complex numbers into complex conjugate pairs.
sos2zp Conversion of second-order sections to zero-pole-gain.
ss2sos Conversion of state-space to second-order sections.
tf2sos Conversion of transfer function to second-order sections.
zp2ss Conversion of zero-pole-gain to state-space.
zp2tf Conversion of zero-pole-gain to transfer function.
References


Purpose
Conversion of zero-pole-gain to state-space.

Syntax
\[ [A,B,C,D] = \text{zp2ss}(z,p,k) \]

Description
zp2ss converts a zero-pole-gain representation of a given system to an equivalent state-space representation.

\[ [A,B,C,D] = \text{zp2ss}(z,p,k) \]

finds a single input, multiple output, state-space representation

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

given a system in factored transfer function form

\[
H(s) = \frac{Z(s)}{P(s)} = k \frac{(s - z_1)(s - z_2) \cdots (s - z_n)}{(s - p_1)(s - p_2) \cdots (s - p_n)}
\]

Column vector \( p \) specifies the pole locations, and matrix \( z \) the zero locations with as many columns as there are outputs. The gains for each numerator transfer function are in vector \( k \). The \( A, B, C, \) and \( D \) matrices are returned in controller canonical form.

\( \text{Inf} \) values may be used as place holders in \( z \) if some columns have fewer zeros than others.

Algorithm
zp2ss, for single-input systems, groups complex pairs together into two-by-two blocks down the diagonal of the \( A \) matrix. This requires the zeros and poles to be real or complex conjugate pairs.

See Also
sos2ss Conversion of second-order sections to state-space.
ss2zp Conversion of state-space to zero-pole-gain.
tf2ss Conversion of transfer function to state-space.
zp2sos Conversion of zero-pole-gain to second-order sections.
zp2tf Conversion of zero-pole-gain to transfer function.
Purpose

Conversion of zero-pole-gain to transfer function.

Syntax

\[ [b,a] = \text{zp2tf}(z,p,k) \]

Description

zp2tf forms transfer function polynomials from the zeros, poles, and gains of a system in factored form.

\[ [b,a] = \text{zp2tf}(z,p,k) \]

finds a rational transfer function:

\[
\frac{B(s)}{A(s)} = \frac{b_1 s^{(nb-1)} + \cdots + b_{(nb-1)} s + b_{nb}}{a_1 s^{(na-1)} + \cdots + a_{(na-1)} s + a_{na}}
\]

given a system in factored transfer function form:

\[
H(s) = \frac{Z(s)}{P(s)} = k \frac{(s-z_1)(s-z_2) \cdot (s-z_m)}{(s-p_1)(s-p_2) \cdot (s-p_n)}
\]

Column vector \( p \) specifies the pole locations, and matrix \( z \) the zero locations, with as many columns as there are outputs. The gains for each numerator transfer function are in vector \( k \). The zeros and poles must be real or come in complex conjugate pairs. The polynomial coefficients are returned in vectors: the denominator coefficients in row vector \( a \) and the numerator coefficients in matrix \( b \), with as many rows as there are columns of \( z \).

Inf values can be used as place holders in \( z \) if some columns have fewer zeros than others.

Algorithm

The system is converted to transfer function form using \text{poly} with \( p \) and the columns of \( z \).

See Also

- \text{sos2tf}  
  Conversion of second-order sections to transfer function.
- \text{ss2tf}  
  Conversion of state-space to transfer function.
- \text{tf2zp}  
  Conversion of transfer function to zero-pole-gain.
- \text{zp2sos}  
  Conversion of zero-pole-gain to second-order sections.
- \text{zp2ss}  
  Conversion of zero-pole-gain to state-space.
Purpose

Zero-pole plot.

Syntax

zplane(z,p)
zplane(b,a)
[hz,hp,ht] = zplane(z,p)

Description

This function displays the poles and zeros of discrete-time systems.

zplane(z,p) plots the zeros specified in column vector z and the poles specified in column vector p in the current figure window. The symbol ‘o’ represents a zero and the symbol ‘x’ represents a pole. The plot includes the unit circle for reference. If z and p are arrays, zplane plots the poles and zeros in the columns of z and p, respectively, in different colors.

You can override the automatic scaling of zplane using

axis([xmin xmax ymin ymax])
or
set(gca,'ylim',[ymin ymax])
or
set(gca,'xlim',[xmin xmax])

after calling zplane. This is useful in the case where one or a few of the zeros or poles have such a large magnitude that the others are grouped tightly around the origin and are hard to distinguish.

zplane(b,a) where b and a are row vectors, first uses roots to find the zeros and poles of the transfer function represented by numerator coefficients b and denominator coefficients a.

[hz,hp,ht] = zplane(z,p) returns vectors of handles to the zero lines, hz, and the pole lines, hp. ht is a vector of handles to the axes/unit circle line and to text objects, which are present when there are multiple zeros or poles. If there are no zeros or no poles, hz or hp is set to the empty matrix [].
Examples

Plot the poles and zeros of a 5th-order Butterworth lowpass digital filter with
cutoff frequency of 0.2:

```
[z,p,k] = butter(5,0.2);
zplane(z,p);
title('5th-Order Butterworth Lowpass Digital Filter');
```

To generate the same plot with a transfer function representation of the filter:

```
[b,a] = butter(5,0.2);  % transfer function
zplane(b,a)
```

See Also

freqz          Frequency response of digital filters.
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